

Estimating water level changes with SAR amplitude altimetry: the case study of lake Trasimeno, Italy

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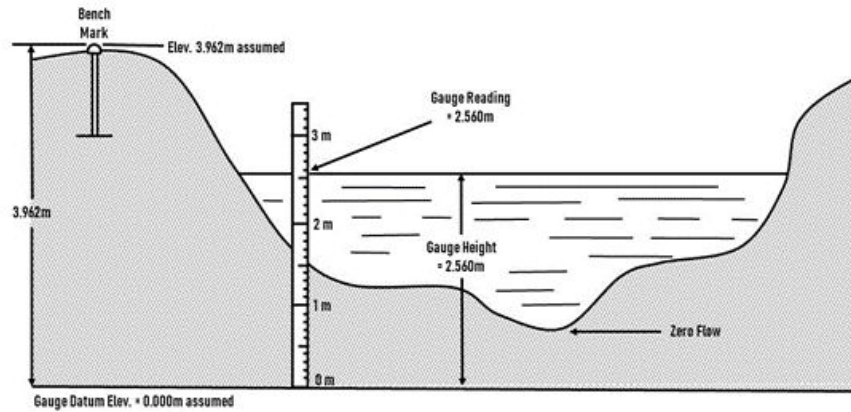


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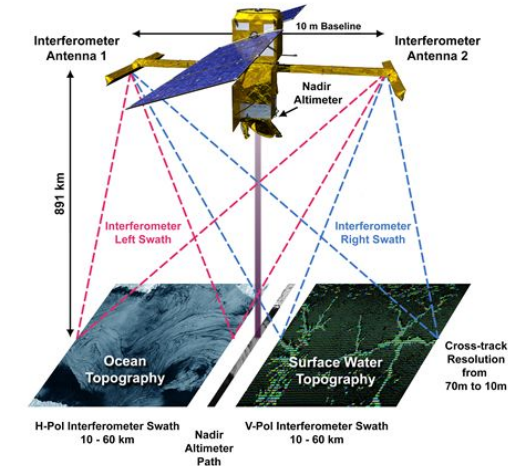


Water level monitoring

Traditional in-situ gauges



Remote sensing techniques



<https://swot.jpl.nasa.gov/resources/104/diagram-of-swot-data-collection/>



Key capability

High accuracy (millimeters)



Key capability

Accuracy of few decimeters, up to few centimeters
Large scale coverage
Height reference frames consistently declared



Main limitation

Limited coverage
High maintenance costs
Often unreported local height reference

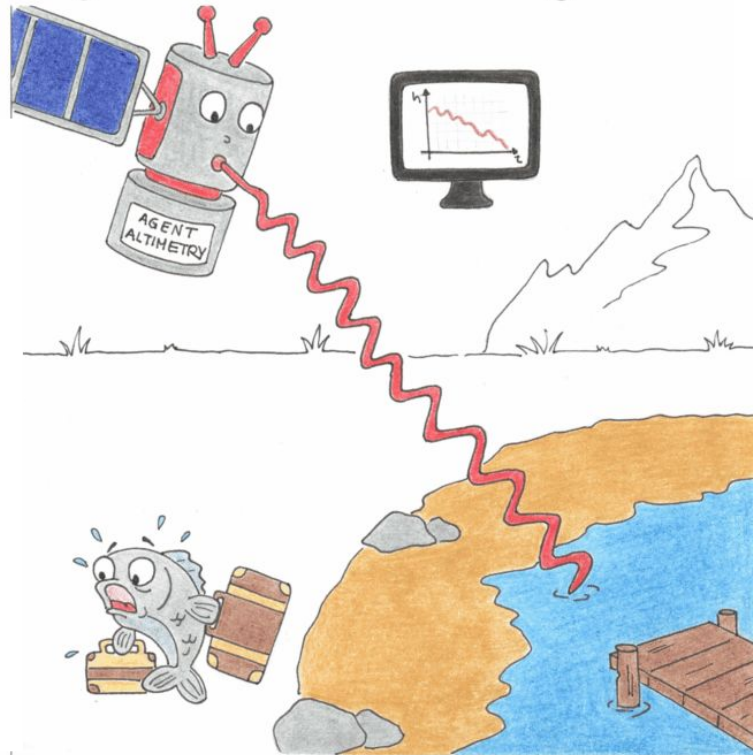


Main limitation

Variable spatial and temporal resolution

Earth Observation for water level monitoring: altimetry

Altimetry in Action: Tracking Water Levels



Satellite altimetry derives water surface height from pulse travel time. Repeated measurements create time series to monitor water level changes.



Key capability

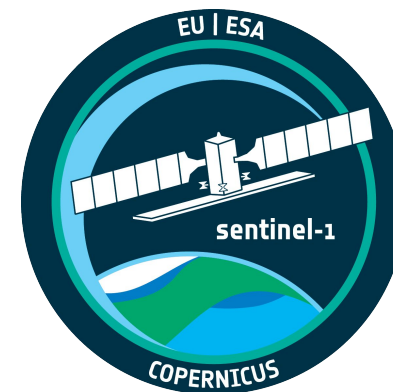
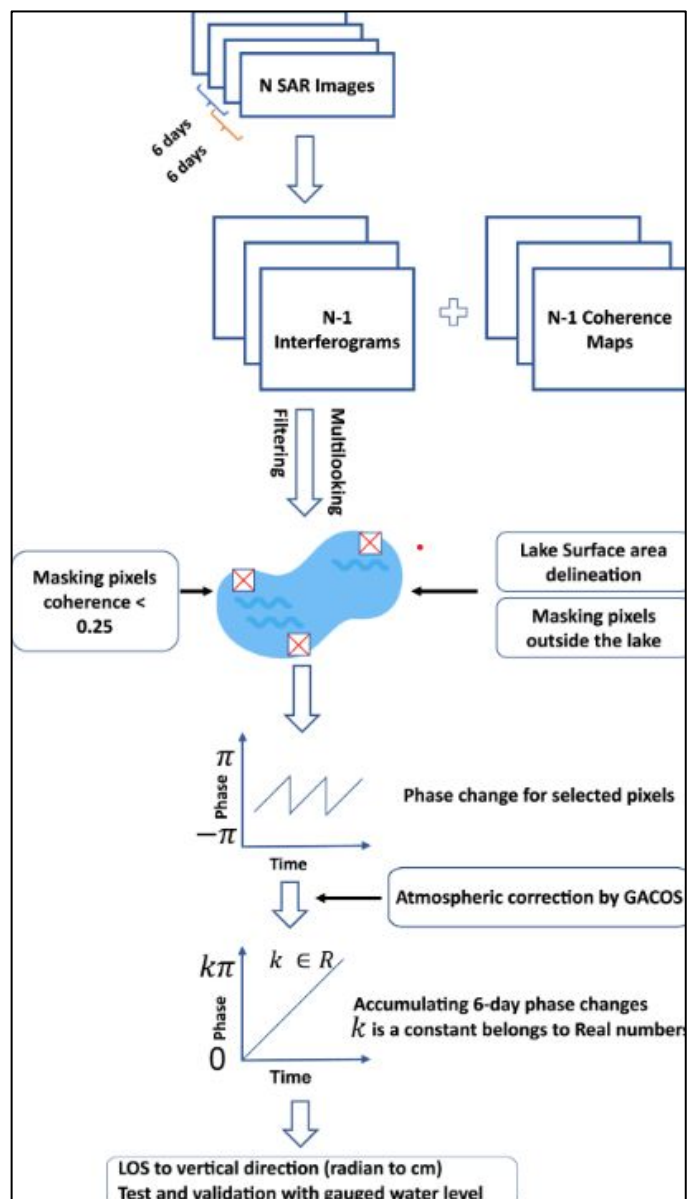
Water level estimation with accuracy of few decimeters, up to few centimeters



Main limitation

Variable spatial and temporal resolution

Earth Observation for water level monitoring: D-InSAR



Key capability

Water level estimation with accuracy below 30 centimeters

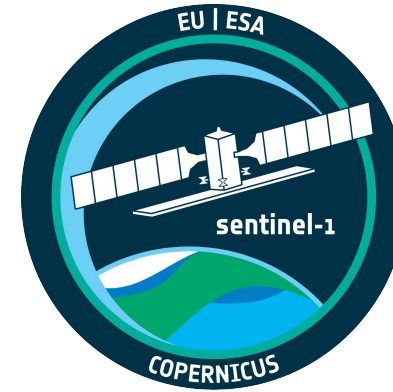


Main limitation

Phase unwrapping and decorrelation
Slow change of water level, less than 2 cm per 6-day period

Earth Observation for water level monitoring: SAR amplitude

What about **SAR**
amplitude data?



Key capability

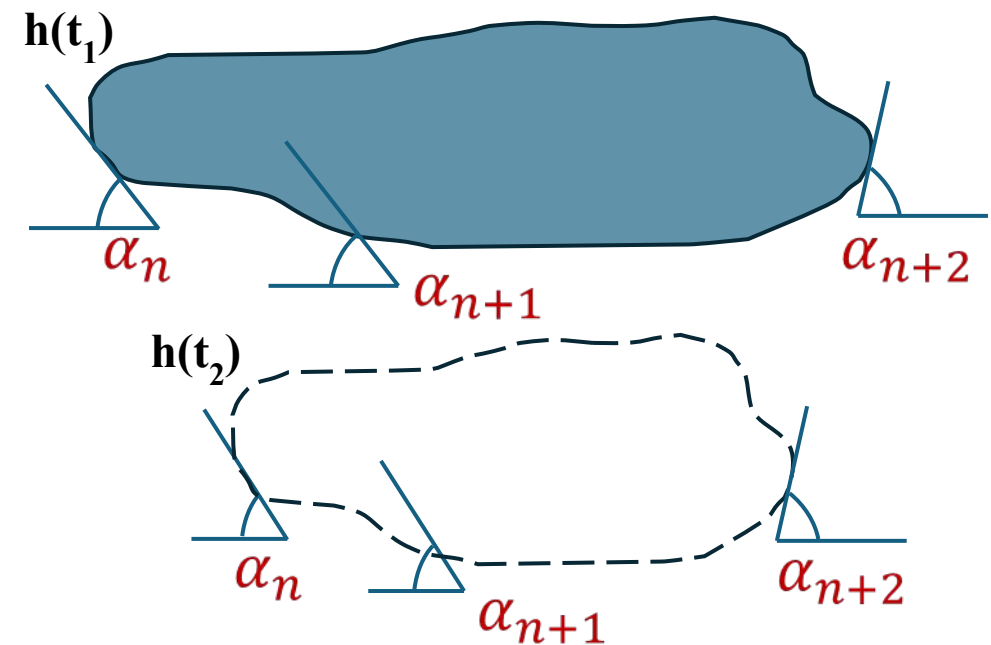
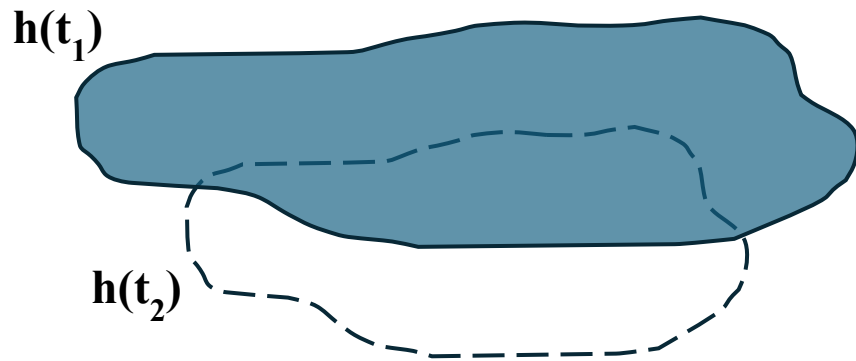
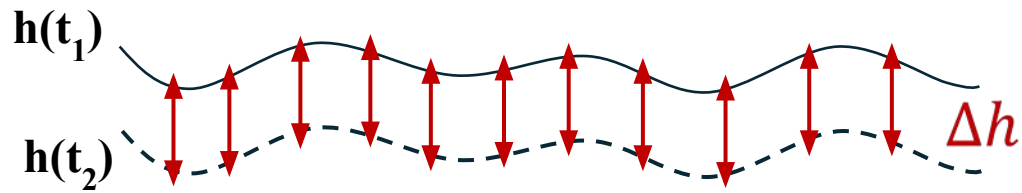


Water level estimation with a few centimeter accuracy
High spatial and temporal resolution
Not affected by phase unwrapping

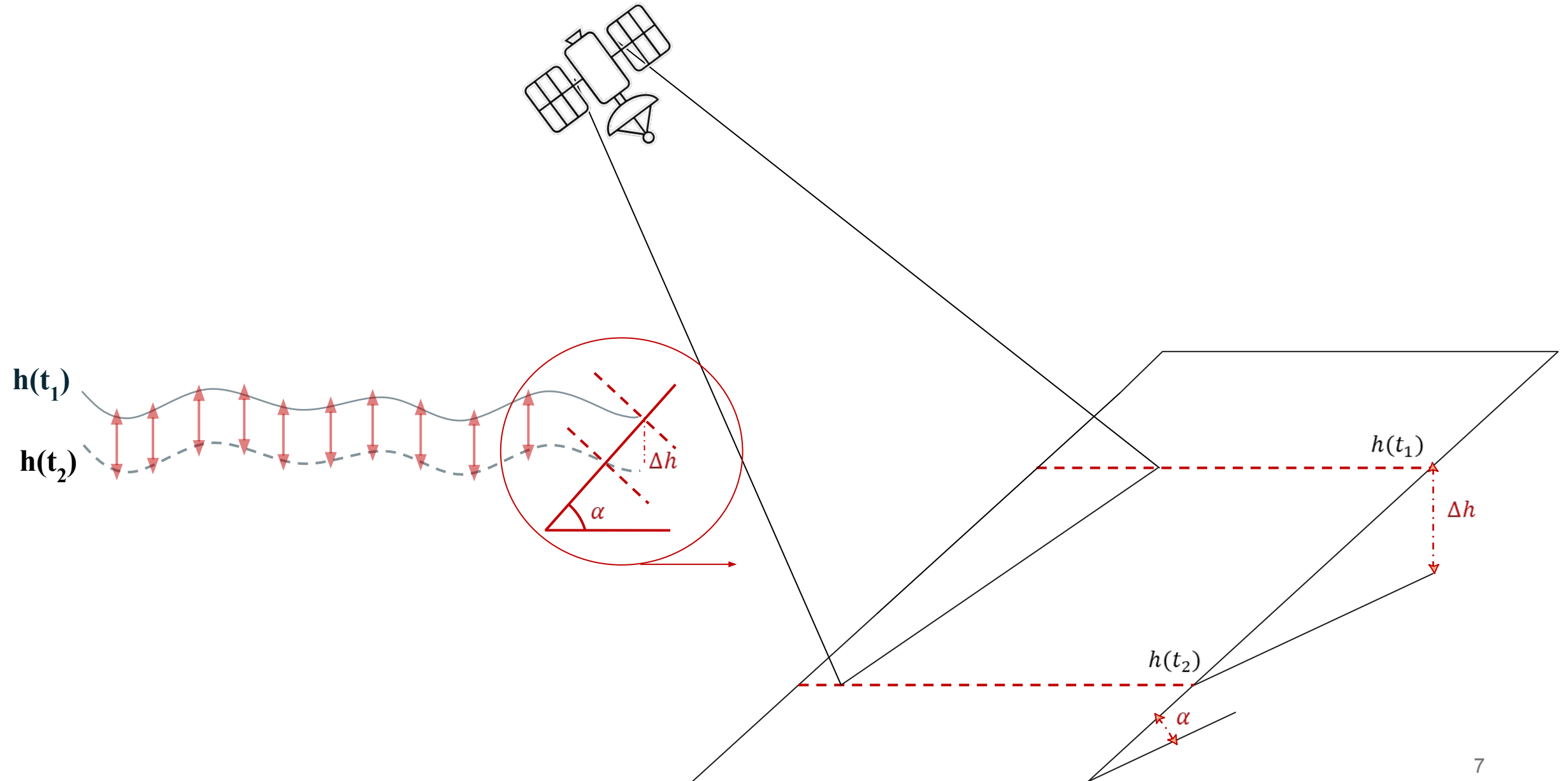
Methodology – Initial assumptions

Between two epochs t_1 and t_2 :

- **Constant water level changes** - the water level across the lake surface changes by a constant value of Δh 
- **Slope stability** – the slopes α_n of each shoreline remains constant if no significant morphological change occur



Methodology – Geometry of the model



Methodology – Geometry of the model

$$\Delta h_e = \Delta \sin(\alpha)$$

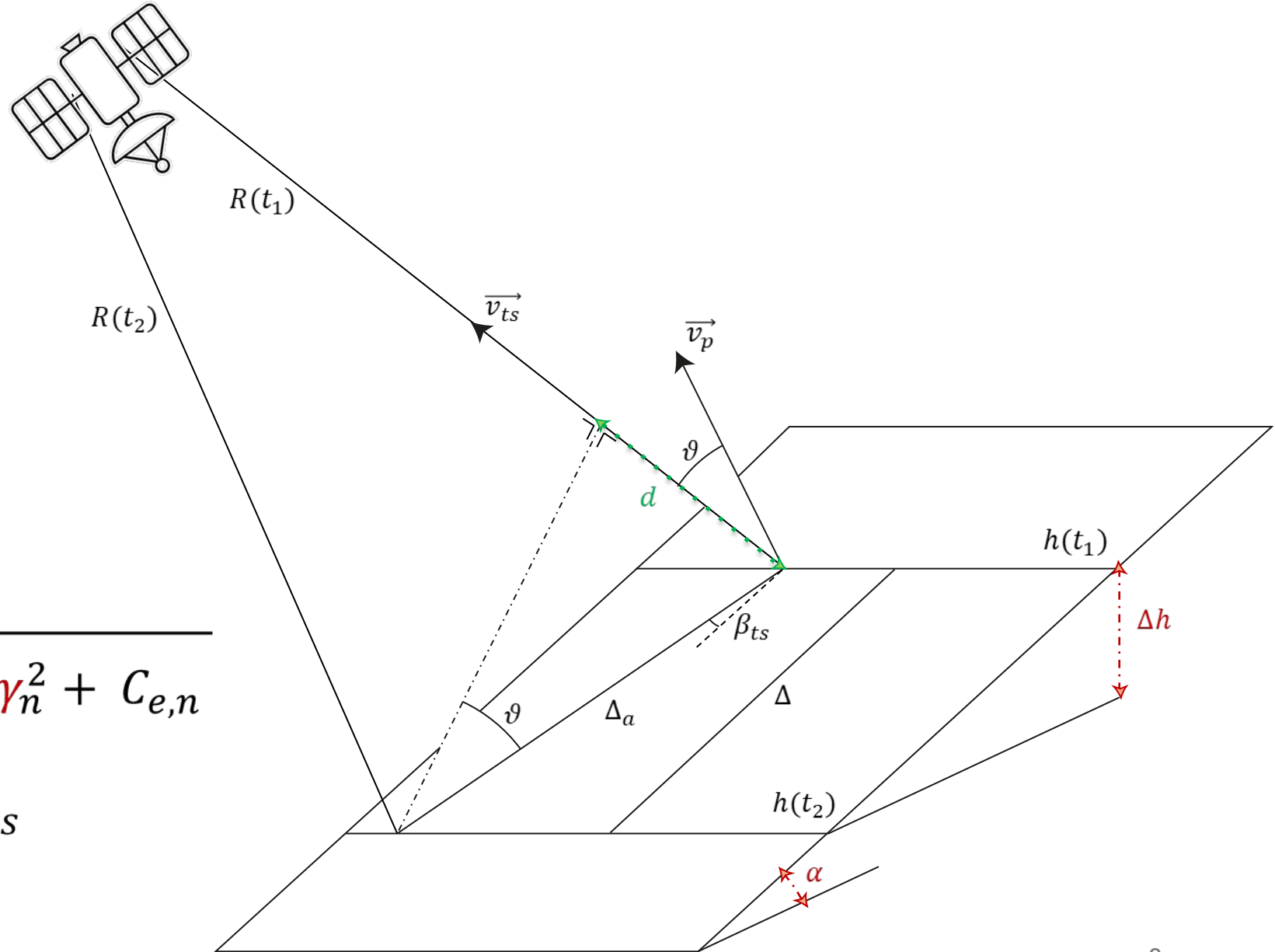
$$d_{e,n} = \frac{\Delta \sin(\vartheta)}{\cos(\beta_{st})}$$

$$\gamma = \frac{1}{\tan(\alpha)}$$

$$d_{e,n} = \frac{\Delta h_e}{\cos(\beta_{ts})} \sqrt{A_{e,n} \gamma_n^2 + B_{e,n} \gamma_n^2 + C_{e,n}}$$

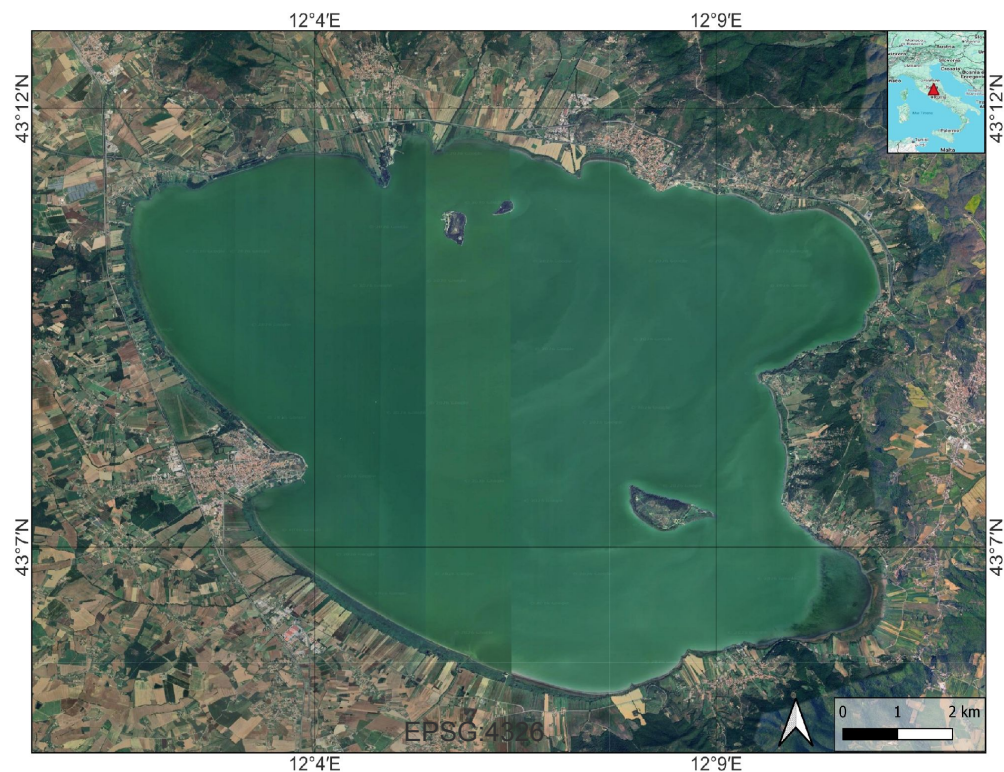
$n = \text{number of coastlines}$

$e = \text{number of epochs}$



Case study and data

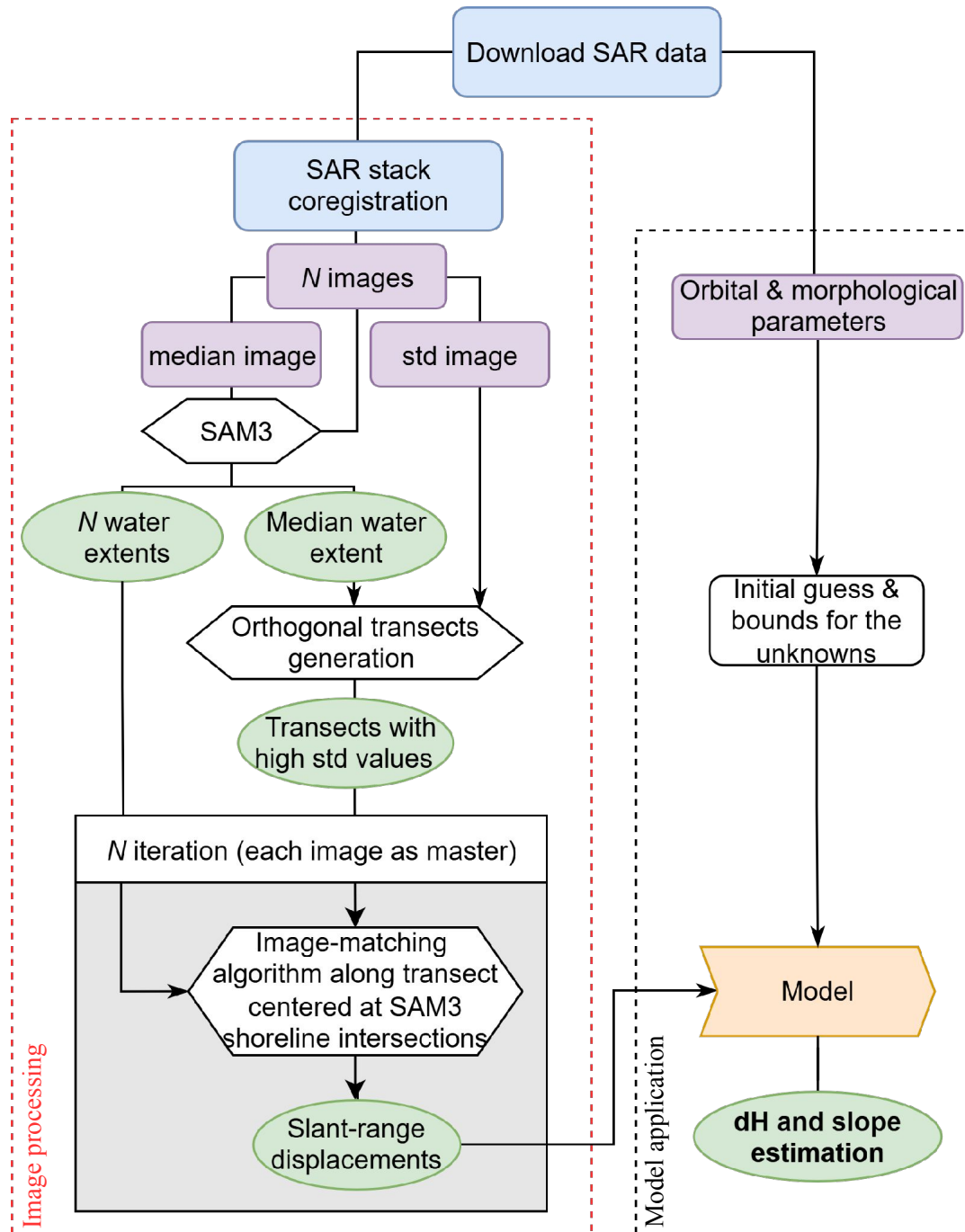
Trasimeno Lake (Umbria Region, Italy): shallow lake without natural outlets with area $\sim 128 \text{ km}^2$ and depth $\sim 4\text{-}6 \text{ m}$



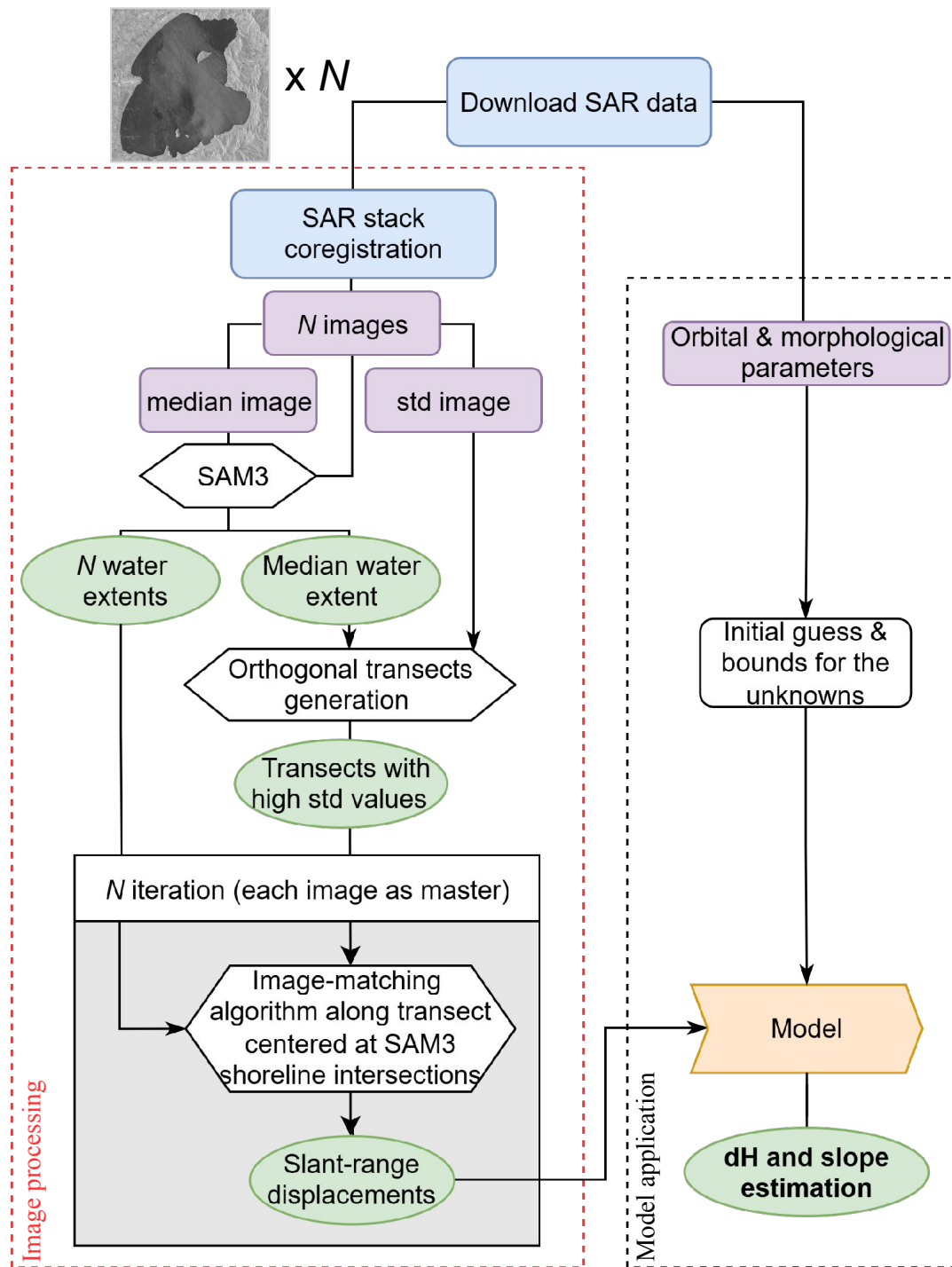
Sensor	Year	Orbit	Polarization	N. images
Sentinel-1	2022 2023 2024	ASC	VV+VH	30 - 29 - 29
		DSC	VV+VH	29 - 30 - 30
COSMO-SkyMed 2 generation	2022	ASC	HH	16

In-situ gauge measurements for 2022, 2023 and 2024 provided by the Water Resources Development and Protection Unit of Umbria Region

Workflow



Workflow

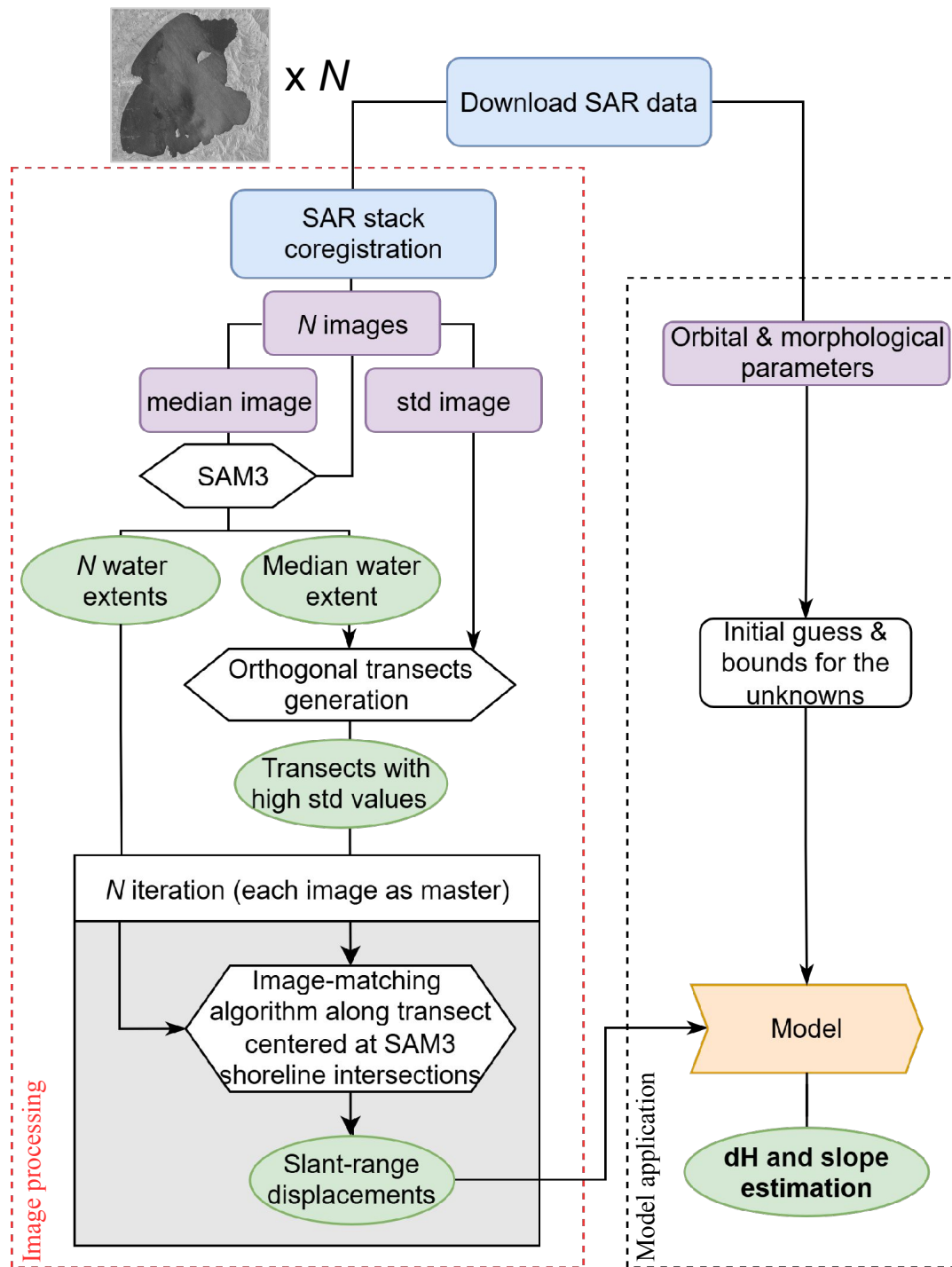
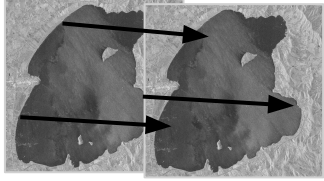


Images download and coregistration:

- MapItaly



Workflow

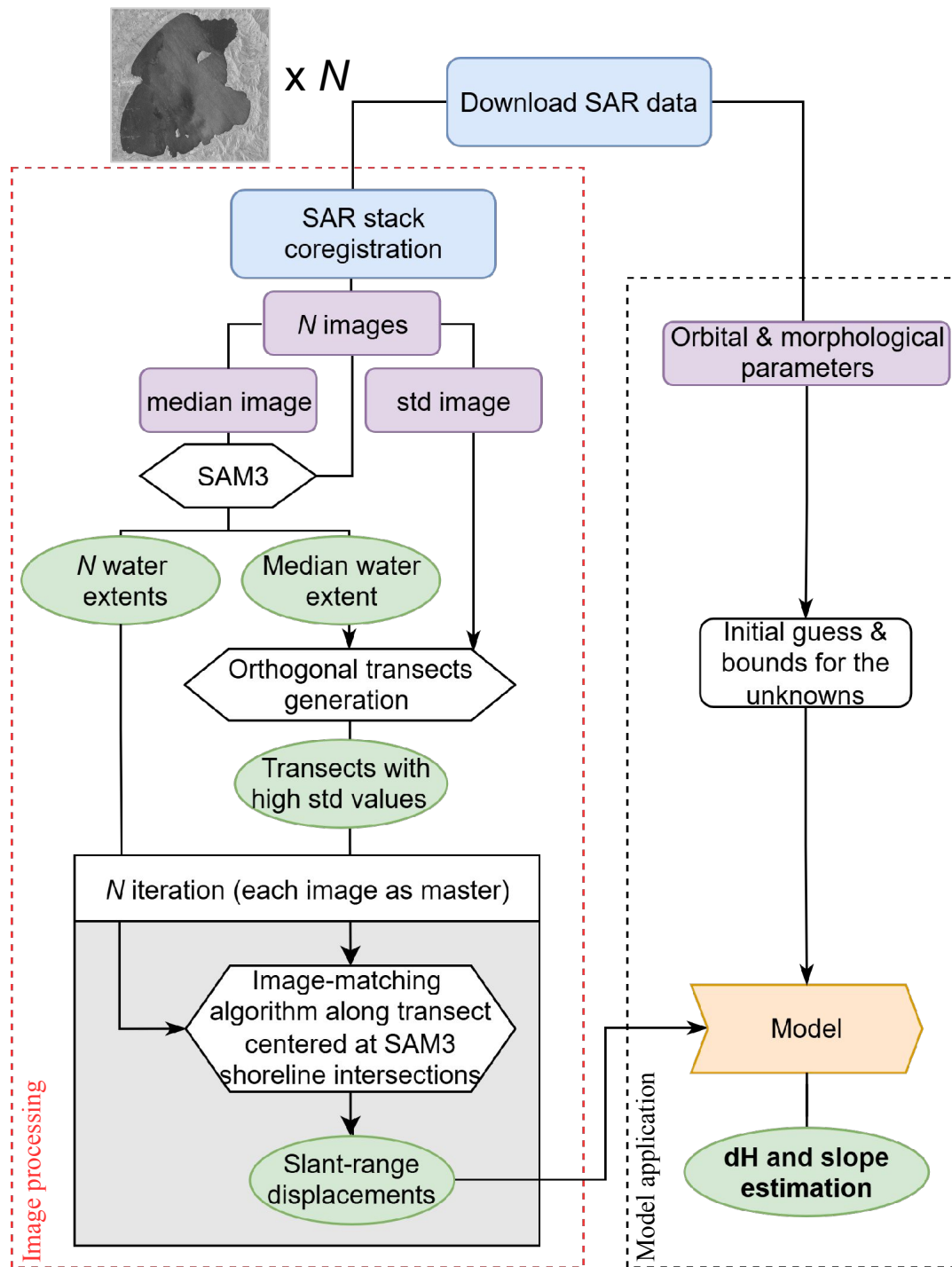
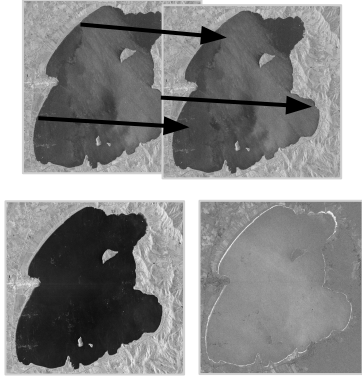


Images download and coregistration:

- MapItaly
- PyGMTSAR



Workflow

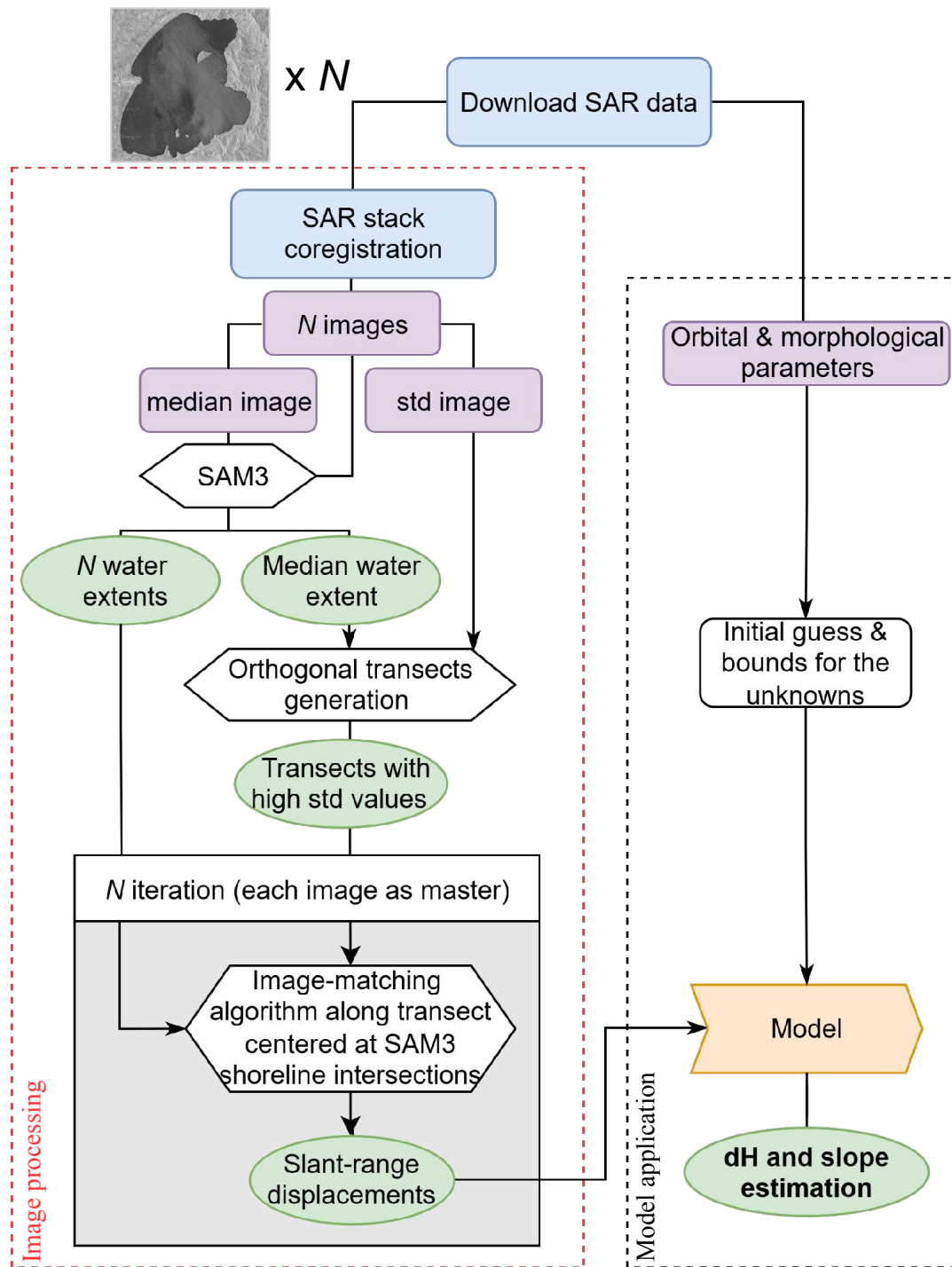
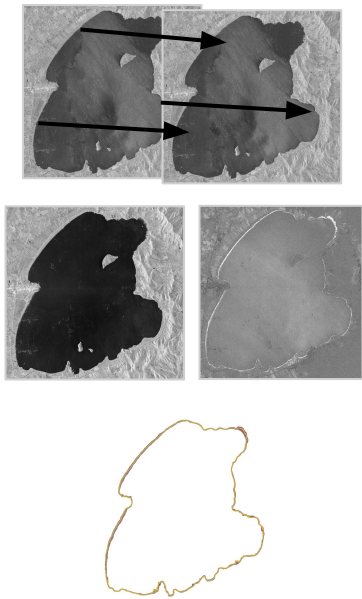


Images download and coregistration:

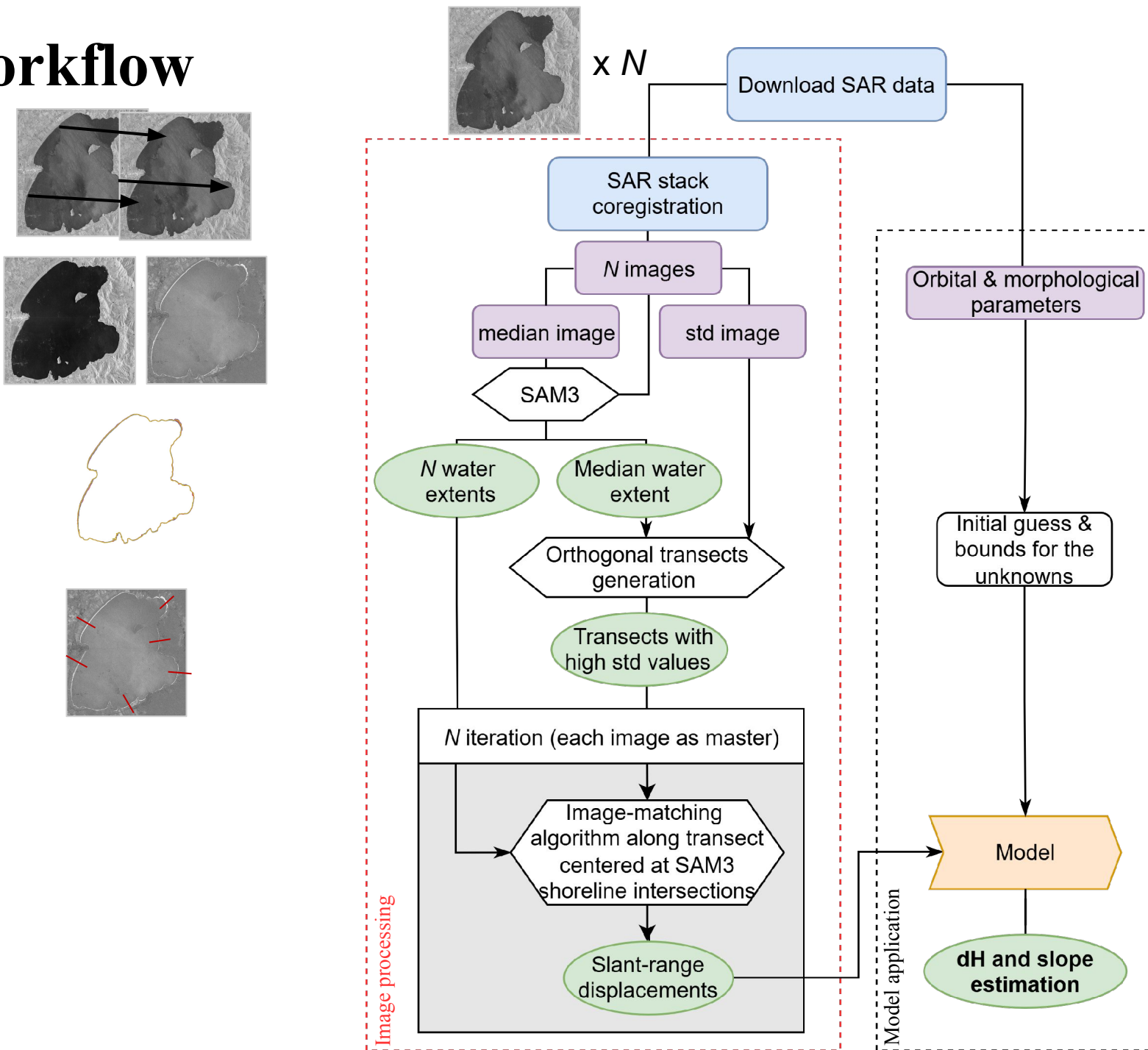
- MapItaly
- PyGMTSAR
- SNAP



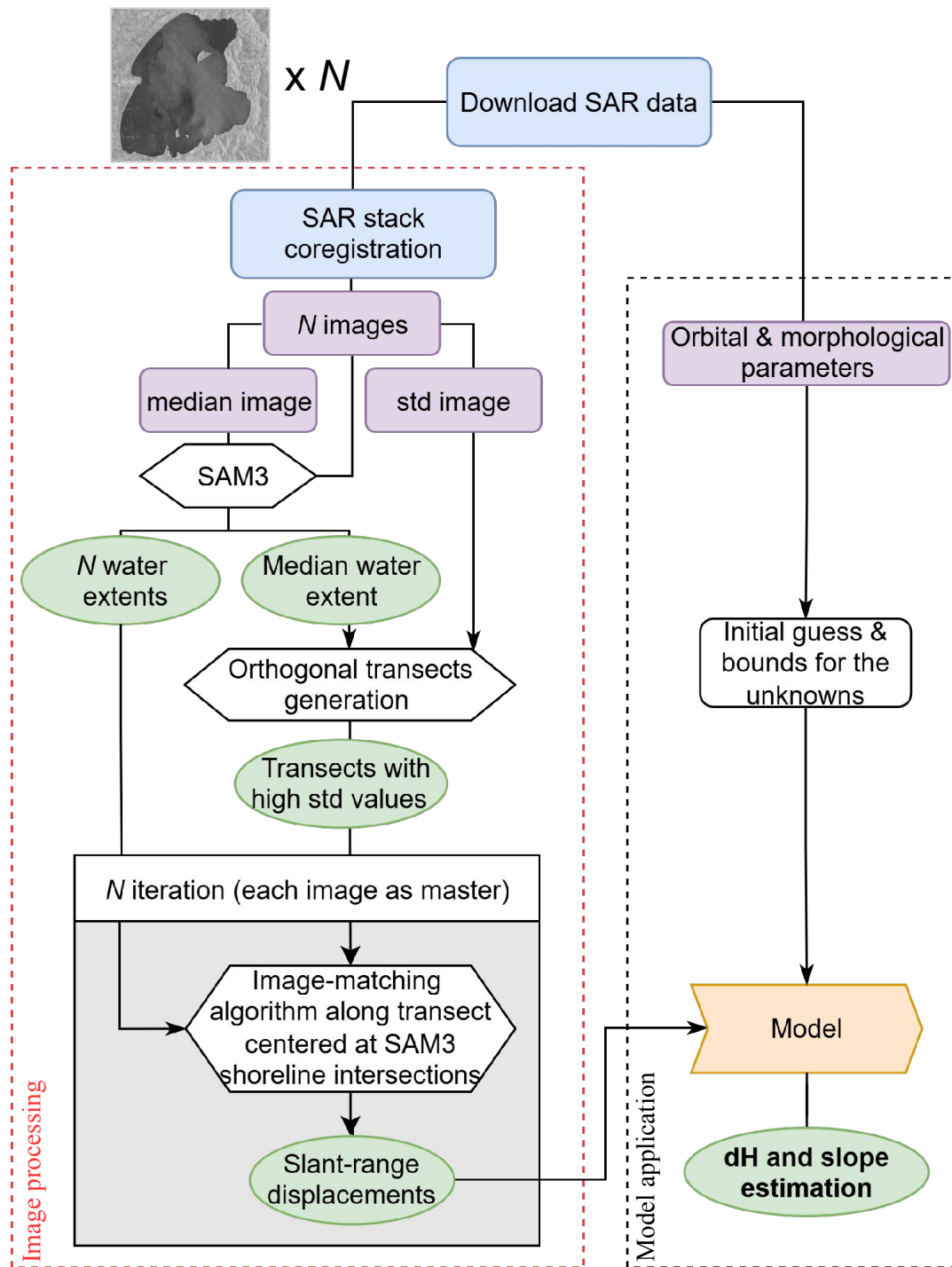
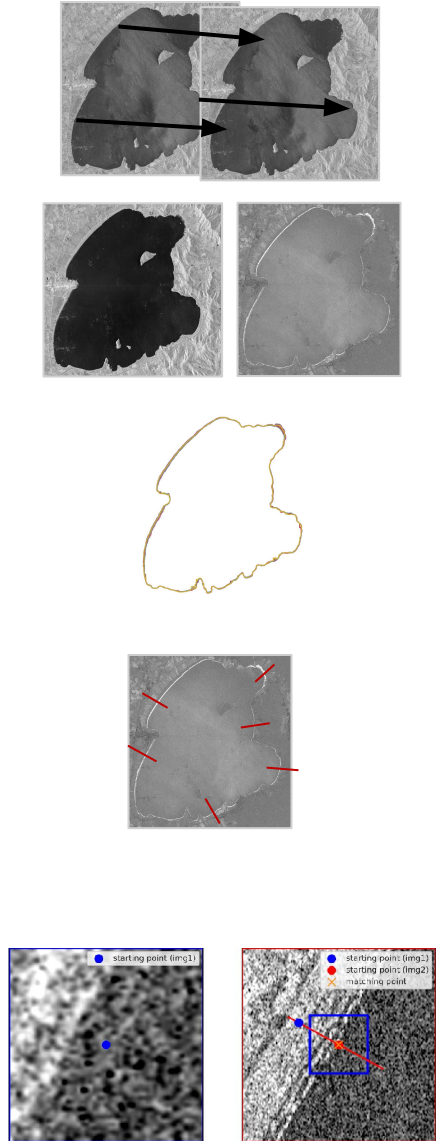
Workflow



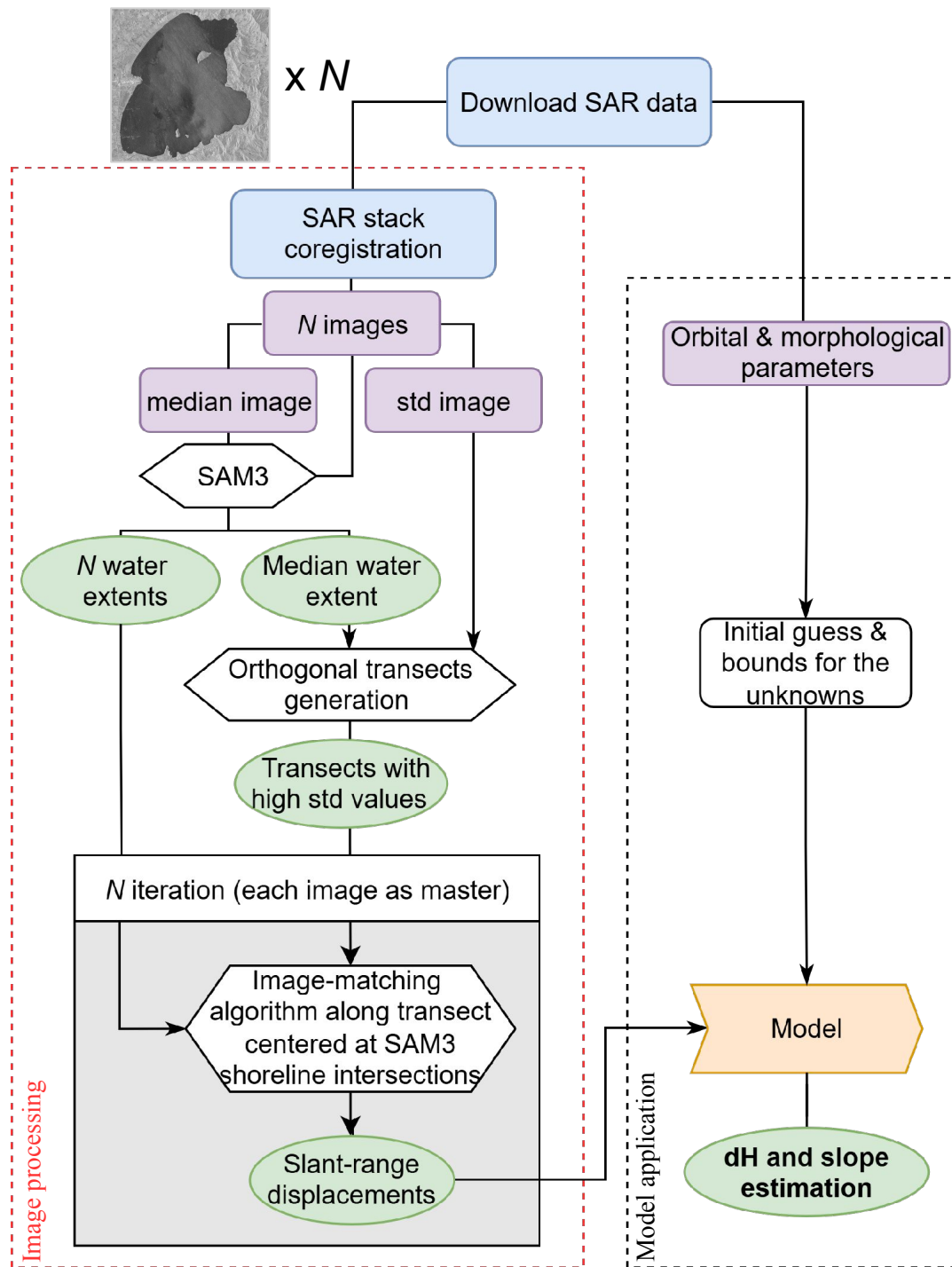
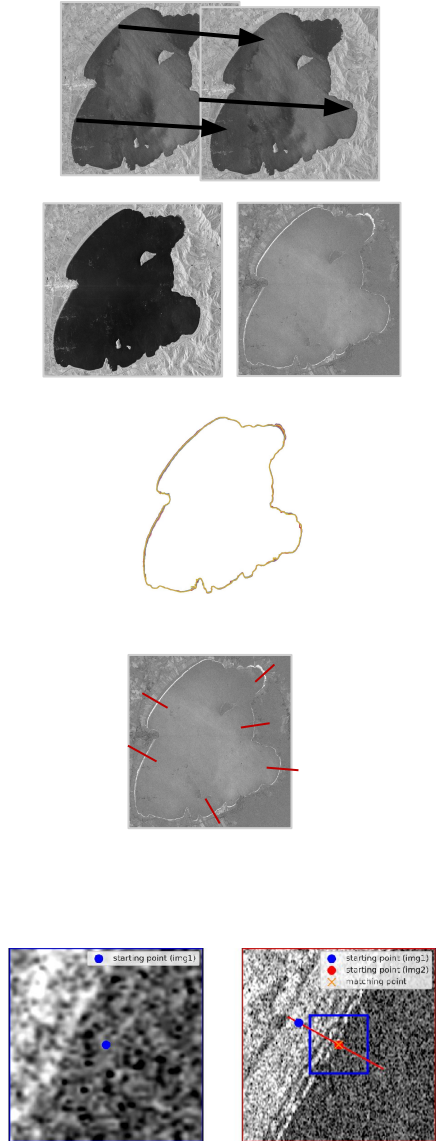
Workflow



Workflow

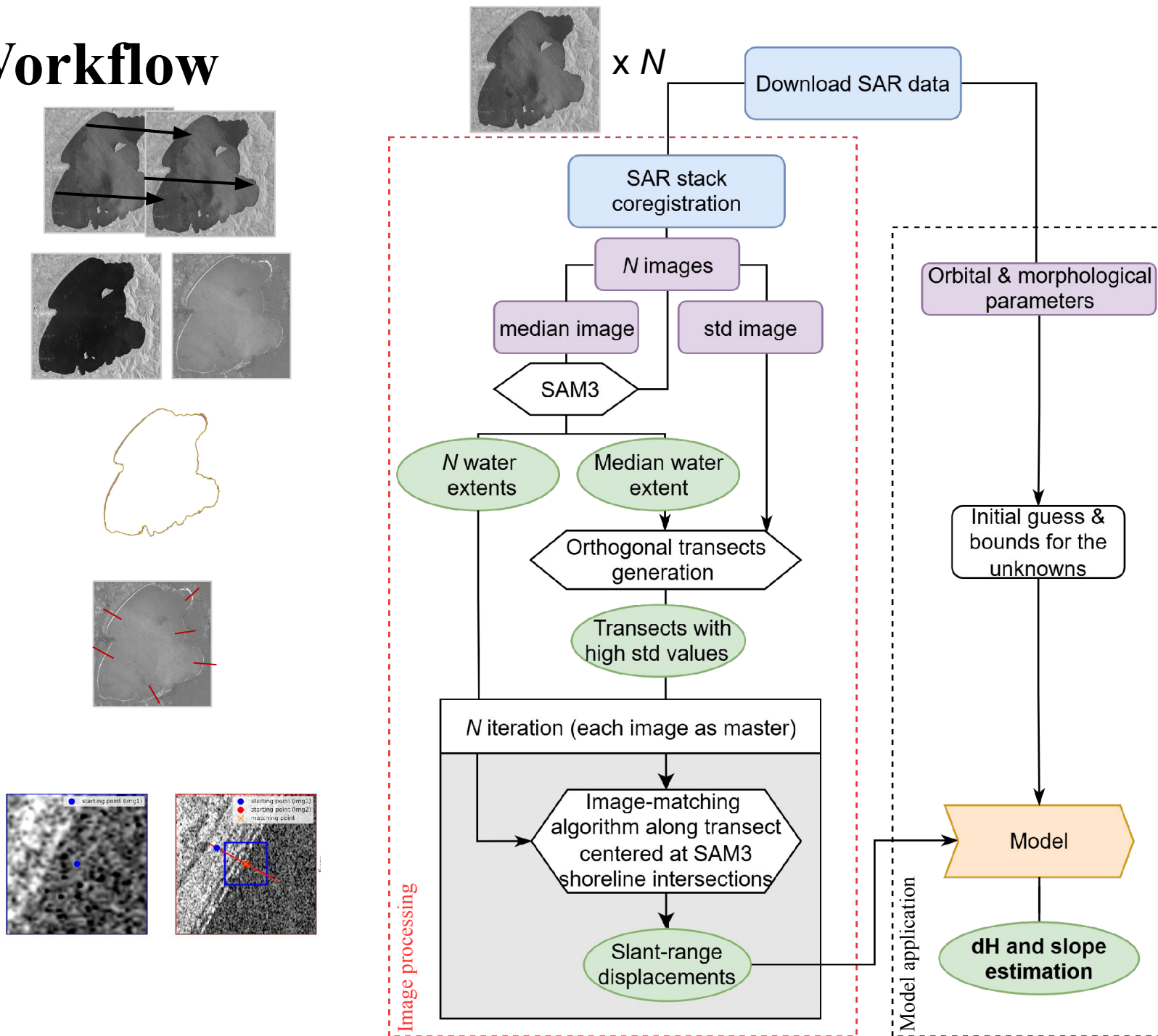


Workflow



$\beta_{ts} \rightarrow \text{aspects}$
 $\theta_{inc} \rightarrow \text{incidence angle}$
 $p, v \rightarrow \text{orbit state vectors}$

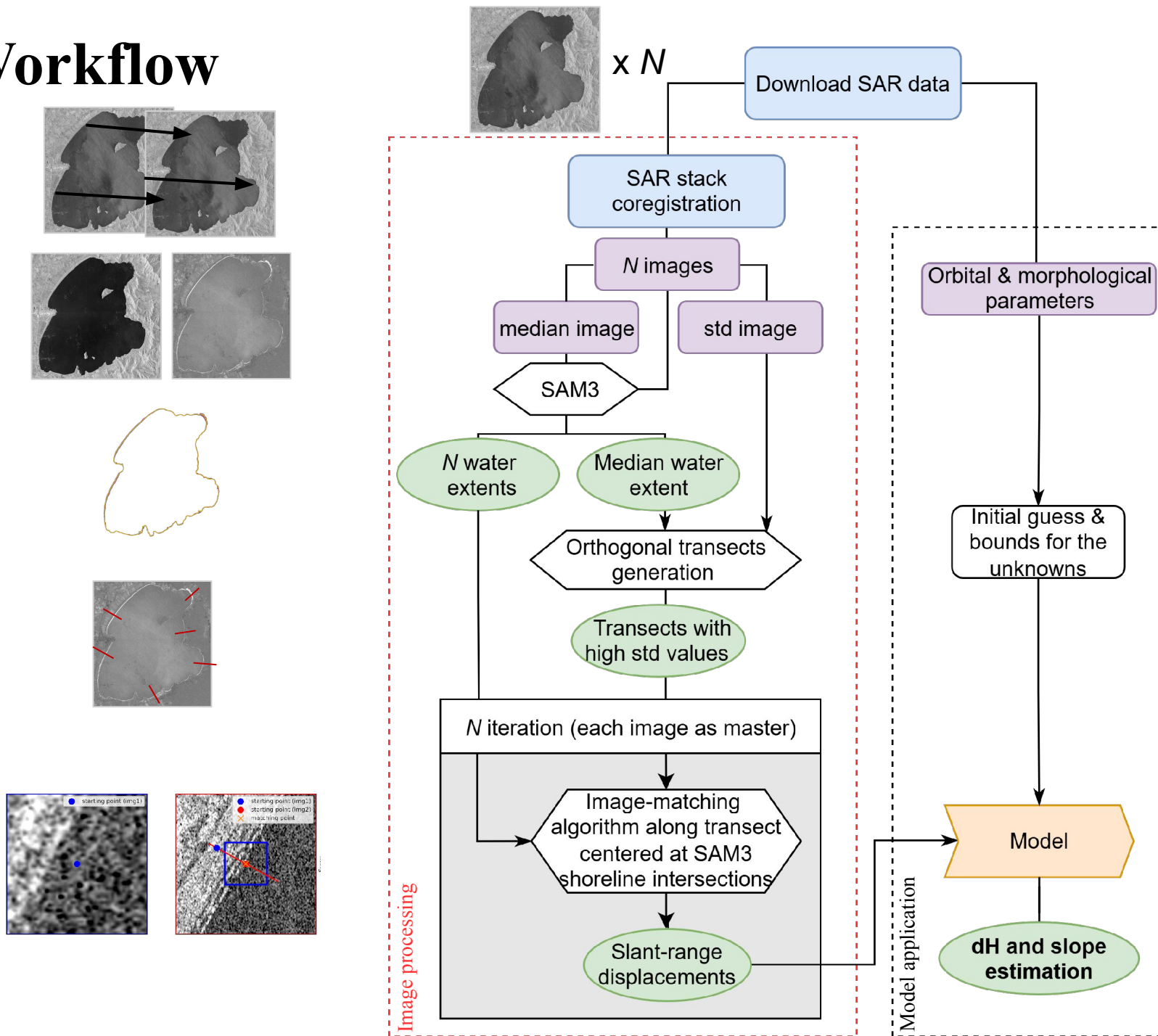
Workflow



$\beta_{ts} \rightarrow \text{aspects}$
 $\theta_{inc} \rightarrow \text{incidence angle}$
 $p, v \rightarrow \text{orbit state vectors}$

$$d_{e,n} = \frac{\Delta h_e}{\cos(\beta_{ts})} \sqrt{A_{e,n} \gamma_n^2 + B_{e,n} \gamma_n^2 + C_{e,n}}$$

Workflow



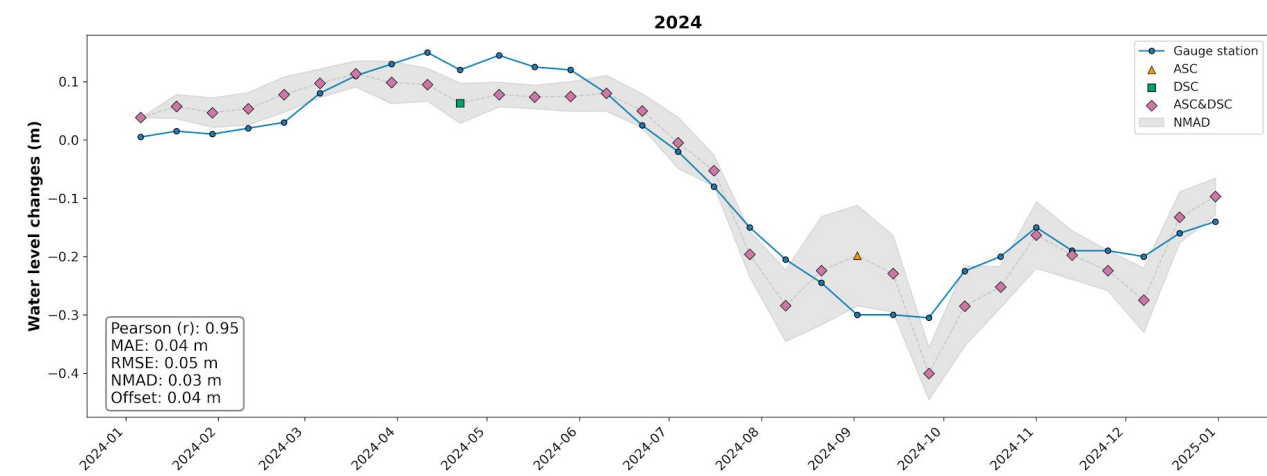
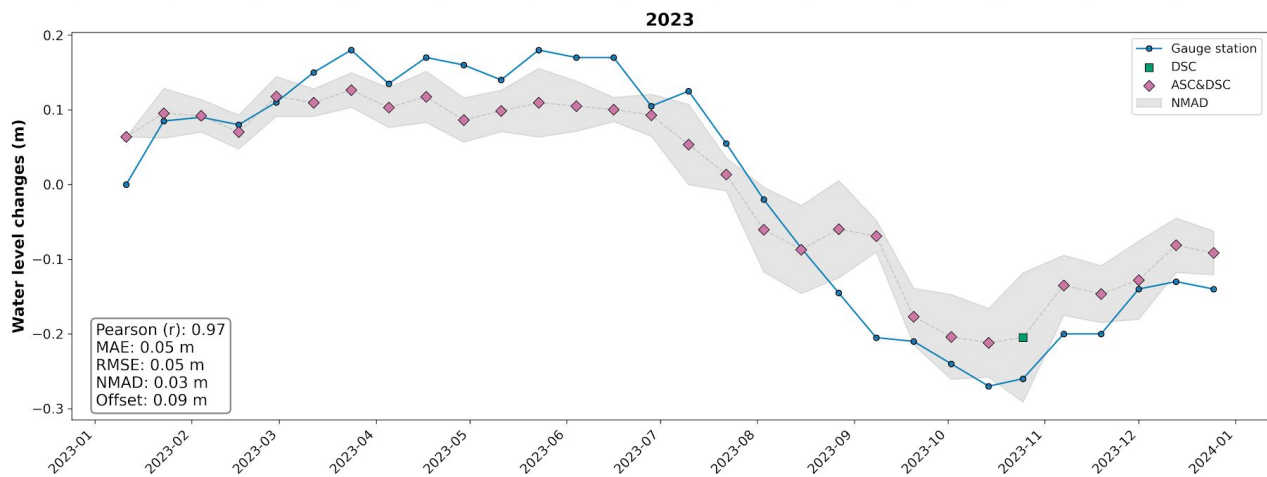
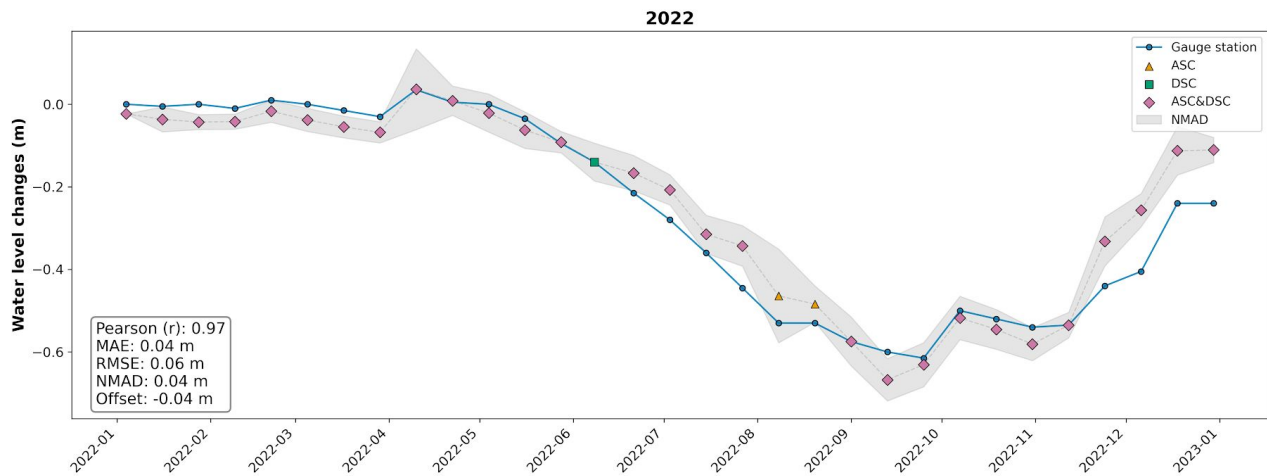
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Δh for each epoch
 α for each transect

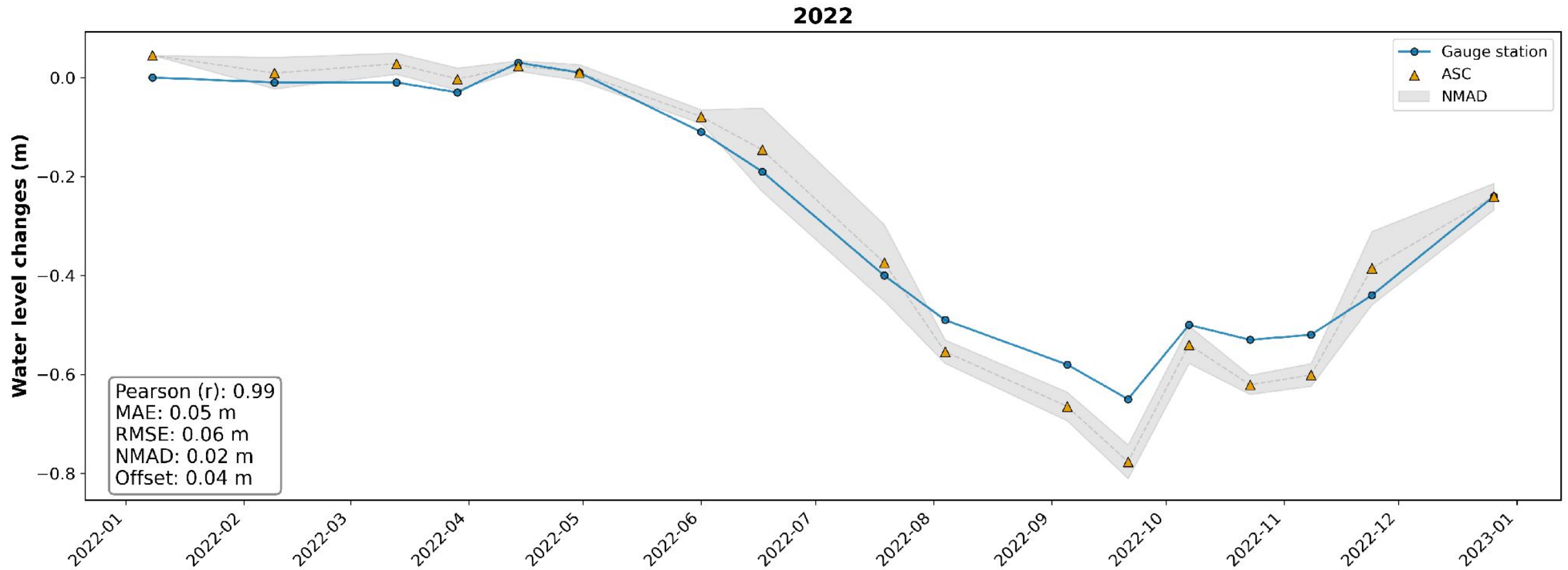
Results – S1 VV+VH ASC & DSC

SAR ascending and descending cumulative Δh relative to the first epoch



Results – CSG HH ASC

SAR ascending cumulative Δh relative to the first epoch



Conclusions and future works

- The method demonstrated a potential accuracy of **5-10 cm** without relying on a priori morphological information with the possibility to be tested with **any SAR sensor** (e.g., Sentinel-1, COSMO-SkyMed, IRIDE)
- The model is limited by:
 - satellite viewing geometry and lake morphology affecting transect selection
 - vegetation coverage and wind effects reducing image quality and affecting the image-matching algorithm
- Future work will focus on assessing the **generalization** capability of the model across **different case studies** and **sensors** to provide an additional water level monitoring solution for diverse scenarios, such as lakes, flood events, or small agricultural basins

Thank you for the attention

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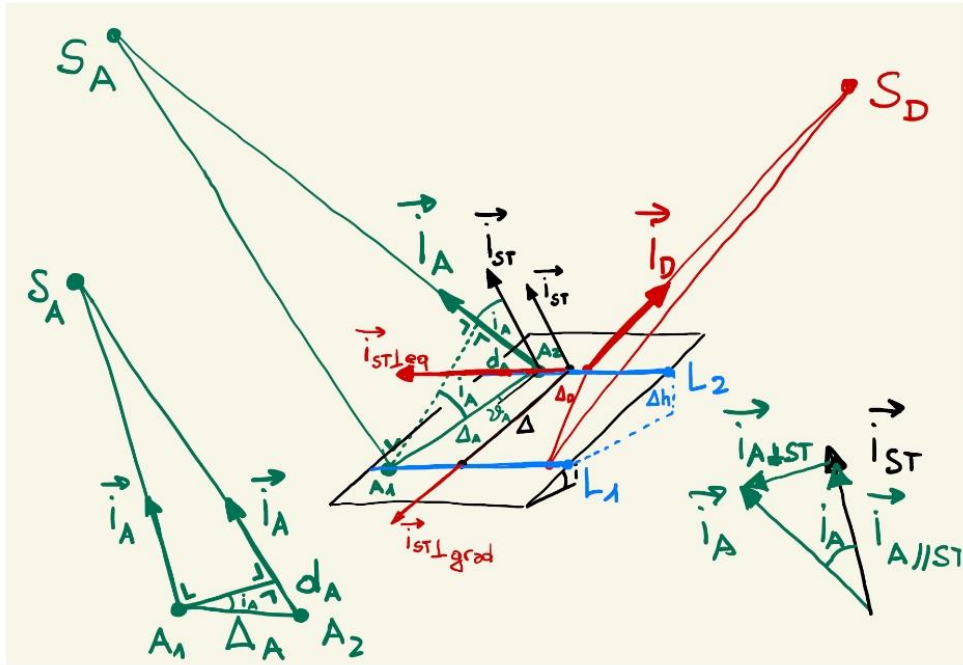
Features of active LiDAR and RADAR satellite altimeters

Table 1. Features of the currently active LiDAR and RADAR satellite altimeters.

Mission	Footprint Size (m)	Scanning Pattern (m)	Temporal Resolution (days)	Sensor Type
ICESat-2	13	90 (across-track spacing)	91 (repeat cycle)	LiDAR
GEDI	25	60 (along-track spacing)	variable (ISS repeat cycle $\approx$ 3 days)	LiDAR
Poseidon-3B	$\sim$ 2–10 km	30 km (along-track resolution)	$\sim$ 10	RADAR
Sentinel-3A	$\sim$ 300	1.67 km (across-track resolution)	27	RADAR
HY-2	16 km	6–7 km (along-track resolution)	14	RADAR
SARAL	$\sim$ 1400	170 (along-track resolution)	35	RADAR
SWOT	$\sim$ 100 (over land)	120 km (along-track swath)	21	RADAR

ICESat-2 across-track spacing for pair beams is 90 m while the two-beam spacing is close to 3 km.

Model



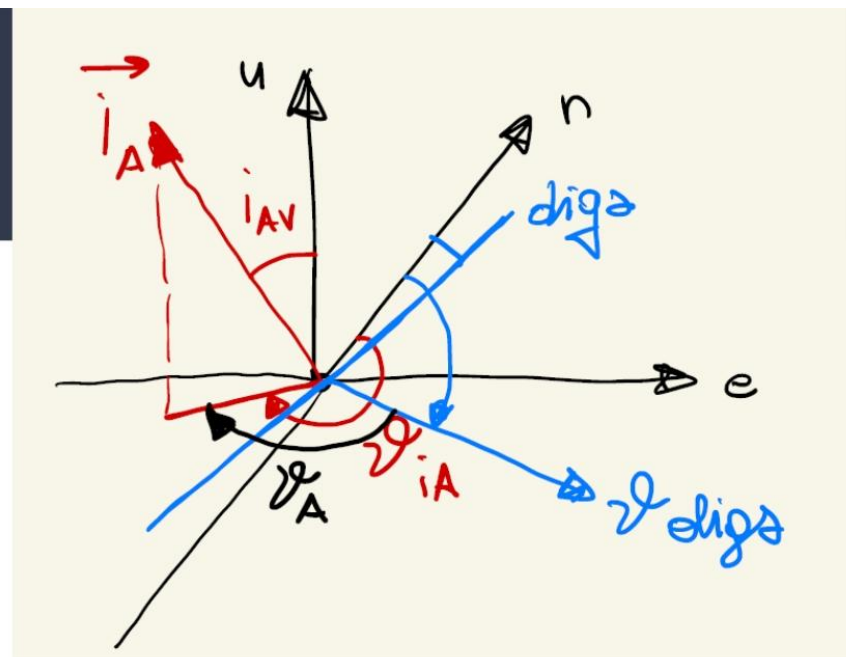
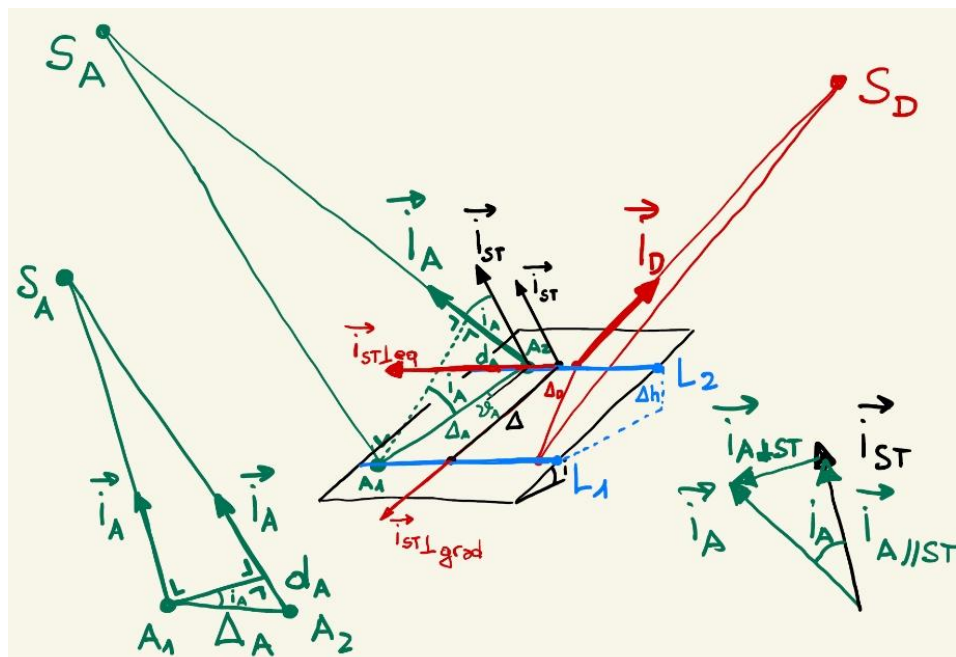
$$\sin i = \sqrt{i_{STx}^2 + i_{STy}^2}$$

incognita

$$\Delta h = \Delta \sin i$$

incognita

$$\Delta = \frac{dA}{\sin i_A} \cos \phi_A$$



$$\sin i = \sqrt{i_{STx}^2 + i_{STy}^2} \quad \text{incognita}$$

$$\Delta h = \Delta \sin i \quad \text{incognita}$$

$$\Delta = \frac{d_A}{\sin i_A} \cos \varphi_A$$

- **i_A** angolo tra direzione sat-target e normale al piano **lst vettore**
- **$TetaA$** angolo tra azimuth target e azimuth piano orbitale satellite (**$Tetala$**)

Model

$$\begin{aligned}\cos i_A &= \vec{i}_A \cdot \vec{i}_{ST} = \\ &= (i_{Ae} \overset{A}{\sin} \vartheta_{i_{ST}}) \sin i + \\ &+ (i_{An} \overset{B}{\cos} \vartheta_{i_{ST}}) \sin i + \\ &+ (i_{Au}) \cos i\end{aligned}$$

$$\Delta h = \Delta \sin i$$

$$\Delta = \frac{d_A}{\sin i_A} \cos \vartheta_A$$

$$\frac{1}{\Delta h} = \frac{1}{\frac{d_A}{\sin i_A} \cos \vartheta_A \sin i}$$

$$\begin{aligned}\frac{d_A \cos \vartheta_A}{\Delta h} &= \frac{\sin i_A}{\sin i} = \\ &= \frac{\sqrt{1 - \cos^2 i_A}}{\sin i}\end{aligned}$$

$$\begin{aligned}K = \left(\frac{d_A^2 \cos^2 \vartheta_A}{\Delta h^2} \right) &= \frac{1 - \cos^2 i_A}{\sin^2 i} = \\ &= \frac{1 - (\overset{D}{(A+B)} \sin i + C \cos i)^2}{\sin^2 i} = \\ &= \frac{1 - (D^2 \sin^2 i + C^2 \cos^2 i + 2CD \sin i \cos i)}{\sin^2 i}\end{aligned}$$

$$= \frac{1}{\sin^2 i} - D^2 - \frac{C^2}{\tan^2 i} - \frac{2CD}{\tan i} =$$

$$= \frac{1}{\sin^2 i} - D^2 - \frac{C^2}{\tan^2 i} - \frac{2CD}{\tan i} =$$

$$\begin{aligned}\left[\sin i = \frac{\tan i}{\sqrt{1 + \tan^2 i}} \right] \\ = \frac{1 + \tan^2 i - D^2 \tan^2 i - C^2 - 2CD \tan i}{\tan^2 i}\end{aligned}$$

$$t = \tan i$$

Model

$$\frac{d_A \cos \vartheta_A}{\Delta h} = \frac{\sin i_A}{\sin i} =$$

$$= \frac{\sqrt{1 - \cos^2 i_A}}{\sin i}$$

$$K = \left(\frac{d_A^2 \cos^2 \vartheta_A}{\Delta h^2} \right) = \frac{1 - \cos^2 i_A}{\sin^2 i} =$$

$$= \frac{1 - (A+B) \sin i + C \cos i)^2}{\sin^2 i} =$$

$$= \frac{1 - (D^2 \sin^2 i + C^2 \cos^2 i + 2CD \sin i \cos i)}{\sin^2 i} =$$

$$= \frac{1}{\sin^2 i} - D^2 - \frac{C^2}{\tan^2 i} - \frac{2CD}{\tan i} =$$

$$= \frac{1}{\sin^2 i} - D^2 - \frac{C^2}{\tan^2 i} - \frac{2CD}{\tan i} =$$

$$\left[\sin i = \frac{\tan i}{\sqrt{1 + \tan^2 i}} \right]$$

$$= \frac{1 + \tan^2 i - D^2 \tan^2 i - C^2 - 2CD \tan i}{\tan^2 i}$$

$$t = \tan i$$

$$d_A = \frac{\Delta h}{\cos \vartheta_A} \sqrt{\frac{1 + t^2 - D^2 - 2CDt - D^2 t^2}{t^2}} =$$

$$= \frac{\Delta h}{\cos \vartheta_A} \sqrt{(1 - C^2) \gamma^2 + 2CD\gamma + (1 - D^2)}$$

$\left[\gamma = \frac{1}{t} \right] \quad 0 \leq \gamma \leq +\infty$

$\left[\begin{aligned} D &= i_{Ae} \sin \vartheta_{1ST} + i_{An} \cos \vartheta_{1ST} \\ C &= i_{An} \end{aligned} \right]$

$\Delta = C^2 D^2 - (1 - D^2)(1 - C^2) = C^2 + D^2 - 1 < 0$

$d_{A1}^1 = f(\Delta h^1, i_1)$
 $d_{A2}^1 = f(\Delta h^1, i_2)$
 $d_{A1}^2 = f(\Delta h^2, i_1)$
 $d_{A2}^2 = f(\Delta h^2, i_2)$

Workflow – Final Δh estimation

ΔH	04/01/2022	16/01/2022	28/01/2022	...
04/01/2022	Δh_{11}	Δh_{12}	Δh_{13}	...
16/01/2022	Δh_{21}	Δh_{22}	Δh_{23}	...
28/01/2022	Δh_{31}	Δh_{23}	Δh_{33}	...
...	...	...	...	...

ΔH matrix (NxN)
for each year

- Matrix $T_{ij} = \frac{\Delta h_{i,j} - \Delta h_{j,i}}{2}$ (NxN)
 - Matrix $L_{ij} = T_{i+1,j+1} - T_{i,j}$ (N-1xN-1)
- Computing consecutive row differences eliminates dependency on the reference epoch, transforming observations into sequential temporal increments

L_{ij}	04/01/2022	16/01/2022	28/01/2022	...	Median value	Final Δh
04/01/2022	L_{11}	L_{12}	L_{13}	...	$M_{1j} = \text{Median}(L_{1j})$	$M_1 = M_{1j}$
16/01/2022	L_{21}	L_{22}	L_{23}	...	$M_{2j} = \text{Median}(L_{2j})$	$M_2 = M_{2j} + M_1$
28/01/2022	L_{31}	L_{23}	L_{33}	...	$M_{3j} = \text{Median}(L_{3j})$	$M_3 = M_{3j} + M_2$
...	...	...	...	...	...	...

$N \times \Delta h$ with respect
to the first epoch