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“Stochastic gravity modelling of finite distributions”

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Session: Modelling of Earth's dynamic

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Deterministic gravity modelling of given distributions



$$V = G\rho \iiint_U \frac{1}{l} dU$$

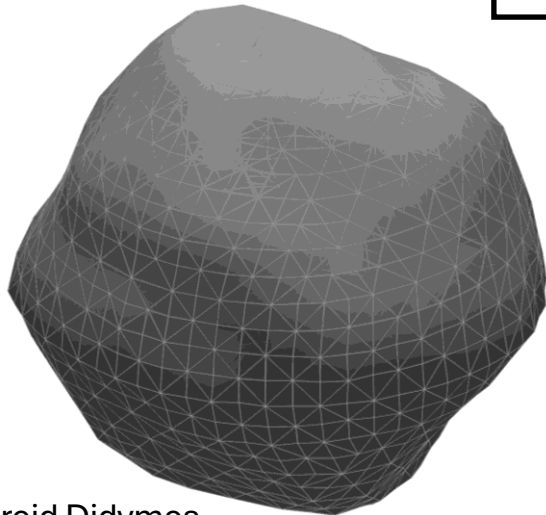
Analytical Solution

Closed expressions including transcendental functions ($\sin(x)$, $\cos(x)$, $\sqrt{x}$, $\ln x$, ...)

Numerical Solution



Asteroid Eros



Asteroid Didymos

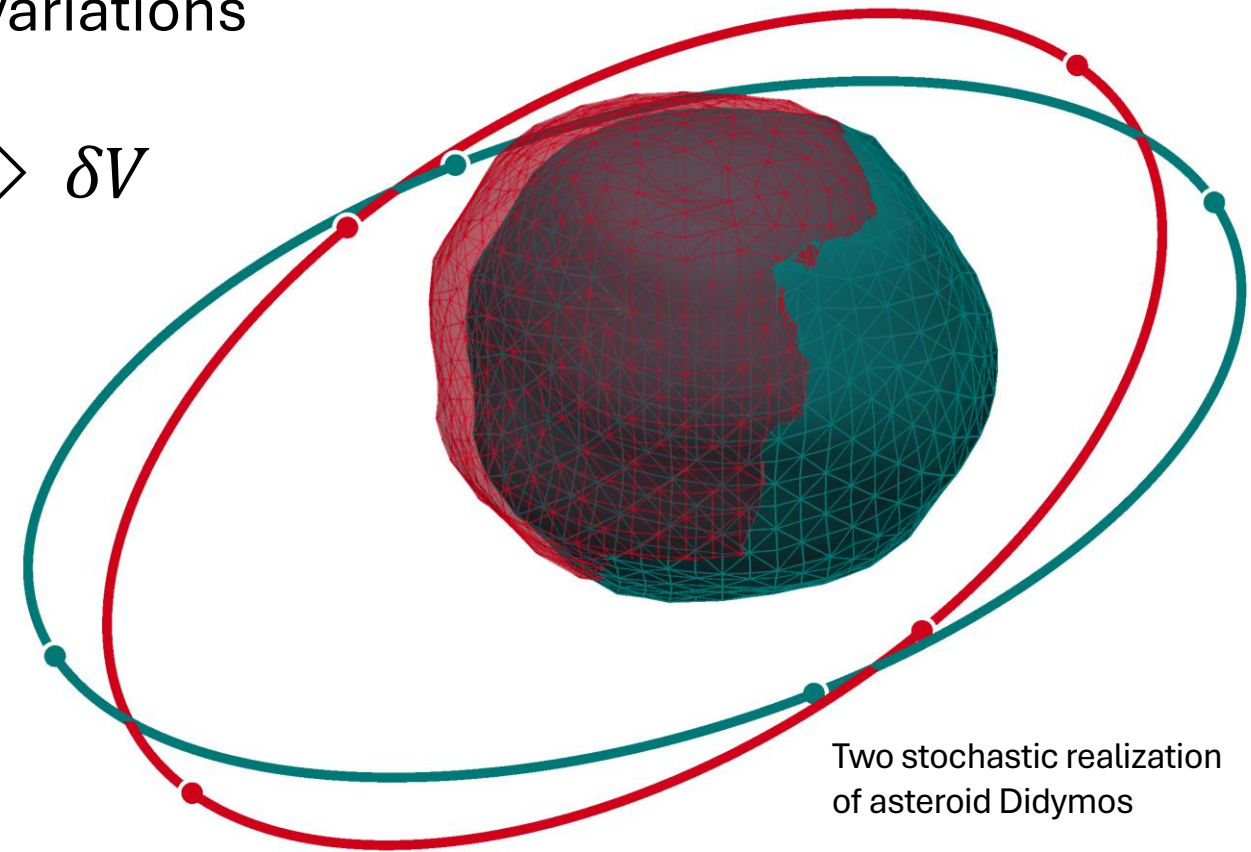
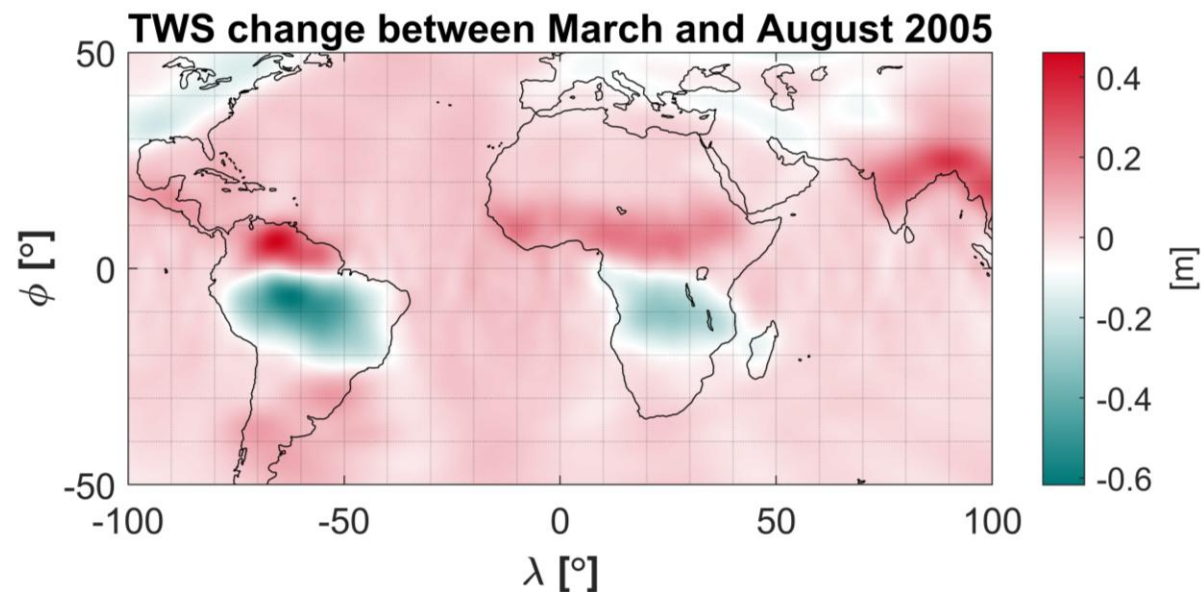
$$V(r, \theta, \lambda) = \frac{GM}{r} \sum_{n=0}^{+\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^n \bar{P}_{n,m}(\cos\theta) [\bar{C}_{n,m} \cos(m\lambda) + \bar{S}_{n,m} \sin(m\lambda)]$$

Existence of shape uncertainties

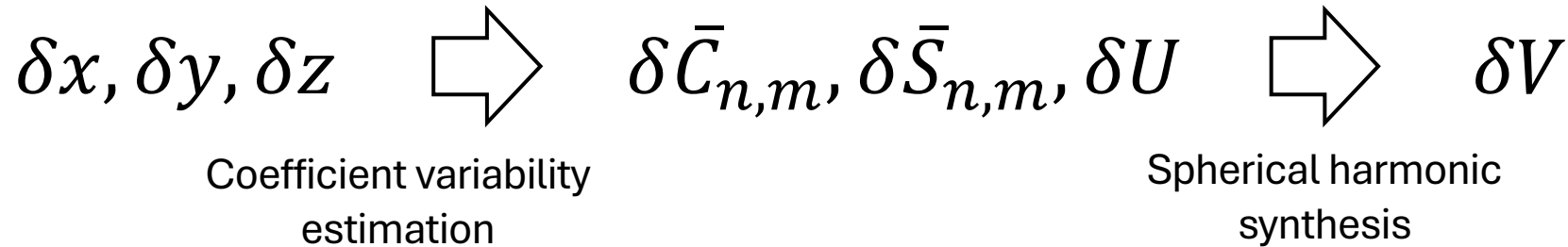
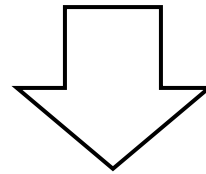


Direct link between shape and gravity variations

$$\delta x, \delta y, \delta z \Rightarrow \delta \bar{C}_{n,m}, \delta \bar{S}_{n,m} \Rightarrow \delta V$$



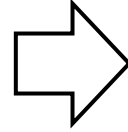
$$V(r, \theta, \lambda) = \frac{G\rho U}{r} \left\{ 1 + \sum_{n=1}^N \left(\frac{a}{r}\right)^n \sum_{m=0}^n \bar{P}_{n,m}(\cos\theta) [\bar{C}_{n,m} \cos(m\lambda) + \bar{S}_{n,m} \sin(m\lambda)] \right\}$$



Propagation Procedure



$$V = \frac{G\rho U}{r} \bar{\Pi}^\top \bar{\Phi}$$



$$\left[\frac{\partial V}{\partial \mathbf{C}} \right] = \frac{G\rho}{r} \left[\bar{\Pi}^\top \bar{\Phi} \frac{\partial U}{\partial \mathbf{C}} + U \bar{\Pi}^\top \frac{\partial \bar{\Phi}}{\partial \mathbf{C}} \right]$$

$$\mathbf{C} = (x_1, y_1, z_1, x_2, y_2, z_2, \dots, x_{\max}, y_{\max}, z_{\max})^\top$$

$$\bar{\Phi} = (\bar{\Phi}_{0,0} \quad \bar{\Phi}_{1,0} \quad \bar{\Phi}_{1,1} \quad \dots \quad \bar{\Phi}_{N,N-1} \quad \bar{\Phi}_{N,N})^\top$$

$$\bar{\Phi}_{n,m} = (\bar{C}_{nm} \quad \bar{S}_{nm})^\top$$

*** Remain constant for each shape model**

Uncertainty propagation law:

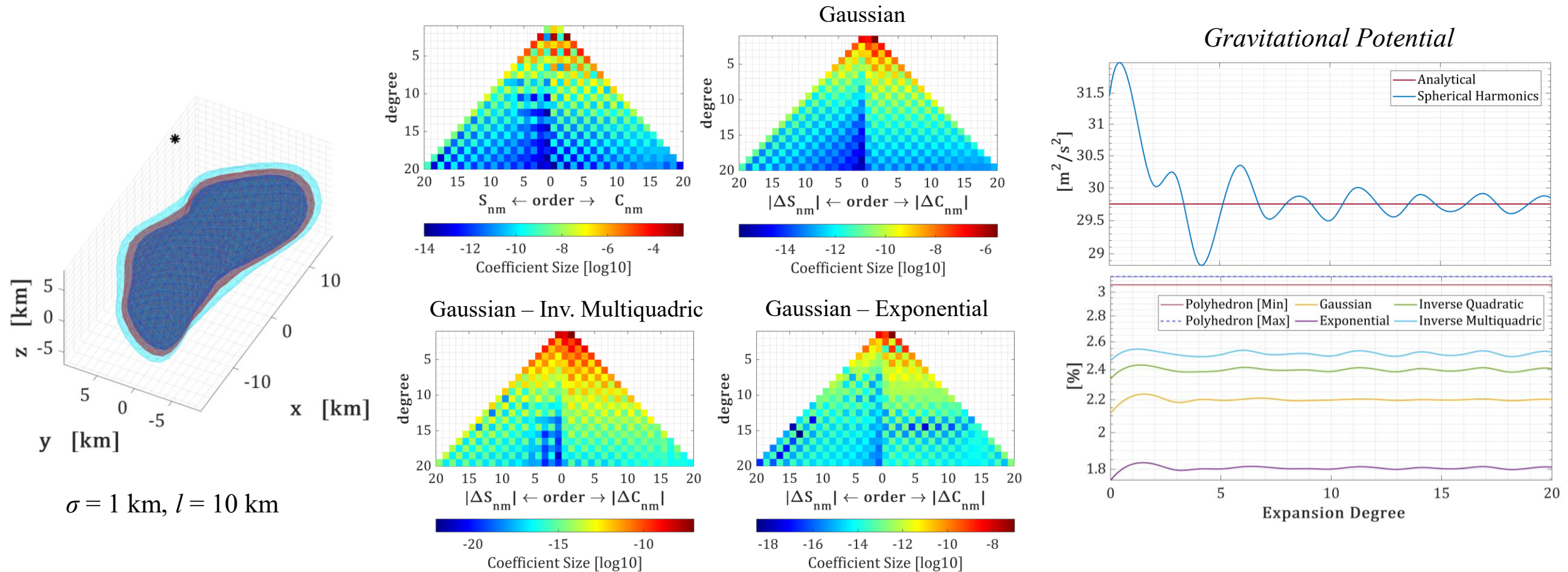
$$P_V = \frac{\partial V}{\partial \mathbf{C}} \mathbf{P}_C \left(\frac{\partial V}{\partial \mathbf{C}} \right)^\top \Rightarrow P_V = \sigma_V^2 = |\delta V|^2$$

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Role of Stochastic Model Selection



- Covariance matrix $\mathbf{P}_C$ includes all the uncertainty information
- Included stochastic parameters: covariance σ^2 and correlation length l

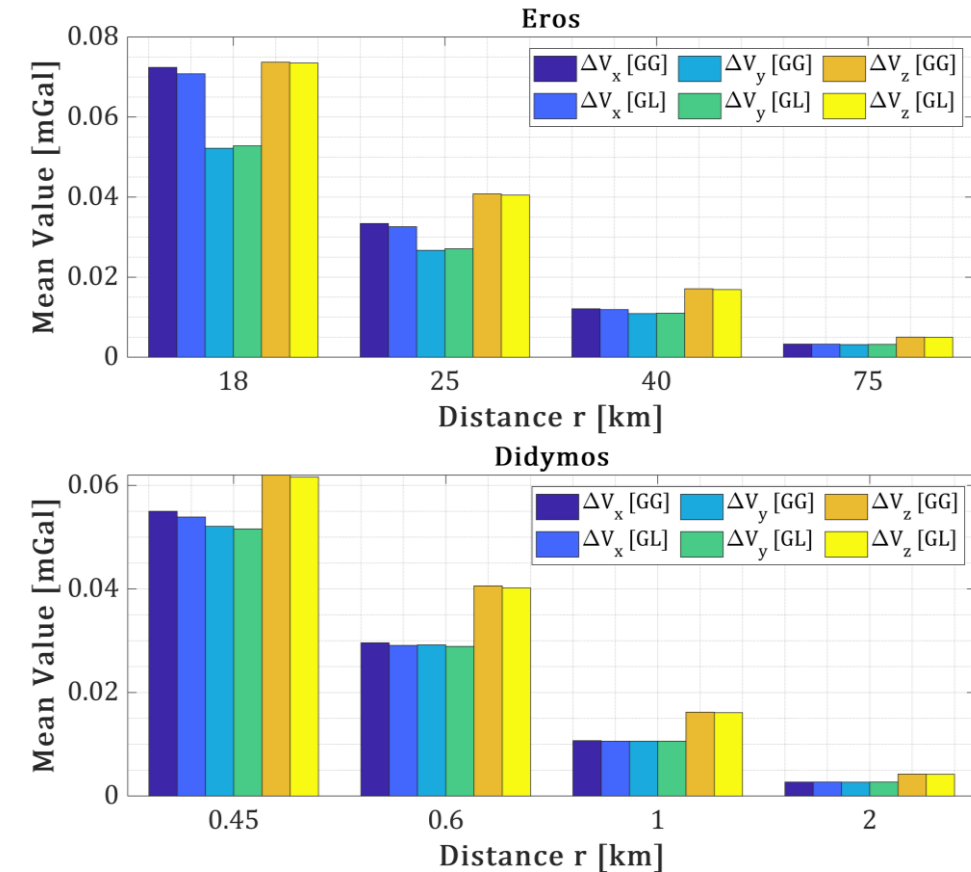
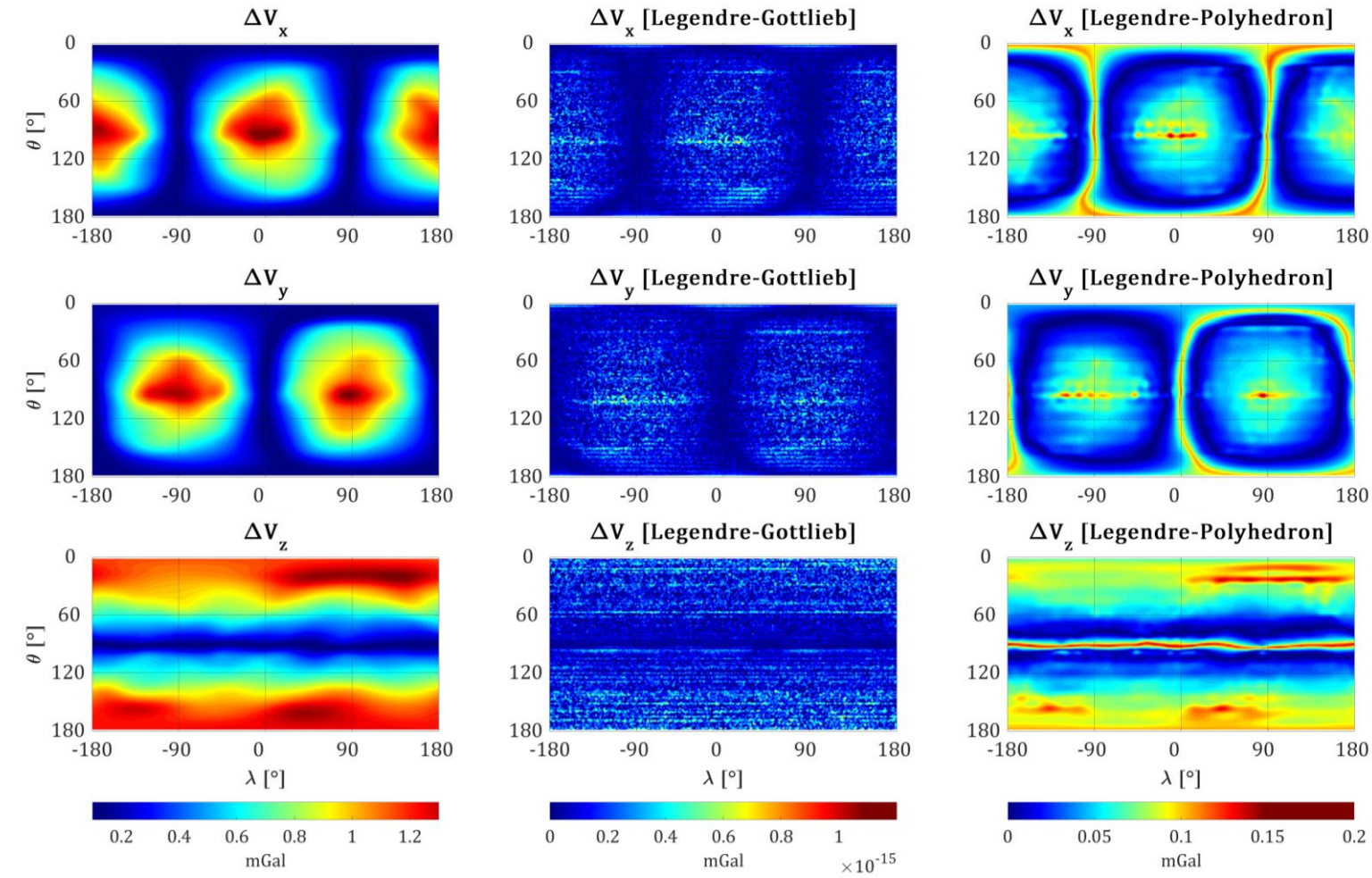


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Extension to First Order Derivatives

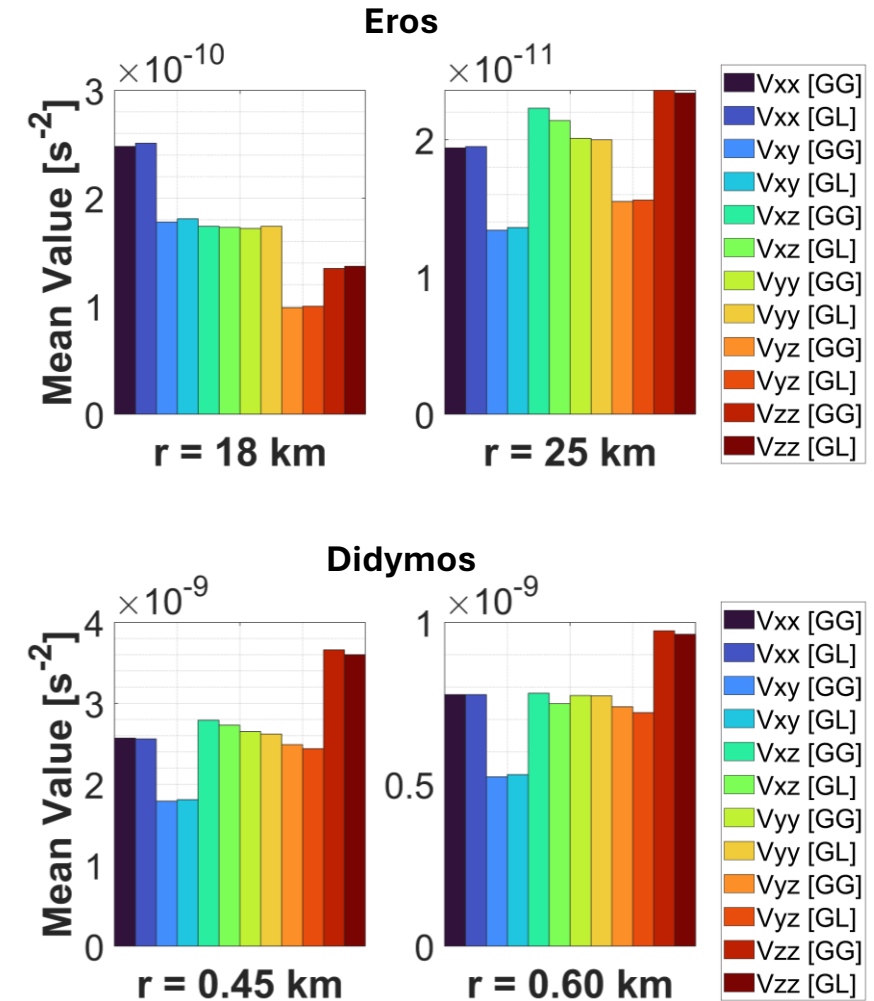
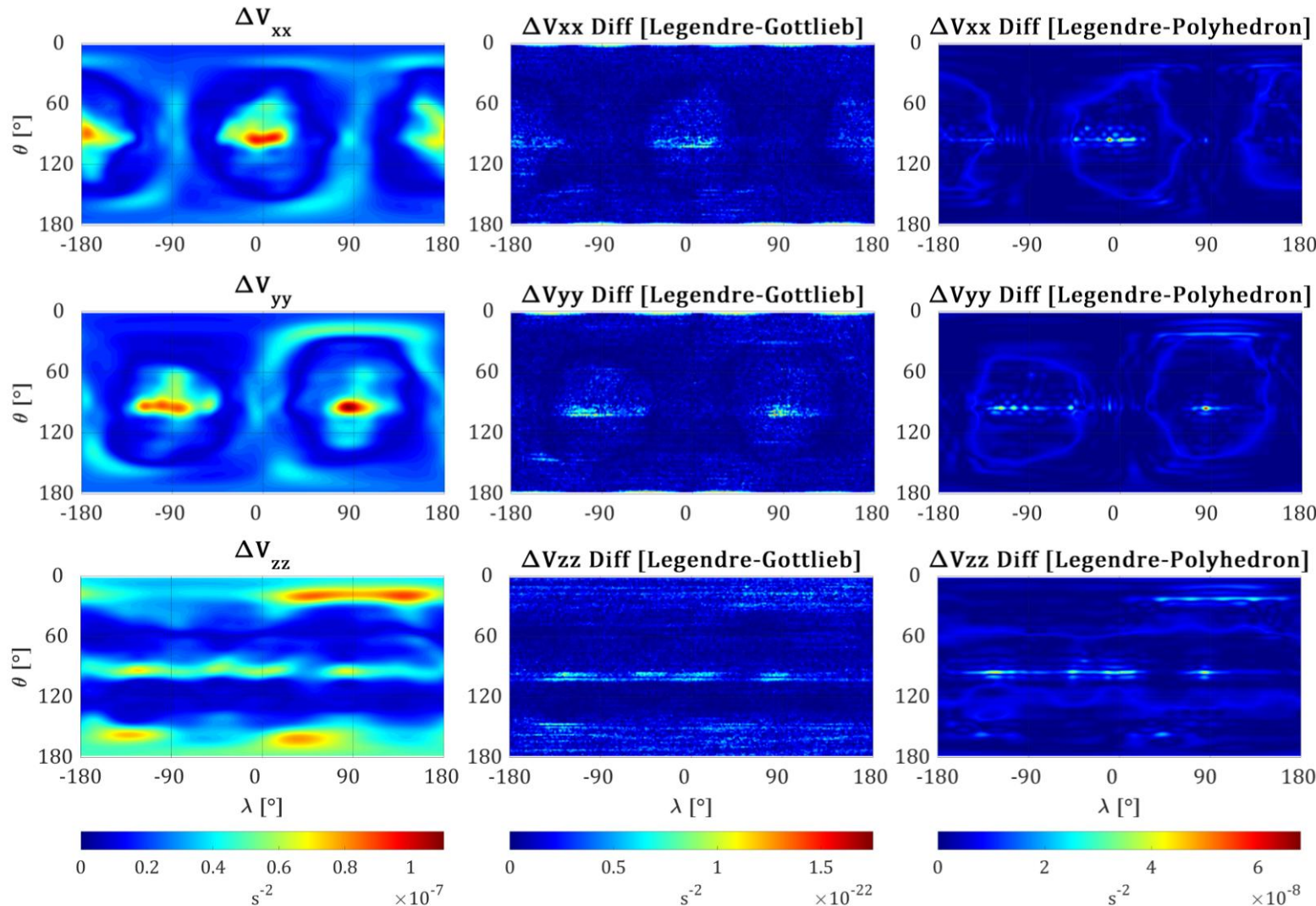


Variations for Didymos at $r = 0.45$ km, using $\sigma = 0.01$ km and $l = 1$ km.



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Extension to Second Order Derivatives



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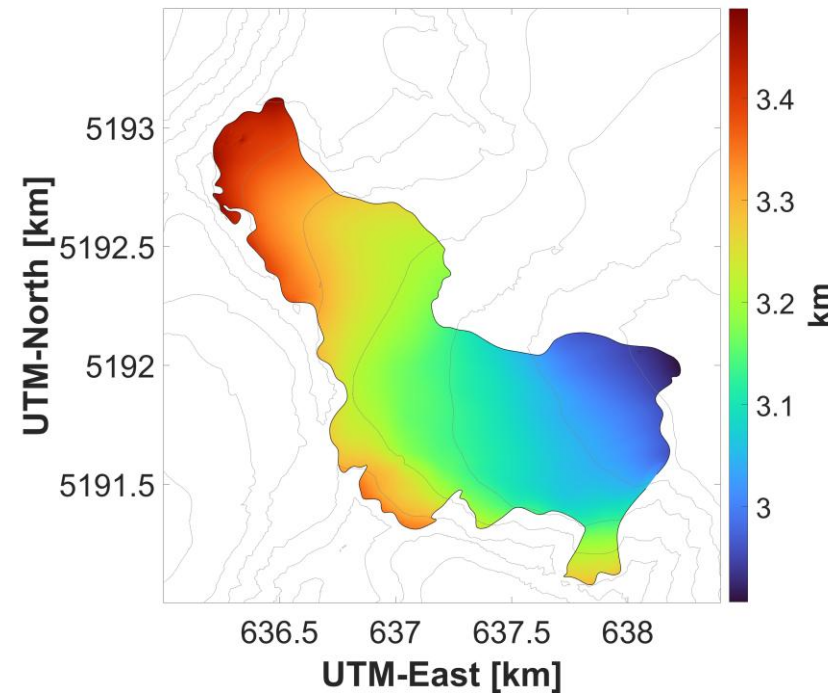
Deterministic Approach

$$\{x, y, z\}_t \rightarrow V_t$$

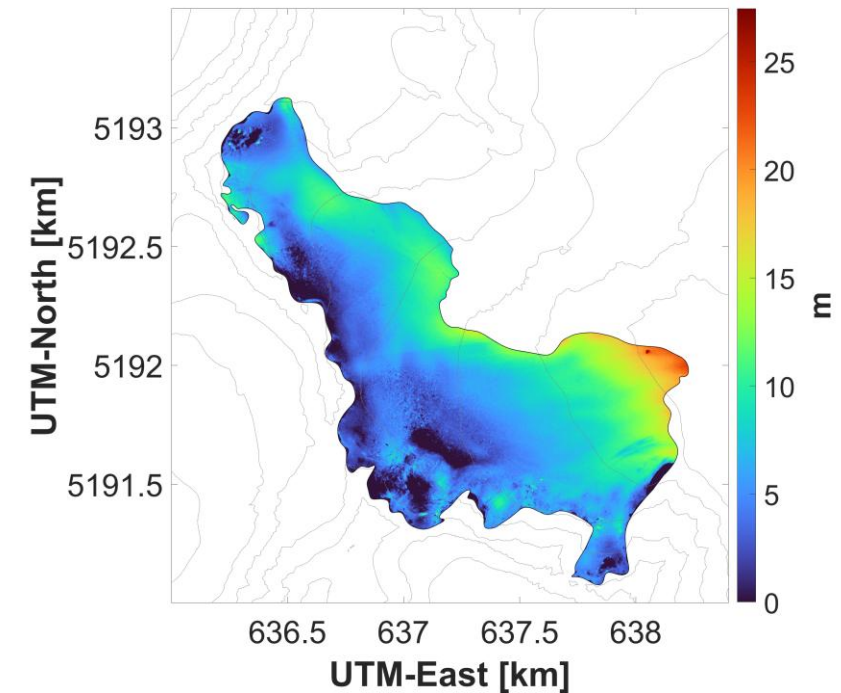
Stochastic Approach

$$\{\Delta x, \Delta y, \Delta z\} \rightarrow \Delta V$$

Glacier surface in 2016

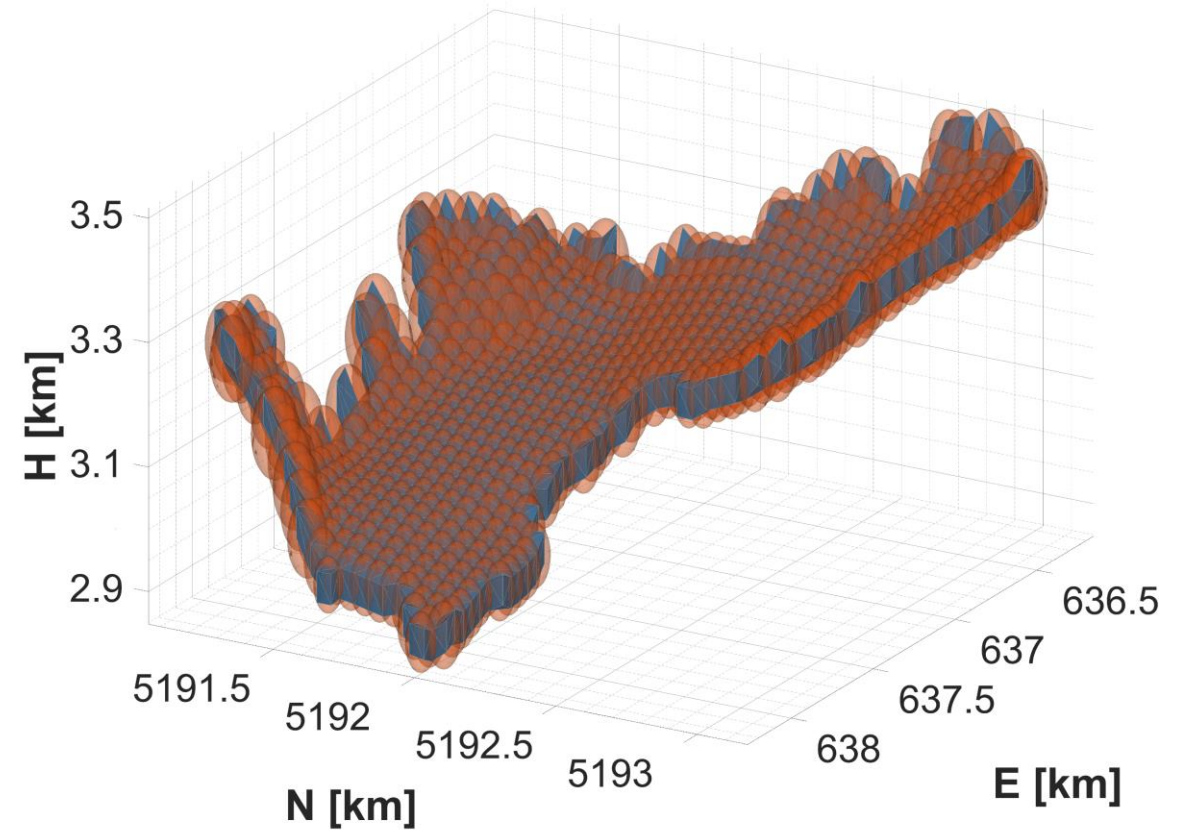
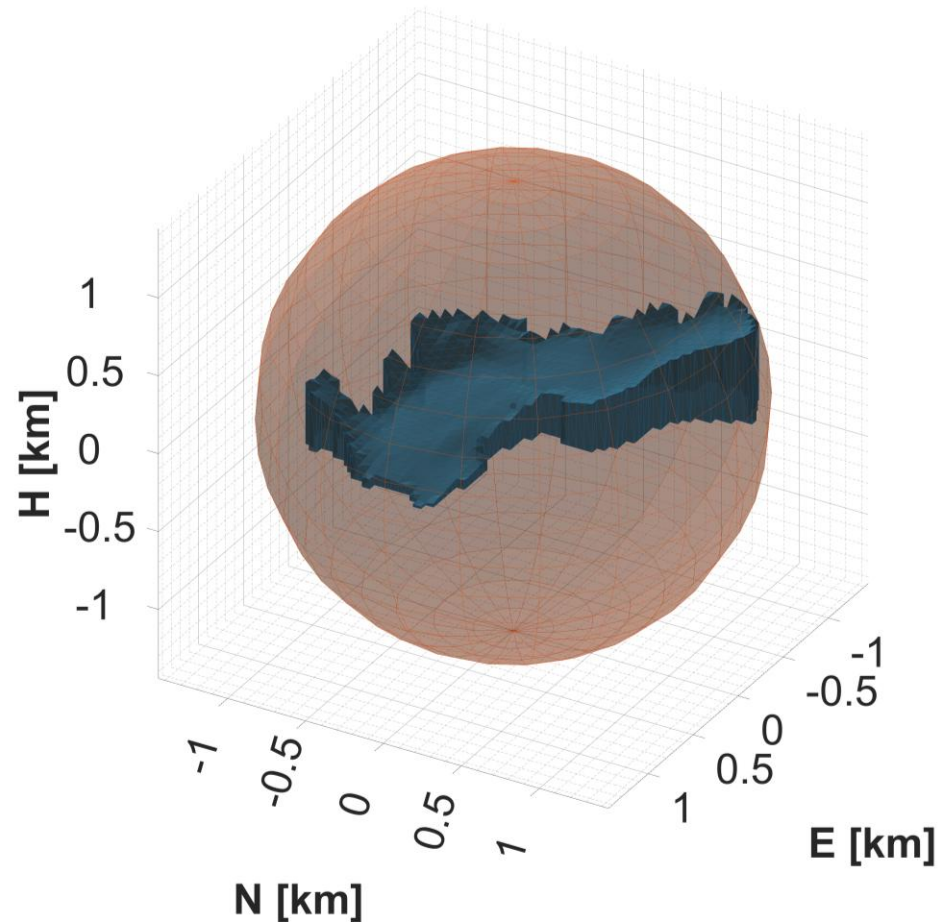


Glacier surface receding between 2009 – 2016



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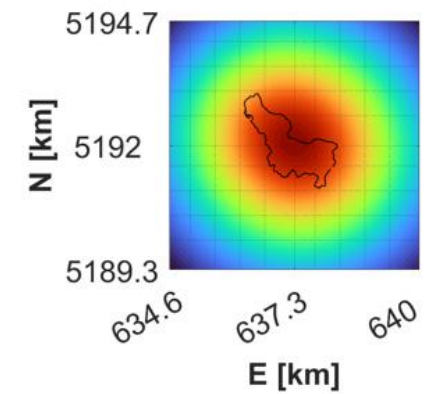
Role of Brillouin Sphere



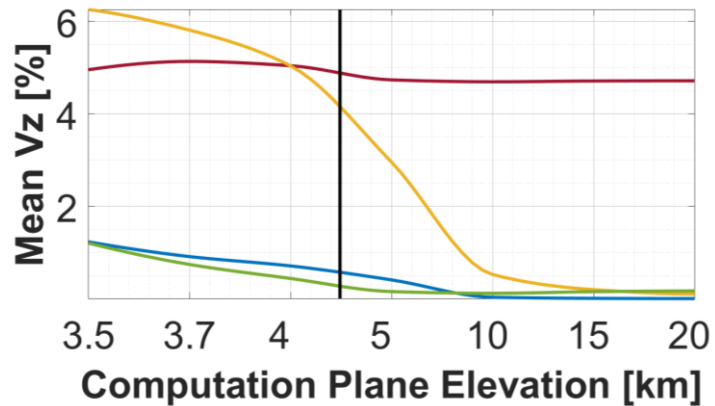
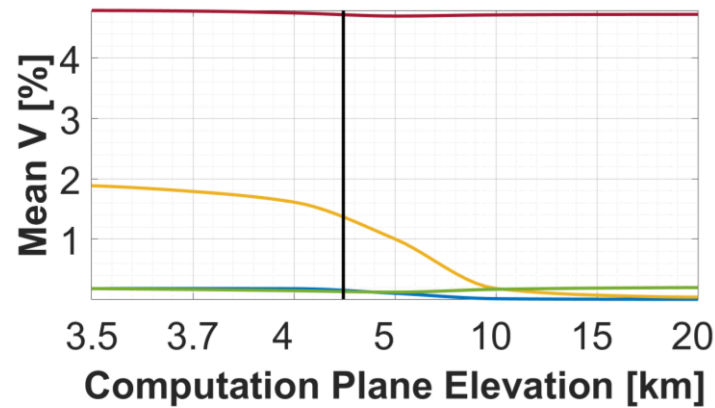
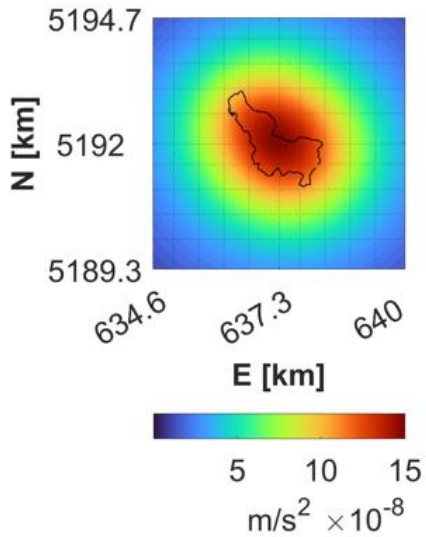
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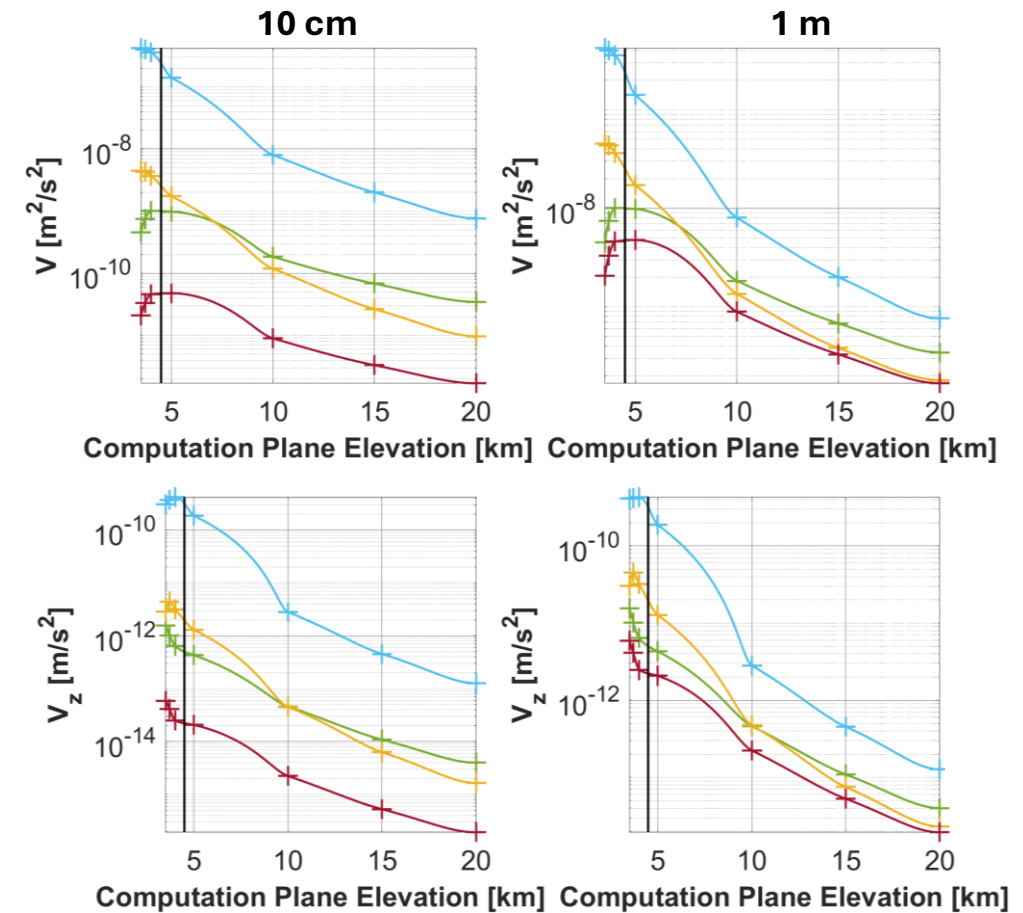
V



V_z



- Analytical - Harmonics
- Analytical - Prismatic
- Analytical - Stochastic polyhedron
- Analytical - Individual polyhedra



- + Mean Dif. from harmonic expansion
- + Mean Dif. from stochastic approach
- + Mean Dif. from right rectangular prism
- + Mean Dif. from individual polyhedra

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- A flexible tool to link shape variations with gravity changes
- Modelling of mass transport and time-varying gravity fields
- Accuracy close to analytical solutions

- Rigorous representation of shape changes
- Model global gravity signals
- Evaluate the uncertainty in shape information