

Vectorial Decorrelation: a ‘new’ perspective for IAR

Lotfi Massarweh,
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Contents

- **Introduction**
 - Mixed-integer models, LAMBDA method
- **Vectorial Decorrelation (VD) principles**
 - From scalar to vectorial formulation
 - Reverse Vectorial Decorrelation solution
- **‘Targeted Insertions’ & implementations**
 - Adjacent block permutations, TI algorithm
 - Numerical examples, further considerations
- **Conclusions**

Introduction

Background information

Mixed-integer least-squares problems

Given $y \in \mathbb{R}^m$ as vector of observables with covariance matrix $Q_{yy} \in \mathbb{R}^{m \times m}$, we consider $a \in \mathbb{Z}^n$ (*ambiguities*) and $b \in \mathbb{R}^p$ (*parameters*), so

$$E\{y\} = Aa + Bb, \quad D\{y\} = Q_{yy} \succ 0$$

where $[A, B] \in \mathbb{R}^{m \times (n+p)}$ is the **full (column) rank** design matrix.

The integer-estimability problem precedes the ambiguity resolution problem!

➤ **References:** Teunissen (2019), Khodabandeh and Teunissen (2019), Teunissen and Khodabandeh (2022).

Ultimately, we seek the ‘fixed’ solution

$$\check{a}, \check{b} = \arg \min_{a \in \mathbb{Z}^n, b \in \mathbb{R}^p} \|y - Aa - Bb\|_{Q_{yy}}^2$$

Primal orthogonal decomposition

Least-squares AMBiguity Decorrelation Adjustment method

A *primal* orthogonal decomposition (Teunissen 1993) is given by

$$\|y - Aa - Bb\|_{Q_{yy}}^2 \triangleq \|\hat{e}\|_{Q_{yy}}^2 + \|\hat{a} - a\|_{Q_{\hat{a}\hat{a}}}^2 + \|\hat{b}(a) - b\|_{Q_{\hat{b}(a)\hat{b}(a)}}^2$$

Step 1	Step 2	Step 3
<u>Float solution:</u> $\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}, \begin{bmatrix} Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}\hat{b}} \end{bmatrix}$ given - ambiguities (float) - baseline parameters	<u>Ambiguity resolution:</u> $\check{a} = \mathcal{I}(\hat{a})$ LAMBDA <pre> graph LR subgraph LAMBDA Decorr[Decorr.] --> subgraph IAIE [] I[I] IA[IA] IE[IE] end end </pre>	<u>Fixed solution:</u> $\check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} (\hat{a} - \check{a})$ considering a success rate $\text{SR} \stackrel{\text{def}}{=} P(\check{a} = a) \approx 1$ i.e. $Q_{\check{a}\check{a}} \approx 0$.

Primal orthogonal decomposition

Least-squares AMBiguity Decorrelation Adjustment method

A *primal* orthogonal decomposition (Teunissen 1993) is given by

$$\|y - Aa - Bb\|_{Q_{yy}}^2 \triangleq \|\hat{e}\|_{Q_{yy}}^2 + \|\hat{a} - a\|_{Q_{\hat{a}\hat{a}}}^2 + \|\hat{b}(a) - b\|_{Q_{\hat{b}(a)\hat{b}(a)}}^2$$

Step 1

Float solution:

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}, \begin{bmatrix} Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}\hat{b}} \end{bmatrix}$$

given

- ambiguities (float)
- baseline parameters

Step 2

Ambiguity resolution:

$$\check{a} = Z^{-T} \mathcal{J} (Z^T \hat{a})$$

LAMBDA

Decorr.



I

IA

IE

Step 3

Fixed solution:

$$\check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} (\hat{a} - \check{a})$$

considering a success rate

$$\text{SR} \stackrel{\text{def}}{=} P(\check{a} = a) \approx 1$$

i.e. $Q_{\check{a}\check{a}} \approx 0$.

LAMBDA decomposition

D is diagonal, L is lower unitriangular

Adoption of LTDL factorization (i.e. last-to-first)

We compute $Q_{\hat{a}\hat{a}} = L^T D L$ for $L \in \mathbb{R}^{n \times n}$ and $D \in \mathbb{R}^{n \times n}$ with $d_i = \sigma_{i|i+1\dots n}^2$, so

$$L = \begin{bmatrix} 1 & & & \\ l_{21} & 1 & & \\ \vdots & \vdots & \ddots & \\ l_{n1} & l_{n2} & \cdots & 1 \end{bmatrix}, \quad D = \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_n \end{bmatrix}$$

In a block-wise form

$$Q_{\hat{a}\hat{a}} = \begin{bmatrix} Q_{11} & " \\ Q_{21} & Q_{22} \end{bmatrix} = \begin{bmatrix} L_{11} \\ L_Q & L_{22} \end{bmatrix}^T \begin{bmatrix} D_1 & \\ & D_2 \end{bmatrix} \begin{bmatrix} L_{11} \\ L_Q & L_{22} \end{bmatrix} = \begin{bmatrix} L_{11}^T D_1 L_{11} + L_Q^T D_2 L_Q & " \\ L_{22}^T D_2 L_Q & L_{22}^T D_2 L_{22} \end{bmatrix}$$

and the Schur complement follows

$$Q_{1|2} = Q_{11} - Q_{12} Q_{22}^{-1} Q_{21} \equiv L_{11}^T D_1 L_{11}$$

LAMBDA decorrelation

Admissible Z -transformation (i.e. integer unimodular matrix)

Given $z = Z^T a$, the matrix Z is admissible iff $Z \in \mathbb{Z}^{n \times n}$ and $|\det(Z)| = 1$, so it follows that we can consider Integer Gauss Transformations (IGTs):

$$Z \stackrel{\text{def}}{=} I_n + \alpha_{\text{IGT}} e_{i+1} e_i^T = \begin{bmatrix} I_{i-1} & & \\ & Z_{\text{IGT}} & \\ & & I_{n-i-1} \end{bmatrix}, \quad Z_{\text{IGT}} = \begin{bmatrix} 1 & \\ \alpha_{\text{IGT}} & 1 \end{bmatrix}, \quad \alpha_{\text{IGT}} \in \mathbb{Z}$$

such that

$$L' = LZ = \begin{bmatrix} L'_{11} & & \\ L'_{21} & L'_{22} & \\ L'_{31} & L'_{32} & L'_{33} \end{bmatrix}, \quad L'_{22} = \begin{bmatrix} 1 & \\ l_{i+1,i} + \alpha_{\text{IGT}} & 1 \end{bmatrix}, \quad L'_{32} = L_{32} Z_{\text{IGT}}$$

size reduction: $|l'_{i+1,i}| \leq 0.5$

 We alternate IGTs with ‘adjacent permutations’ for a Lovász-like condition:

$$\tilde{d}_{i+1} = d_i + (l'_{i+1,i})^2 d_{i+1} < d_{i+1} \cdot \delta, \quad \delta = 1 \text{ (LAMBDA)}$$

Vectorial Decorrelation (VD) principles

From scalar to vectorial formulation

Generalized IGTs

We consider a conformal block partitioning of the Z -transformation matrix, i.e.

$$Z = \begin{bmatrix} Z_{11} & \textcolor{red}{Z}_{12} \\ \textcolor{green}{Z}_{21} & Z_{22} \end{bmatrix} \in \mathbb{Z}^{(n_1+n_2) \times (n_1+n_2)}$$

where – via Schur's determinant identity – we require

$$|\det(Z)| \equiv |\det(Z_{22}) \cdot \det(Z_{11} - \textcolor{red}{Z}_{12} Z_{22}^{-1} \textcolor{green}{Z}_{21})| = 1$$

Assuming $\textcolor{red}{Z}_{12} = 0$, we obtain $|\det(Z)| = |\det(Z_{22}) \cdot \det(Z_{11})| = 1$, with

$$Z = \begin{bmatrix} Z_{11} & 0 \\ \textcolor{green}{Z}_{21} & Z_{22} \end{bmatrix} \rightarrow Z^{-1} = \begin{bmatrix} Z_{11}^{-1} & 0 \\ -Z_{22}^{-1} \textcolor{green}{Z}_{21} Z_{11}^{-1} & Z_{22}^{-1} \end{bmatrix}$$

so admissible iff $Z_{11} \in \mathbb{Z}^{n_1 \times n_1}$ and $Z_{22} \in \mathbb{Z}^{n_2 \times n_2}$ are also admissible, but...

From scalar to vectorial formulation

Key principle

Starting with this lower triangular block matrix Z , we compute $\tilde{Q} = Z^T Q Z$ as

$$\begin{bmatrix} \tilde{Q}_{11} & " \\ \tilde{Q}_{21} & \tilde{Q}_{22} \end{bmatrix} = \begin{bmatrix} Z_{11}^T & Z_{21}^T \\ & Z_{22}^T \end{bmatrix} \begin{bmatrix} Q_{11} & " \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} Z_{11} & \\ Z_{21} & Z_{22} \end{bmatrix}$$

then

$$\tilde{Q}_{11} \stackrel{\text{def}}{=} \tilde{Q}_{1|2} + \tilde{Q}_{21}^T \tilde{Q}_{22}^{-1} \tilde{Q}_{21} \quad \rightarrow \quad \begin{cases} \tilde{Q}_{1|2} = Z_{11}^T Q_{1|2} Z_{11} \\ \tilde{Q}_{21} = Z_{22}^T Q_{22} (Z_{21} + Q_{22}^{-1} Q_{21} Z_{11}) \\ \tilde{Q}_{22} = Z_{22}^T Q_{22} Z_{22} \end{cases}$$

Assuming $Z_{11} = I_{n_1}$, we seek $\tilde{Q}_{11} \stackrel{?}{\leq} Q_{11}$ such that

$$\text{TU Delft} \quad Q_{1|2} \leq \tilde{Q}_{11} \stackrel{\text{def}}{=} Q_{1|2} + \underbrace{(Z_{21} + Q_{22}^{-1} Q_{21})^T Q_{22} (Z_{21} + Q_{22}^{-1} Q_{21})}_{\text{?}} \stackrel{?}{\leq} Q_{11}$$

Vectorial Decorrelation problem

Mathematical formulation

A natural optimization problem follows

$$Z_{21}^* = \arg \min_{Z_{21} \in \mathbb{Z}^{n_2 \times n_1}} \text{tr}[(Z_{21} + Q_{22}^{-1} Q_{21})^T Q_{22} (Z_{21} + Q_{22}^{-1} Q_{21})]$$

or equivalently ‘column-wise’ we can write n_1 separated ILS problems

$$z_j^* = \arg \min_{z \in \mathbb{Z}^{n_2}} \|z - \hat{z}_j\|_{Q_{22}^{-1}}^2, \quad \hat{z}_{21} = -[Q_{22}^{-1} Q_{21}]_j \equiv -[L_{22}^{-1} L_Q]_j$$

Potential solutions:

- Integer Rounding: $Z_{21}^{\text{IR}} = -[L_{22}^{-1} L_Q]$ is not much effective...
- Integer Bootstrapping: $Z_{21}^{\text{IB}} =$ 
- Integer Least-Squares: Z_{21}^{ILS} optimum, but potentially very costly!

Reverse Vectorial Decorrelation

Construction RVD matrix

- Initialize $Z_{\text{RVD}} = I_n$
- For $s = 2, \dots, n$

$$Z_{\text{RVD}}(s, 1:s-1) = -[L(s, 1:s-1) Z_{\text{RVD}}(1:s-1, 1:s-1)]$$
- Return $Z_{\text{RVD}} \in \mathbb{Z}^{n \times n}$

$$Z_{11}^{\{i-1\}} = \begin{bmatrix} Z_{11}^{\{i\}} \\ Z_{21}^{\{i\}} & 1 \end{bmatrix}, \quad Z_{21}^{\{i\}} = -[L_Q^{\{i\}} Z_{11}^{\{i\}}]$$

Advantage

- $L' = LZ_{\text{RVD}}$ is bounded!

Disadvantage

- $D' = D$ is not affected...

1	0	0	0	0	0	0	0
$Z_{21}^{\{7\}}$	1	0	0	0	0	0	0
$Z_{21}^{\{6\}}$		1	0	0	0	0	0
$Z_{21}^{\{5\}}$			1	0	0	0	0
$Z_{21}^{\{4\}}$				1	0	0	0
$Z_{21}^{\{3\}}$					1	0	0
$Z_{21}^{\{2\}}$						1	0
$Z_{21}^{\{1\}}$							1

1	0	0	0	0	0	0	0
l_{21}	1	0	0	0	0	0	0
l_{31}	l_{32}	1	0	0	0	0	0
l_{41}	l_{42}	l_{43}	1	0	0	0	0
l_{51}	l_{52}	l_{53}	l_{54}	1	0	0	0
l_{61}	l_{62}	l_{63}	l_{64}	l_{65}	1	0	0
l_{71}	l_{72}	l_{73}	l_{74}	l_{75}	l_{76}	1	0
l_{81}	l_{82}	l_{83}	l_{84}	l_{85}	l_{86}	l_{87}	1

NOTE: it can be constructed via IGTs.

‘Targeted Insertions’ & implementations

Adjacent 'block' permutations

General formulation

We consider a permutation matrix $P^T \in \mathbb{Z}^{n \times n}$ leading to $\tilde{Q} = P^T Q P$, such that

$$P^T = \begin{bmatrix} I_s & & \\ & R^T & \\ & & I_q \end{bmatrix}, \quad R^T = \begin{bmatrix} & I_{t_{II}} \\ I_{t_I} & \end{bmatrix} \equiv R^{-1}$$

After some manipulations we obtain:

$$[\tilde{L}_{42} \ \tilde{L}_{43}] = [L_{43} \ L_{42}]$$

$$\begin{aligned} \begin{bmatrix} \tilde{L}_{21} \\ \tilde{L}_{31} \end{bmatrix} &= \begin{bmatrix} \tilde{L}_{22} & \tilde{L}_{32} \\ \tilde{L}_{32} & \tilde{L}_{33} \end{bmatrix} R^T \begin{bmatrix} L_{22} \\ L_{32} L_{33} \end{bmatrix}^{-1} \begin{bmatrix} L_{21} \\ L_{31} \end{bmatrix} \\ &= \begin{bmatrix} \tilde{L}_{22} & \tilde{L}_{32} \\ \tilde{L}_{32} & \tilde{L}_{33} \end{bmatrix} \begin{bmatrix} -L_{33}^{-1} L_{32} L_{22}^{-1} & L_{33}^{-1} \\ L_{22}^{-1} & \end{bmatrix} \begin{bmatrix} L_{21} \\ L_{31} \end{bmatrix} \\ &= \dots \end{aligned}$$

$$\begin{aligned} L &= \begin{bmatrix} L_{11} & & & \\ L_{21} & L_{22} & & \\ L_{31} & L_{32} & L_{33} & \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix}, D = \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & D_3 & \\ & & & D_4 \end{bmatrix} \\ &\quad \downarrow \\ \tilde{L} &= \begin{bmatrix} \tilde{L}_{11} & & & \\ \tilde{L}_{21} & \tilde{L}_{22} & & \\ \tilde{L}_{31} & \tilde{L}_{32} & \tilde{L}_{33} & \\ \tilde{L}_{41} & \tilde{L}_{42} & \tilde{L}_{43} & \tilde{L}_{44} \end{bmatrix}, \tilde{D} = \begin{bmatrix} \tilde{D}_1 & & & \\ & \tilde{D}_2 & & \\ & & \tilde{D}_3 & \\ & & & \tilde{D}_4 \end{bmatrix} \end{aligned}$$

Adjacent ‘block’ permutations

Targeted Insertion (TI)

For the general case $t_I \geq 1, t_{II} \geq 1$ we need to compute

$$\begin{cases} \tilde{D}_3, \tilde{L}_{33} \xleftarrow{L^T D L} L_{22}^T D_2 L_{22} + L_{32}^T D_3 L_{32} \\ \tilde{L}_{32} = \tilde{D}_3^{-1} \tilde{L}_{33}^{-1} L_{32}^T D_3 L_{33} \\ \tilde{D}_2, \tilde{L}_{22} \xleftarrow{L^T D L} L_{33}^T D_3 L_{33} - \tilde{L}_{32}^T \tilde{D}_3 \tilde{L}_{32} \end{cases}$$

For the simple case $t_I = 1$, given any $t_{II} \geq 1$, then

$$\begin{cases} \tilde{d}_3 = d_2 + \ell_{32}^T D_3 \ell_{32} \\ \tilde{\ell}_{32}^T = \tilde{d}_3^{-1} \ell_{32}^T D_3 L_{33} \\ \tilde{D}_2, \tilde{L}_{22} \xleftarrow{L^T D L} L_{33}^T D_3 L_{33} - \tilde{d}_3 \tilde{\ell}_{32} \tilde{\ell}_{32}^T \end{cases} \quad (\text{via a rank-one downdate})$$

Known as ‘deep insertions’

Schnorr, C.P. & Euchner, M., 1994.

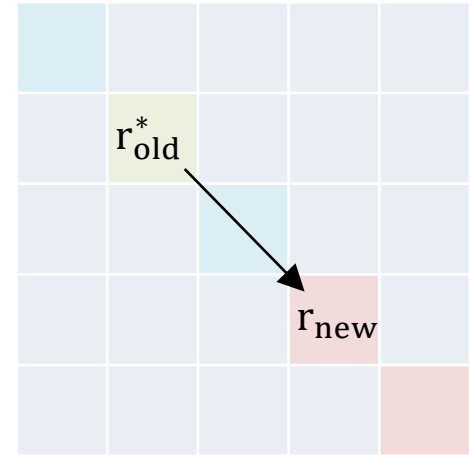
Lattice basis reduction: improved practical algorithms and solving subset sum problems. Mathematical Programming, 66 (1–3), 181–199

TI algorithmic implementation

TI-based decorrelation

Define a target $\bar{g}$ for the conditional std.dev., then

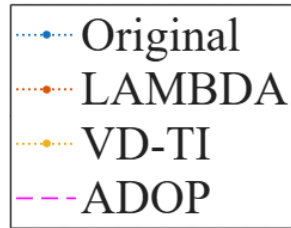
- For $r_{\text{new}} = n, n-1, \dots, 2$
 - For $r_{\text{old}} = 1, \dots, r_{\text{new}}$
 - $g(r_{\text{old}}) = \sum_{r=r_{\text{old}}}^{r=r_{\text{new}}} D(r) L(r, r_{\text{old}})^2$
 - End
 - Compute $r_{\text{old}}^* = \arg \min_{r \leq r_{\text{new}}} [\sqrt{g(r)} - \bar{g}]$
 - If $r_{\text{old}}^* \neq r_{\text{new}}$
 - **TI-algorithm** | Update L, D matrices after moving r_{old}^* into r_{new} .
 - **RVD-transform** | Size reduction for L matrix, also possible via IGTs.
 - End
- End



TI algorithmic example #1

See **LAMBDA 4.0**
(GPS, GF-3f, 5 Rx)

Case $\bar{\mathbf{g}} = \mathbf{0}$ | Aims at smallest conditional std.dev. (last-to-first)

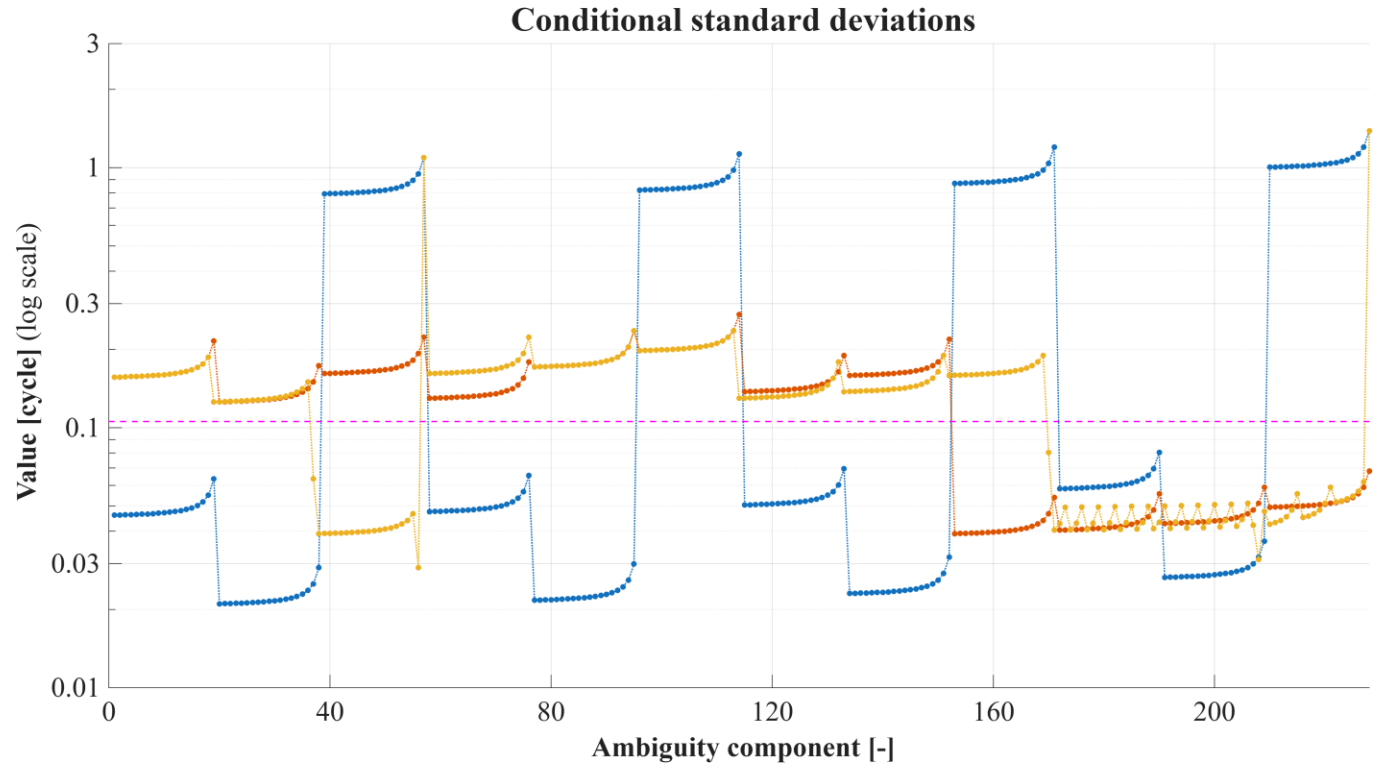


SR ~ 0%

SR = 47.99%

SR = 5.36%

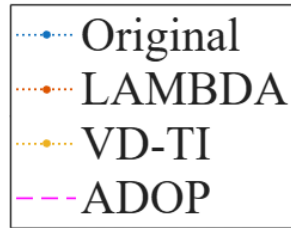
SR = 99.95%



TI algorithmic example #1

See **LAMBDA 4.0**
(GPS, GF-3f, 5 Rx)

Case $\bar{\mathbf{g}} = \mathbf{0}$ | Aims at smallest conditional std.dev. (last-to-first)

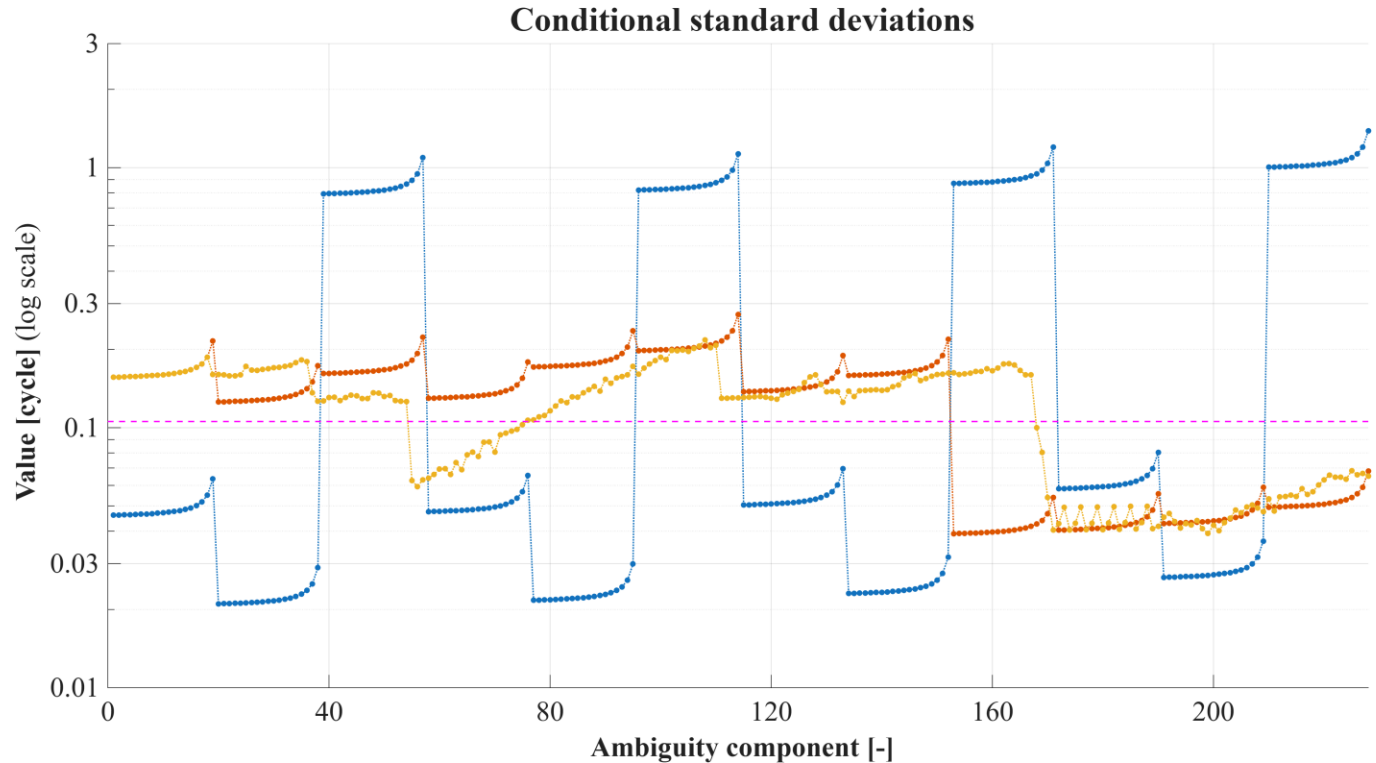


SR $\sim$ 0%

SR = 47.99%

SR = 75.31%

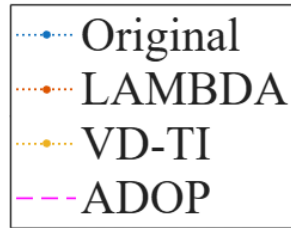
SR = 99.95%



TI algorithmic example #2

See **LAMBDA 4.0**
(GPS, GF-3f, 5 Rx)

Case $\bar{g} = \text{ADOP}$ | Aims at conditional std.dev. (for FAR)

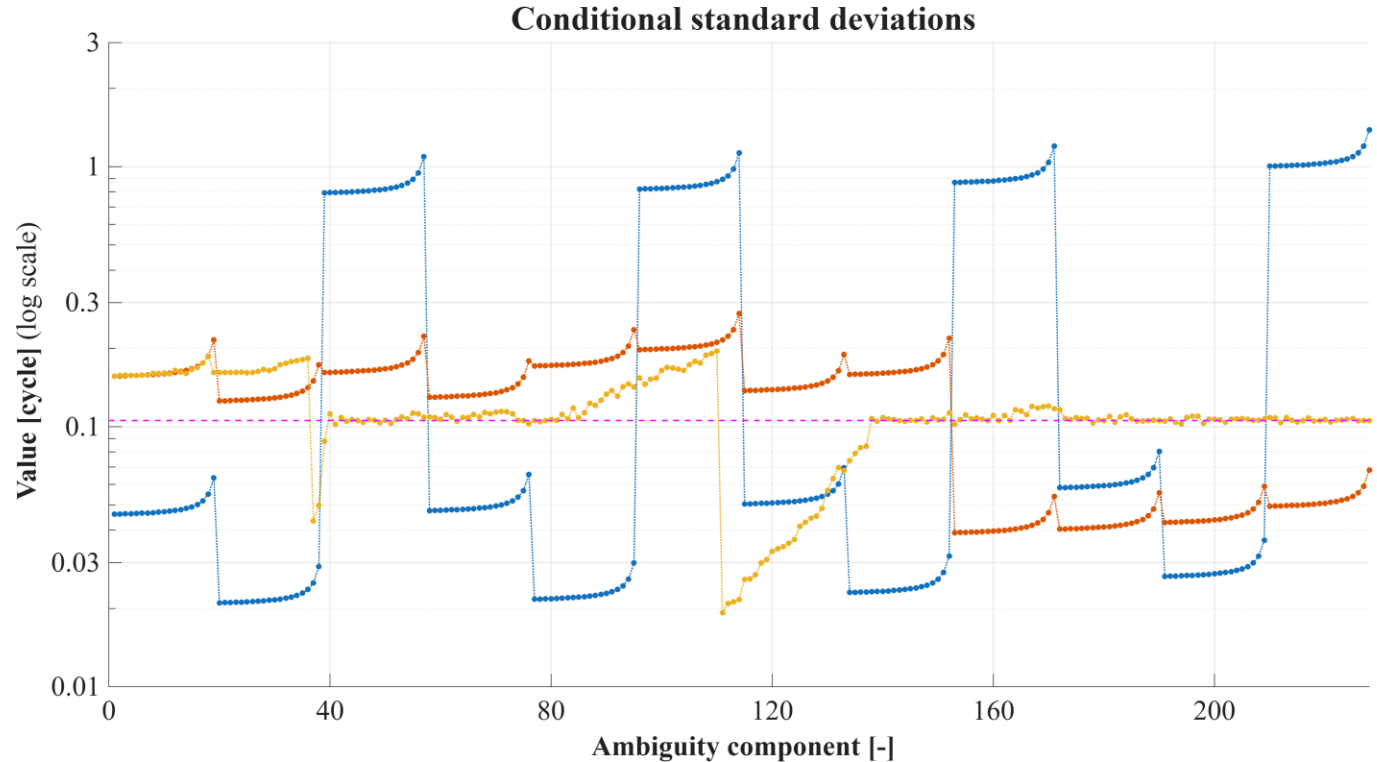


SR $\sim$ 0%

SR = 47.99%

SR = 85.26%

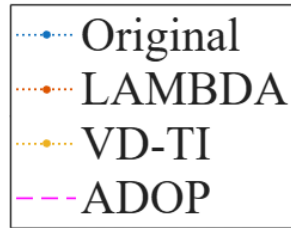
SR = 99.95%



TI algorithmic example #2b

See **LAMBDA 4.0**
(GPS, GF-3f, 5 Rx)

Case $\bar{g} = 0.15$ cycles | Aims at conditional std.dev. (for PAR)

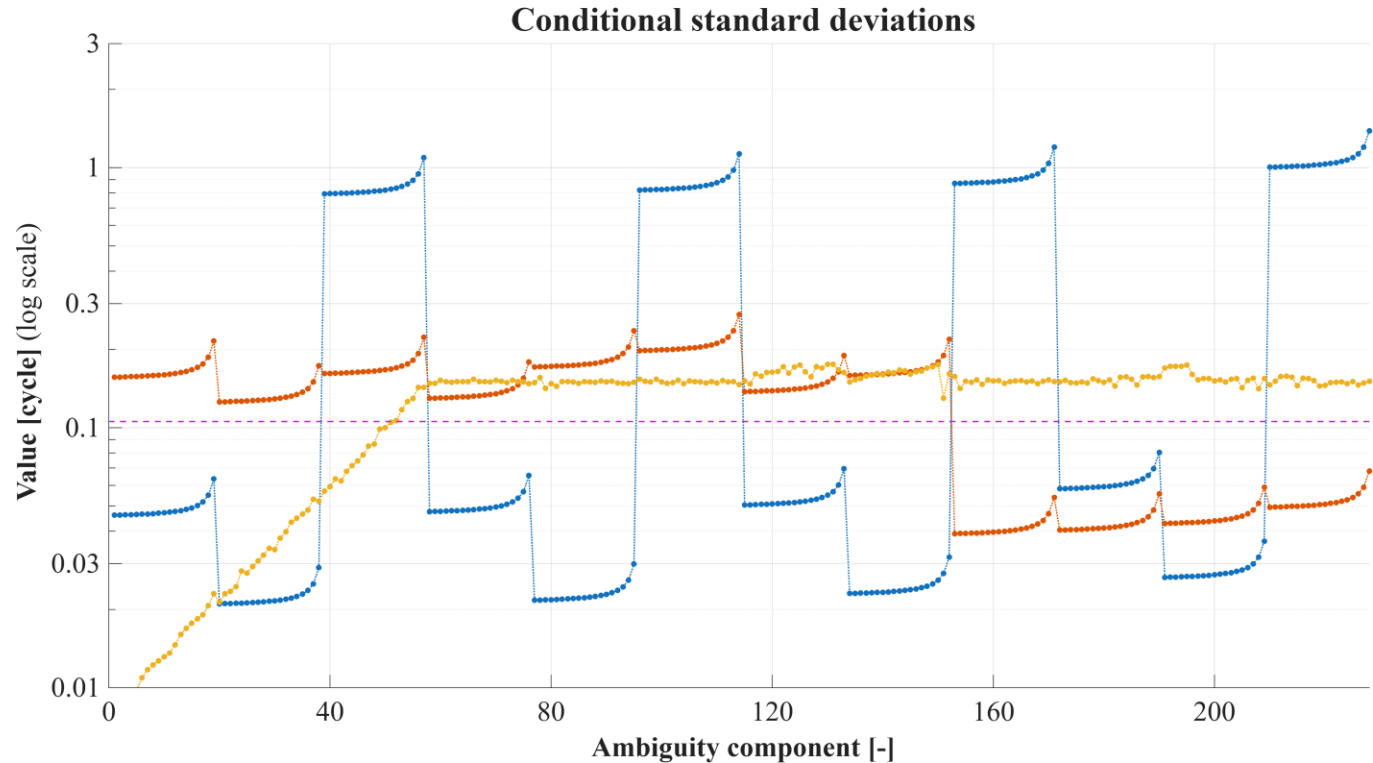


SR $\sim$ 0%

SR = 47.99%

SR = 79.70%

SR = 99.95%



Further considerations

Possible extension into parameter domain

Starting with the following LTDL-decomposition

$$\begin{bmatrix} Q_{\hat{b}\hat{b}} & " & " \\ Q_{\hat{b}1} & Q_{11} & " \\ Q_{\hat{b}1} & Q_{21} & Q_{22} \end{bmatrix} = \begin{bmatrix} L_{\hat{b}\hat{b}} \\ L_{1\hat{b}}L_{11} \\ L_{2\hat{b}}L_{21}L_{22} \end{bmatrix}^T \begin{bmatrix} D_{\hat{b}} & & \\ & D_1 & \\ & & D_2 \end{bmatrix} \begin{bmatrix} L_{\hat{b}\hat{b}} \\ L_{1\hat{b}}L_{11} \\ L_{2\hat{b}}L_{21}L_{22} \end{bmatrix}$$

then

$$\hat{b}(\check{a}_2) = \hat{b} - L_{2\hat{b}}^T L_{22}^{-T} (\hat{a}_2 - \check{a}_2), \quad Q_{\hat{b}|2} = L_{\hat{b}\hat{b}}^T D_{\hat{b}} L_{\hat{b}\hat{b}} + L_{1\hat{b}}^T D_1 L_{1\hat{b}}$$

The **precision improvement** after fixing a_2 is

$$\Delta Q_{\hat{b}|2} \stackrel{\text{def}}{=} Q_{\hat{b}\hat{b}} - Q_{\hat{b}|2} = L_{2\hat{b}}^T D_2 L_{2\hat{b}}$$

subject to

$$P_{S,IB} \stackrel{\text{def}}{=} P(\check{a}_2 = a_2) \geq P_0$$



Conclusions

Summary

- ❑ In this work we *revisit* ‘**Vectorial Decorrelation**’ formulations for Integer Ambiguity Resolution (IAR).
- ❑ While the size-reduction of L is trivially achieved by different principles, an **efficient** flattening of D entries is the issue.
- ❑ Still, TIs or ‘deep insertions’ present one very practical way to tackle the problem under potentially new different criteria.

Some relevant literature

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A satellite with large solar panels is shown in space, with the Earth visible in the background. The satellite is oriented diagonally, with its solar panels extended. The Earth shows the continents of Europe and Africa, with clouds visible over the oceans.

Questions?

Thanks for your attention

Courtesy of ASI (Agenzia Spaziale Italiana)