



Università
di Genova

hotinemarussi2026
XI Hotine-Marussi Symposium

Hierarchical Ambiguity Resolution approach using Vectorial Integer Bootstrapping for multi-GNSS PPP-RTK networks



Yishi Wang¹, Lotfi Massarweh², Pengyu Hou¹, Baocheng Zhang¹

¹Innovation Academy for Precision Measurement Science and Technology, Wuhan, China

² Department of Geoscience and Remote Sensing, Delft University of Technology, Delft,
The Netherlands

September 2026 Italy



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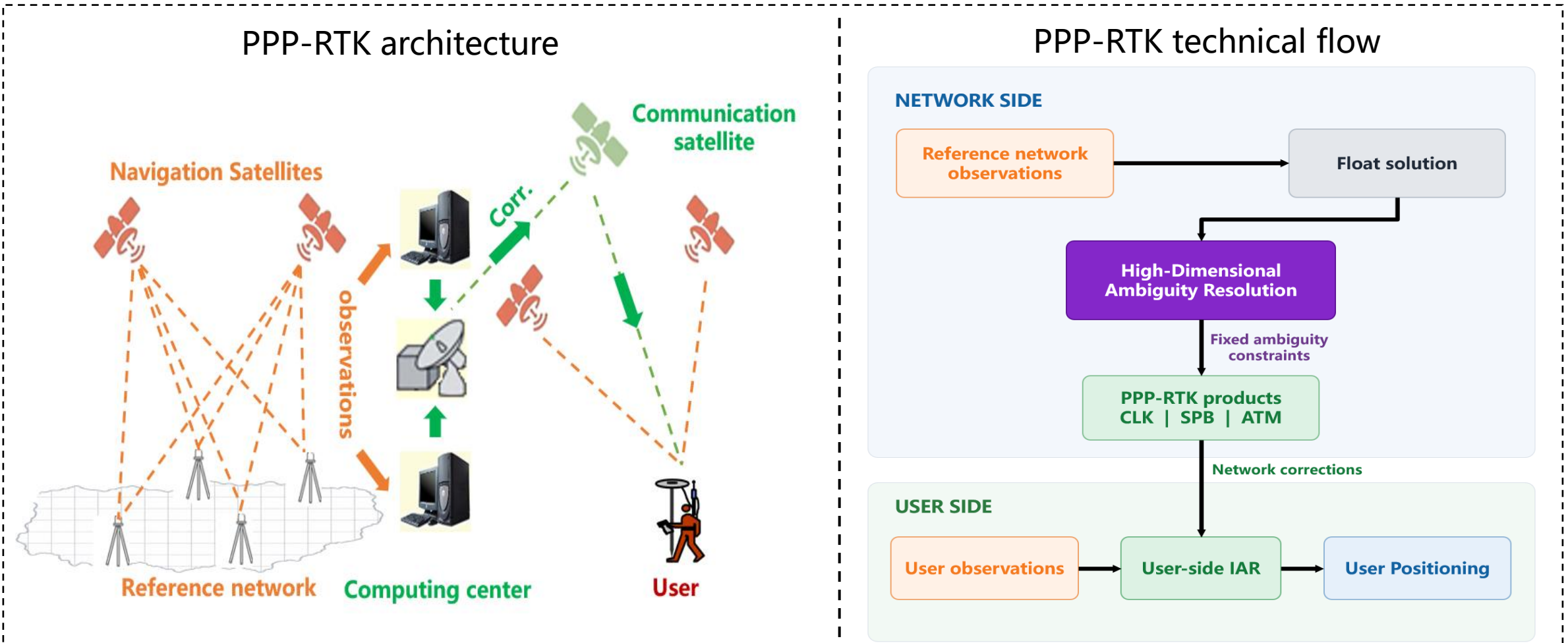
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Introduction

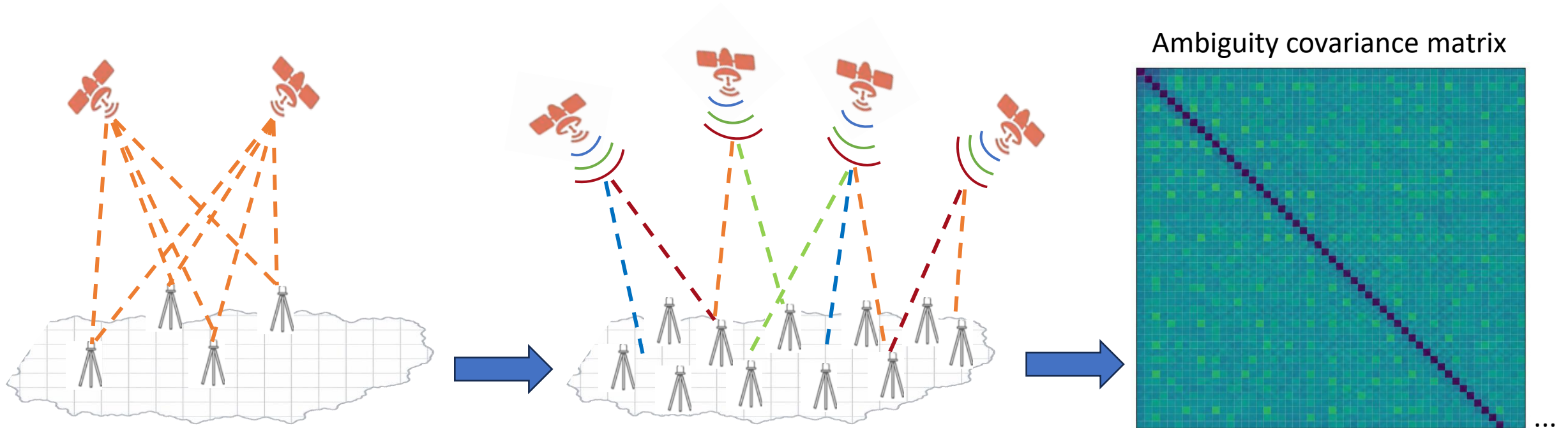
■ Integer-ambiguity resolution-enabled PPP (Wübbena et al. 2005 & Teunissen 2020)



Usually, Network-side ambiguity fixing directly affects the quality of the products.

Introduction

- The number of ambiguities keeps increasing with multi-GNSS/multi-frequency data



- Multi-GNSS and multi-frequency observations rapidly increase the ambiguity dimension
- High-dimensional ambiguity resolution (HDAR) bottlenecks in real-time network processing

Introduction

■ Commonly Used Ambiguity Estimators

Estimator	Advantage	Limitation
IR	Computationally efficient	Neglect ambiguity correlations
IB	Fast sequential estimation	Conditional wrong fixes propagate
ILS-FAR	Optimal (Success Rate)	Search cost grows rapidly with dimension
ILS-PAR	High SR for subset	Discards remaining integer information
BIE	Optimal (Mean Squared Error)	Prohibitive cost for high dimensions
RT-based IA	Rejects unreliable fixes	No failure-rate control
FFRT-based IA	Controls failure rate	Model-dependent threshold

Challenging to balance efficiency and accuracy





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Methods

Vectorial Integer Bootstrapping framework (Teunissen et al. 2021)

1

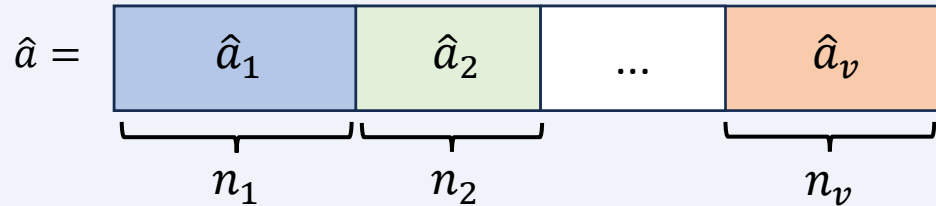
Float ambiguity solution

$$\hat{a} \in \mathbb{R}^n \quad Q_{\hat{a}\hat{a}} \in \mathbb{R}^n$$

2

Partition into blocks_v

$$\hat{a} = (\hat{a}_1^T, \dots, \hat{a}_v^T)^T \quad n = \sum_{i=1}^v n_i$$



3

Sequential conditional estimation

$$\hat{a}_{i|I} = \hat{a}_i - \sum_{j=1}^{i-1} Q_{ij|I} Q_{jj|I}^{-1} (\hat{a}_{j|I} - \check{a}_j)$$

4

Block-wise integer estimation

$$\check{a}_i = J_i(\hat{a}_{i|I}), \quad J_i: \mathbb{R}^n \mapsto \mathbb{Z}^n$$

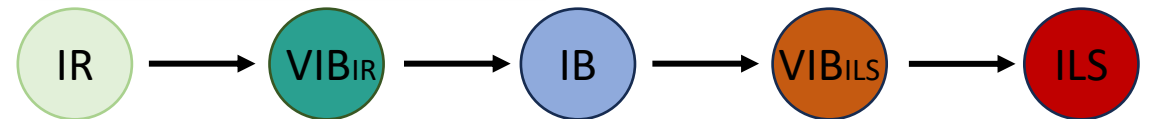
estimators can be chosen block by block

5

VIB fixed ambiguities

$$\check{a}_{VIB} = (\check{a}_1^T, \dots, \check{a}_v^T)^T$$

Performance

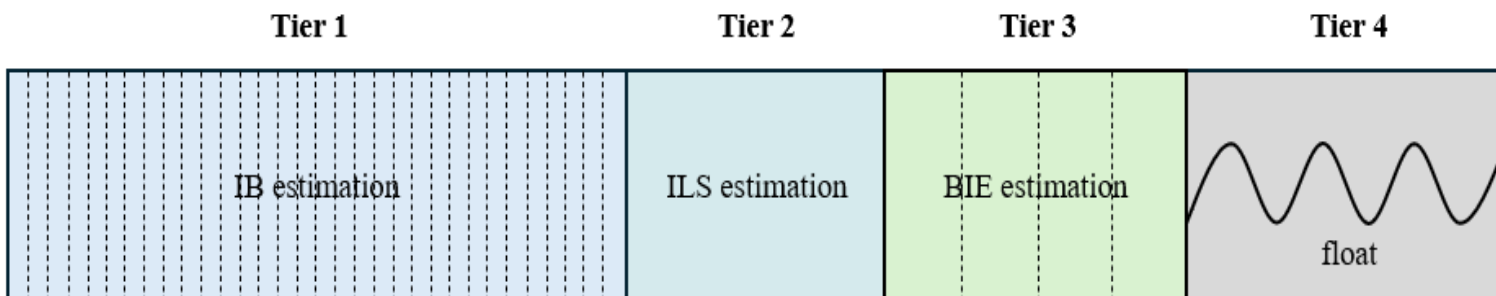
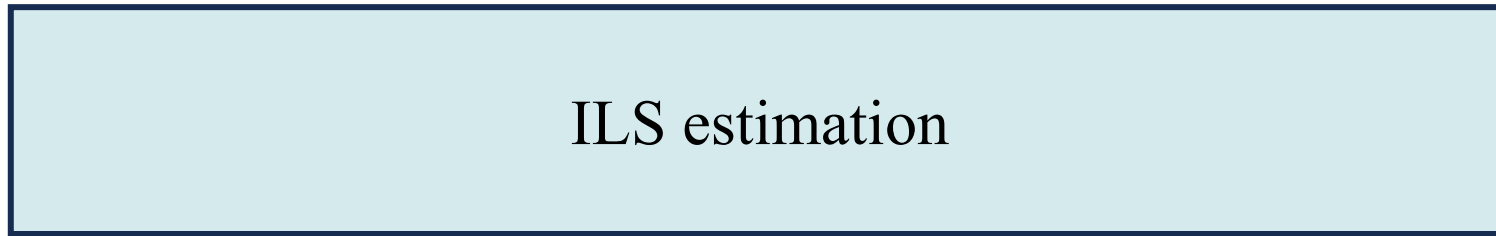


Success rate increases

Computational efficiency increases

Methods

Hierarchical Ambiguity Resolution



(Dashed lines: sub-block boundaries)

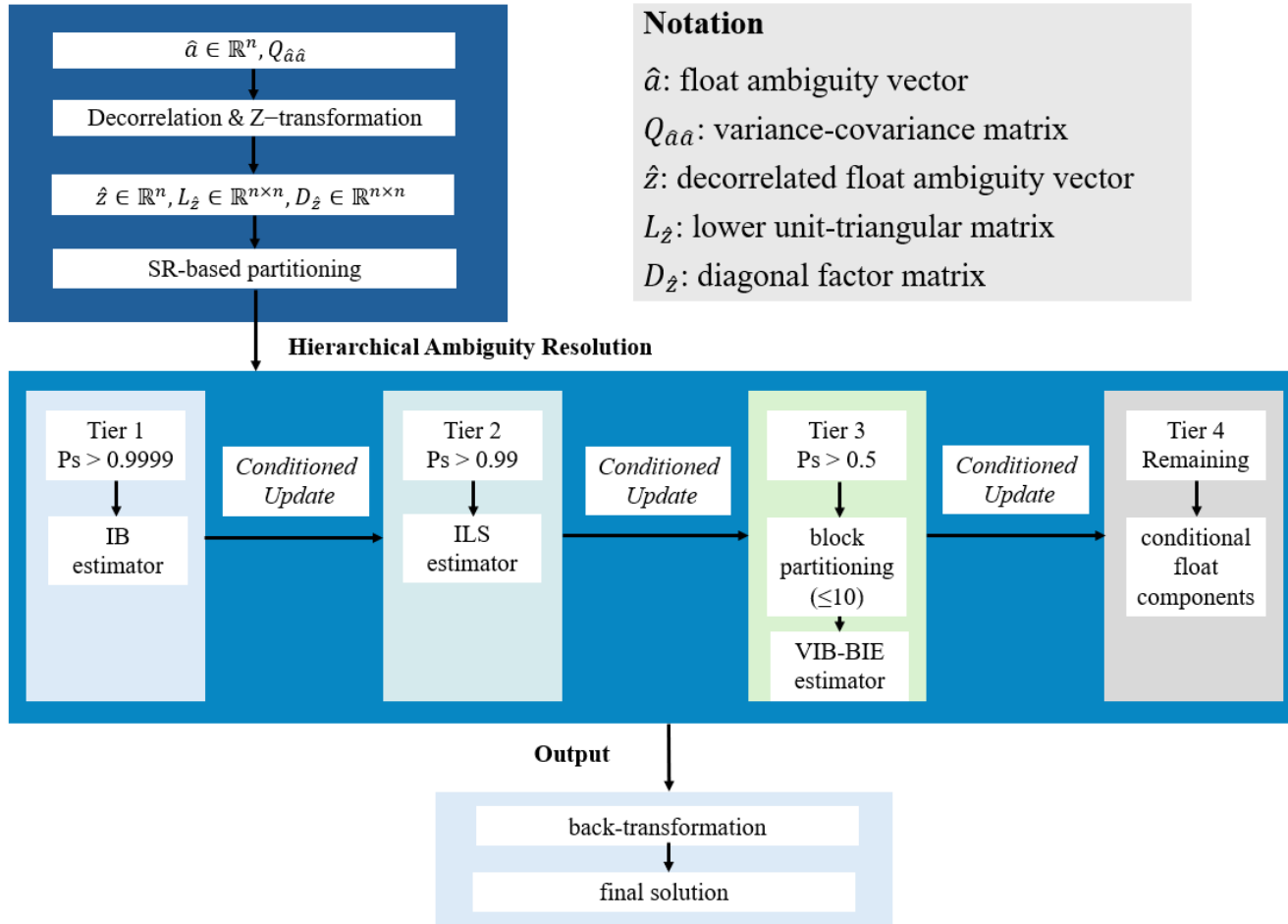
- One estimator for the full ambiguity vector

- Different (classes of) estimators for subsets with different precision levels

IA estimators are not considered here, as their inclusion would require redesigning the sequential conditional estimation framework.

Methods

VIB-HAR Workflow



$$P_s \stackrel{\text{def}}{=} P(\check{z} = z)$$

$$P_s^{\text{IB}}(\hat{z}) = \prod_{i=1}^n \left[2\Phi\left(\frac{1}{2\sigma_{\hat{z}_{i|I}}}\right) - 1 \right]$$

$$P_k^{\text{IB}} \stackrel{\text{def}}{=} P_s^{\text{IB}}(\hat{z}_{\Omega_k}), \quad k = 1, 2, 3, 4$$

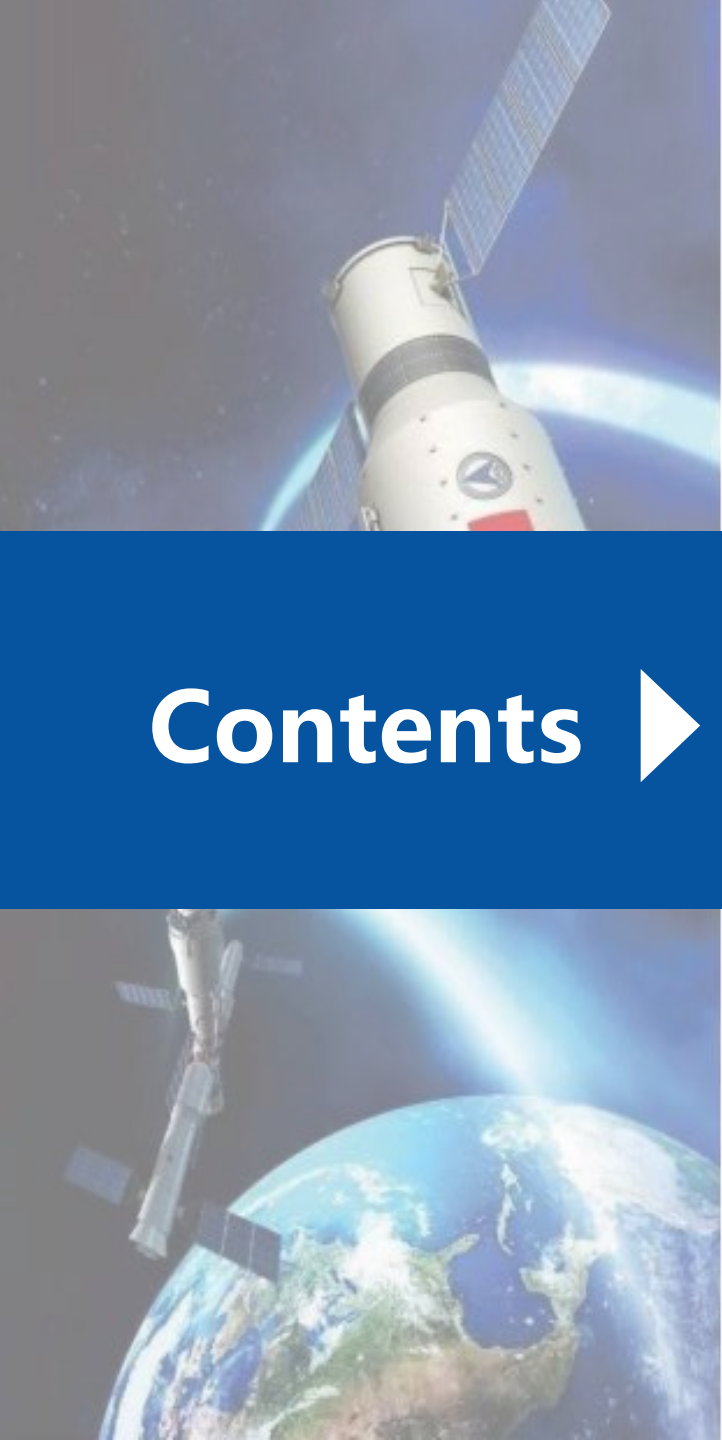
$$\Omega_1 = \{1, \dots, n_1\}, \quad P_1^{\text{IB}} \geq P_{\text{th},1},$$

$$\Omega_2 = \{n_1 + 1, \dots, n_2\}, \quad P_2^{\text{IB}} \geq P_{\text{th},2},$$

$$\Omega_3 = \{n_2 + 1, \dots, n_3\}, \quad P_3^{\text{IB}} \geq P_{\text{th},3},$$

$$\Omega_4 = \{n_3 + 1, \dots, n\}.$$

The thresholds are flexible and can be adapted to different applications

A vertical banner on the left side of the slide. The top half shows a close-up of a space shuttle launch with a large plume of fire and smoke. The bottom half shows a satellite in orbit above the Earth, with the planet's curvature and clouds visible.

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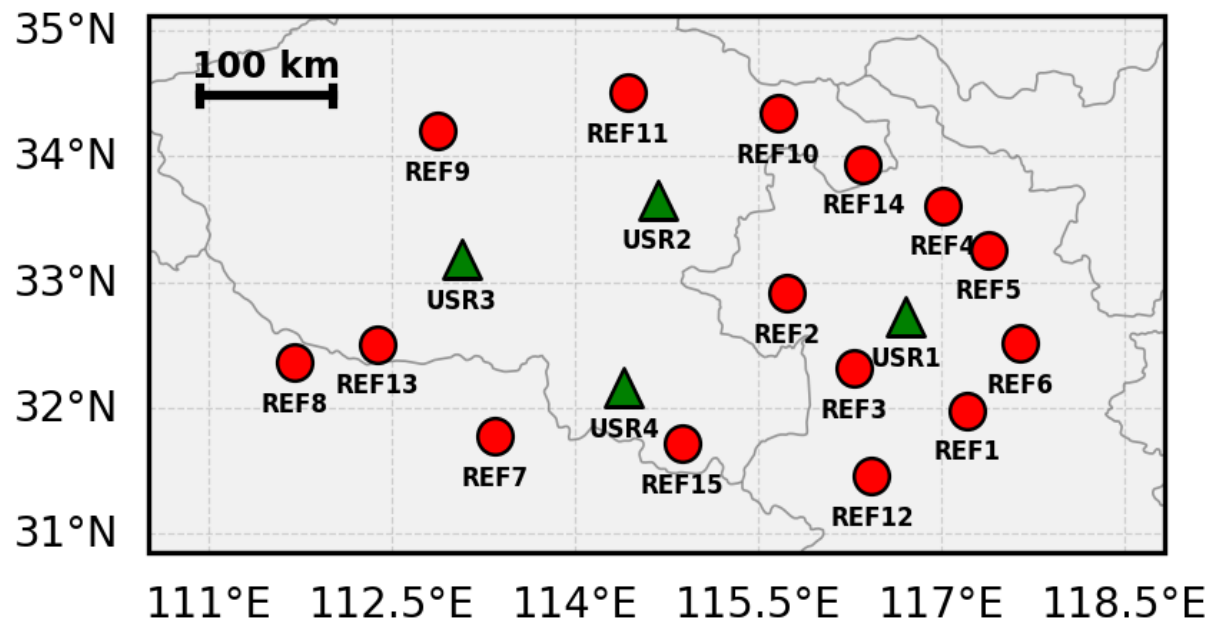
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Experiments

■ Data and experiment design



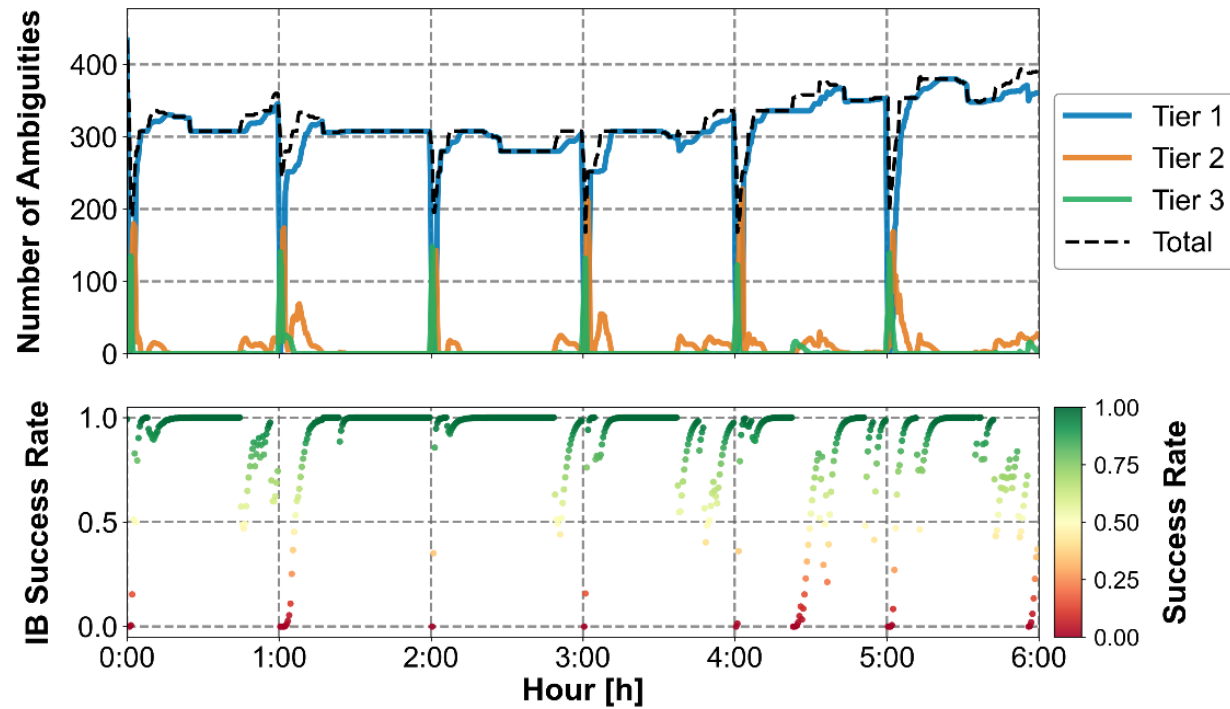
Distribution of network and user stations

- 19 stations, one-week data (DOY210-216)
 - Network side: 15
 - User side: 4
 - Sampling rate: 30 s / 1 s
 - Observations: G/E/C dual-frequency
 - re-start every hour
- Network-side experiment groups:
 - **Float**
 - **IR and IB**
 - **ILS-FAR & ILS-PAR** (with 99% SR threshold)
 - **VIB-HAR**

Results

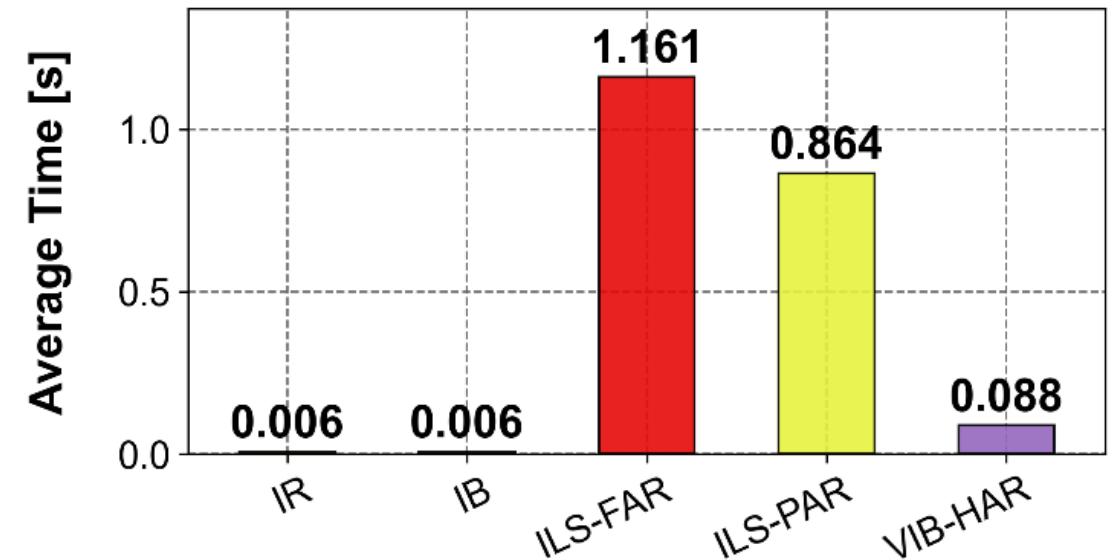
■ Network computational efficiency

Ambiguities dimensionality



~350 dimensions

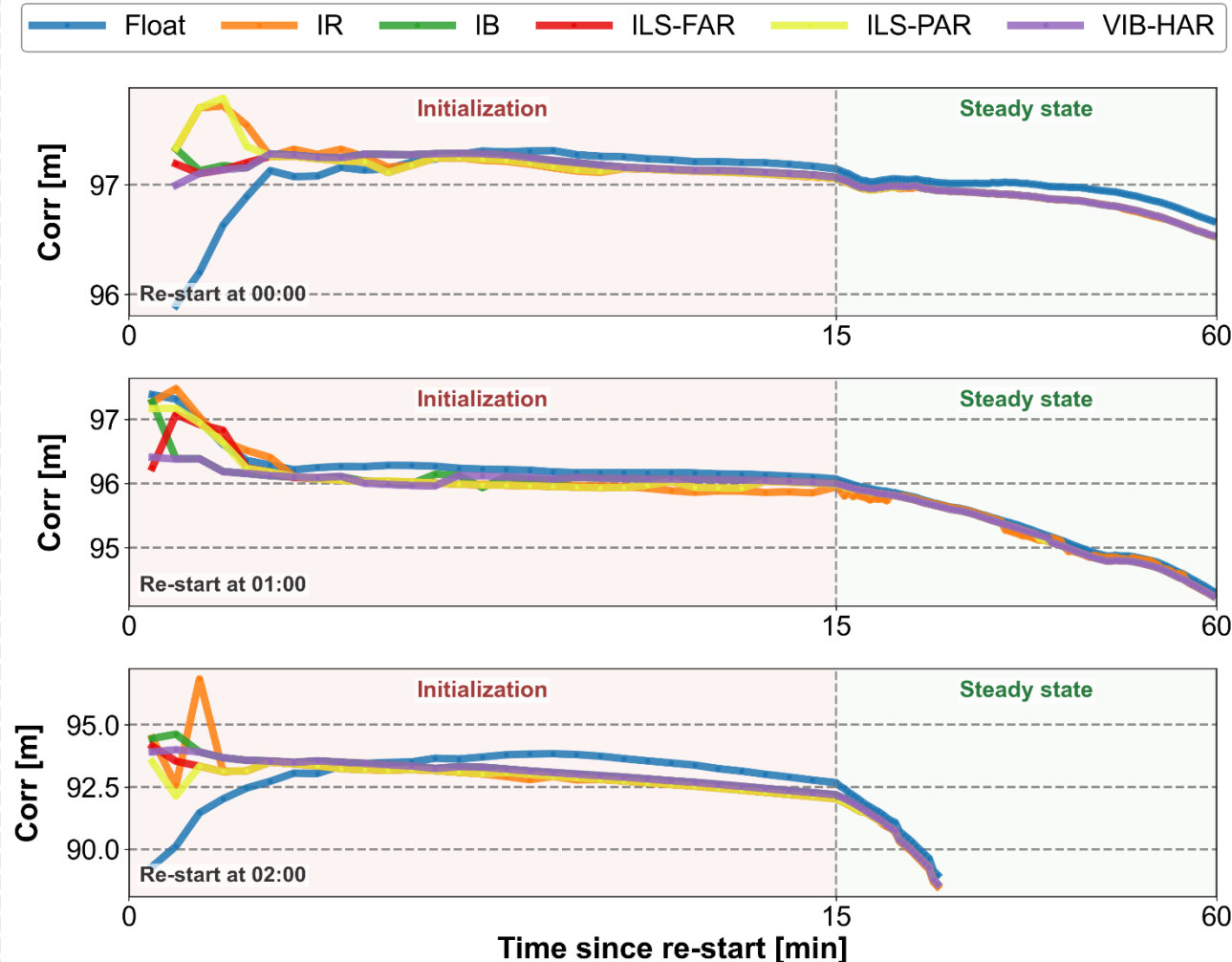
Consumed time



~13× faster than ILS-FAR

Results

■ Network products



AR methods differ mainly during filter initialization

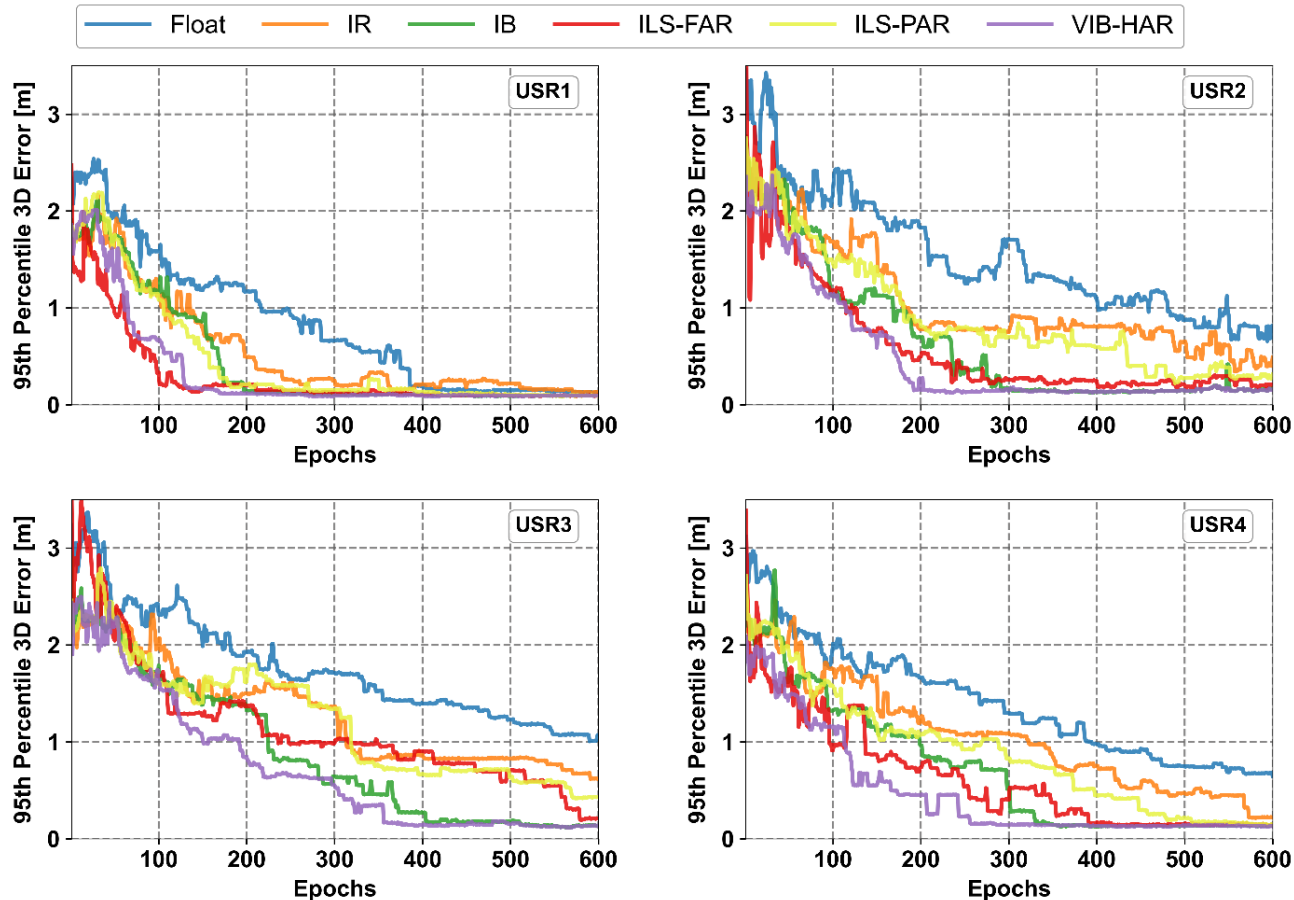
■ Hourly re-starts: 168 one-hour windows over 7 days

■ Initial product differences reach the dm-to-m level

■ The combined corrections become consistent once the network filter reaches a steady state

Results

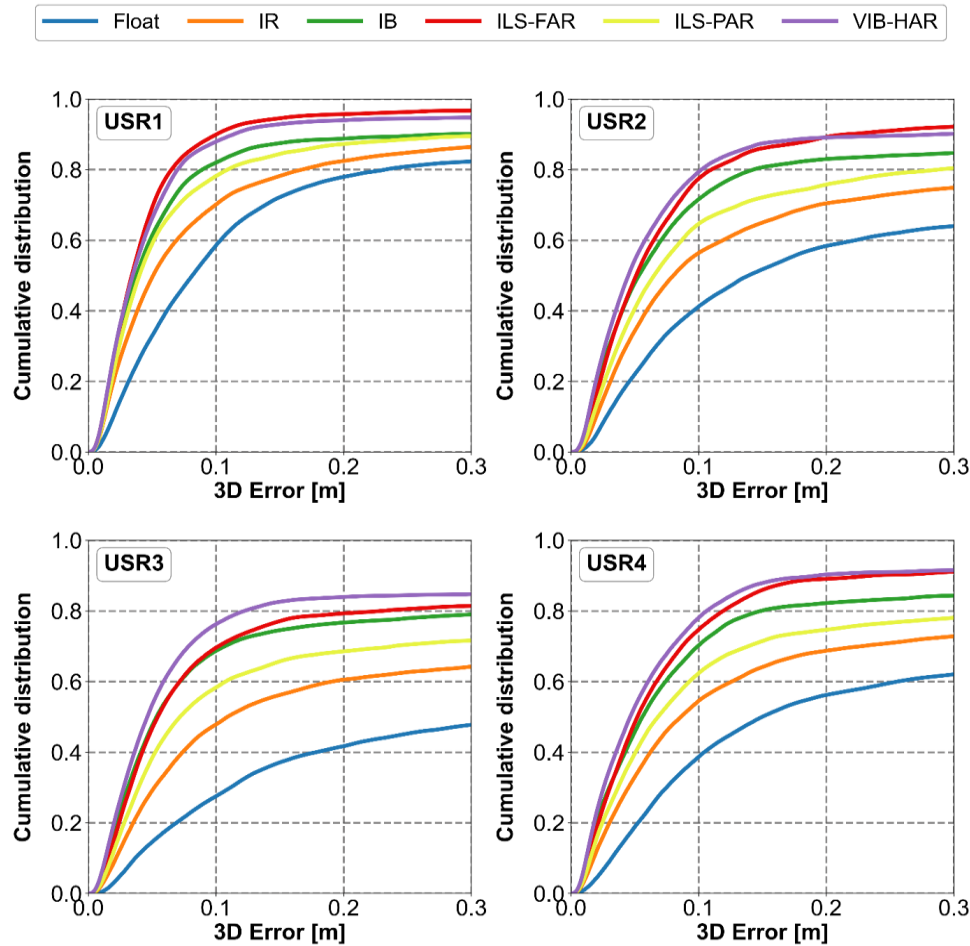
■ User positioning: convergence time (95-percentile)



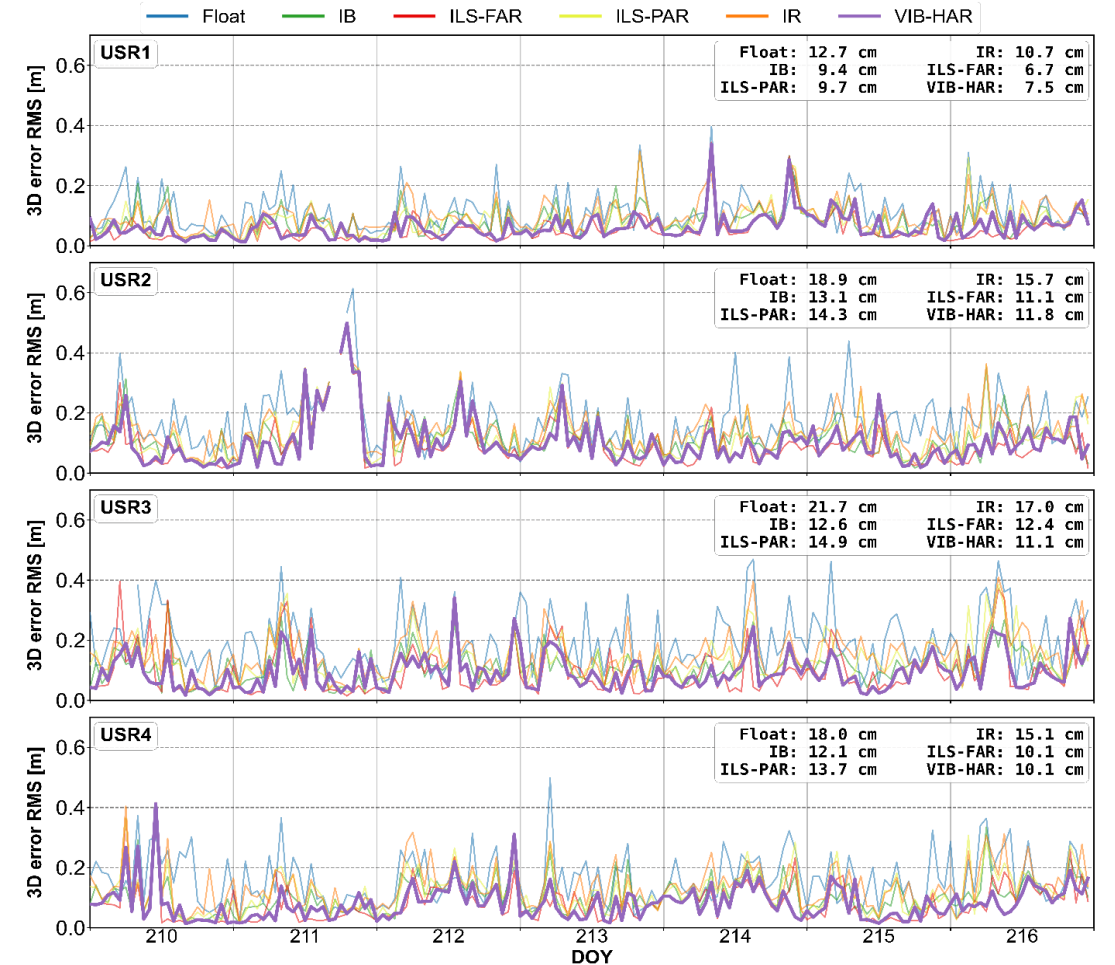
- Hourly re-start: 168 samples over 7 days
- Convergence is achieved when the 3D error remains below 10 cm for at least 10 min
- Achieves, in average, the shortest convergence time of around 3 minutes

Results

■ User positioning: precision



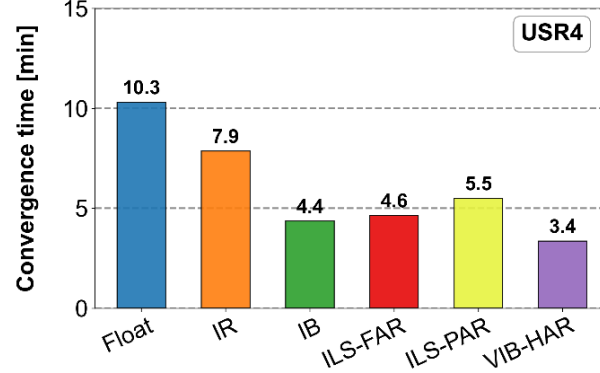
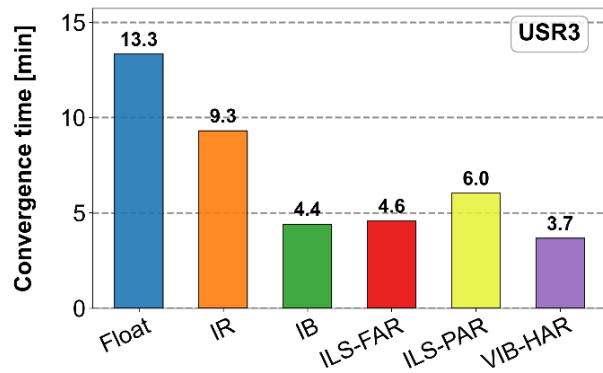
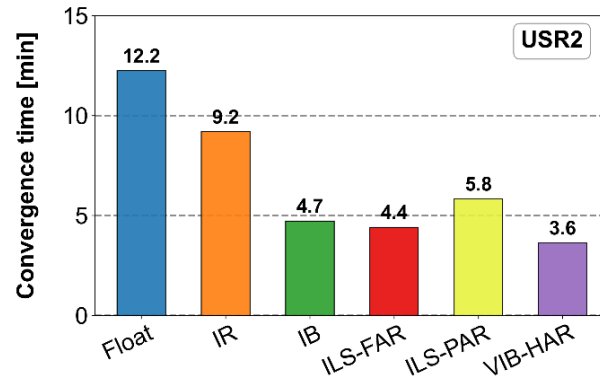
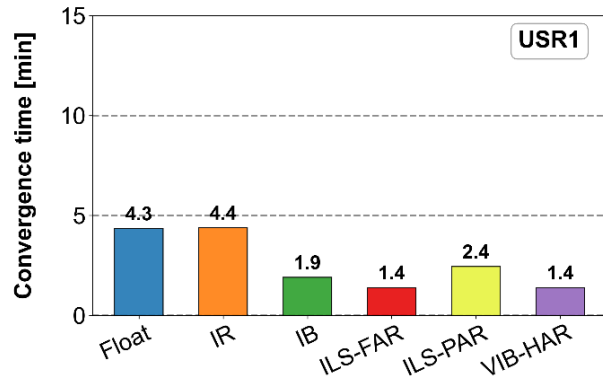
smaller errors with higher efficiency



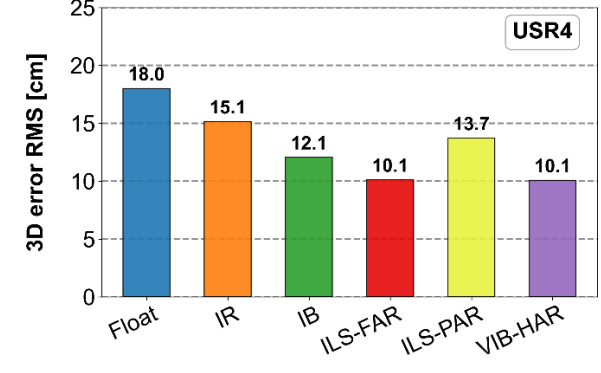
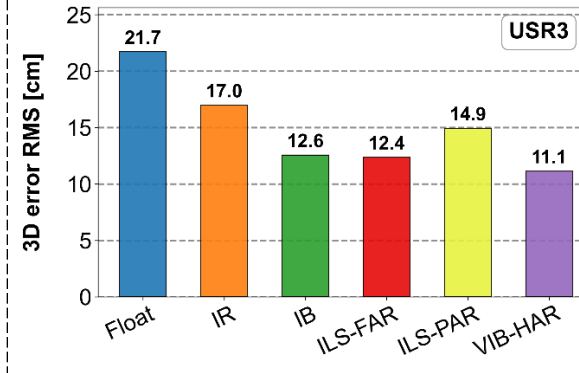
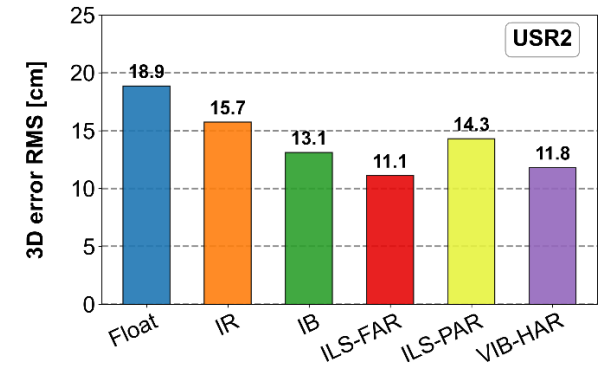
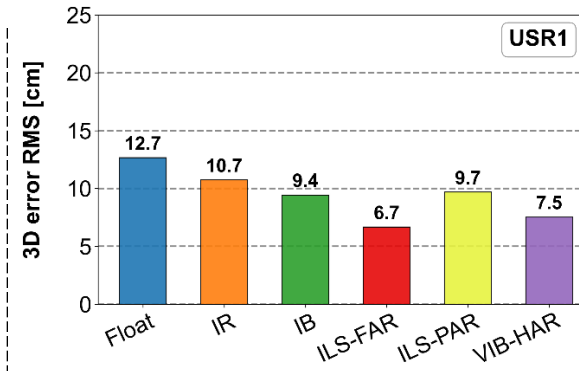
Overall accuracy: VIB-HAR $\approx$ ILS-FAR

Results

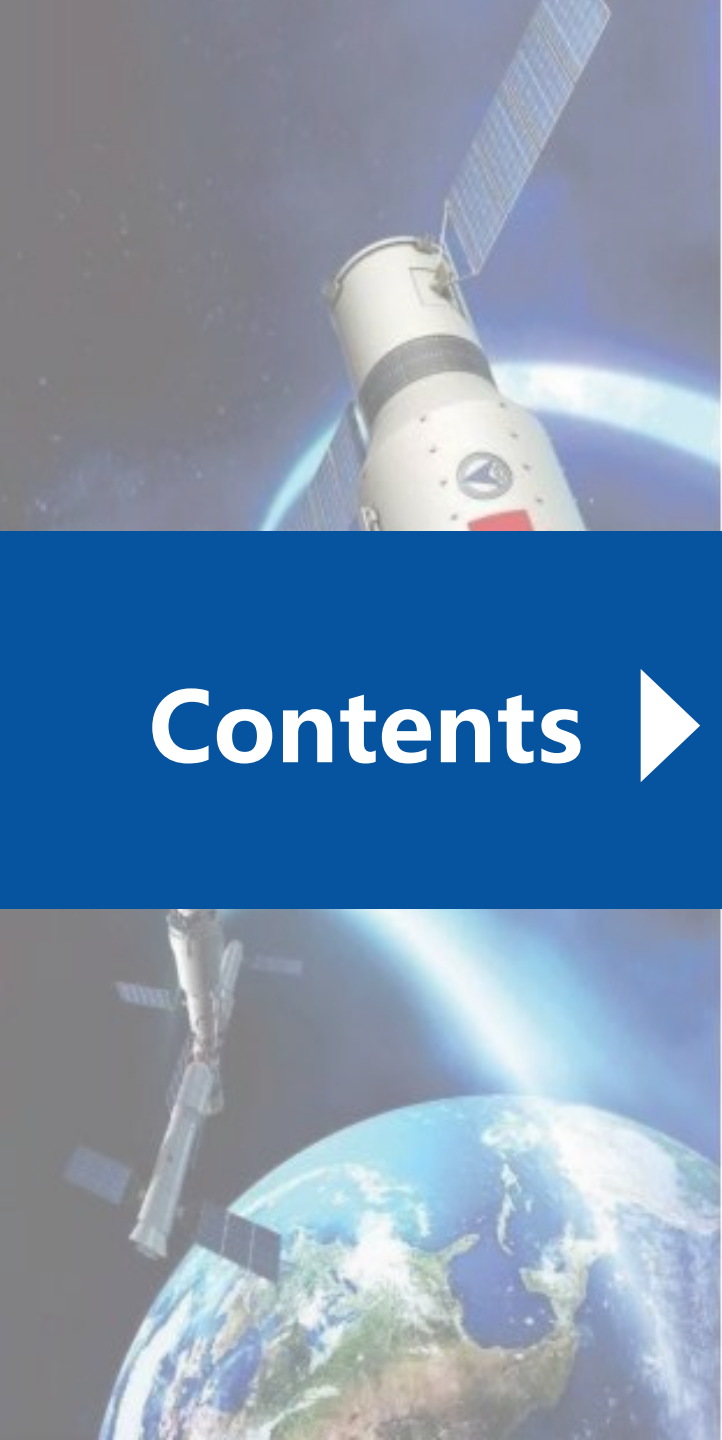
■ Summary



Shorter mean convergence time



Quasi-optimal positioning accuracy

A vertical banner on the left side of the slide. The top half shows a close-up of a space shuttle's nose and solar panels against the blackness of space. The bottom half shows a view of the Earth from space, with a satellite in orbit and a bright light source creating a lens flare effect.

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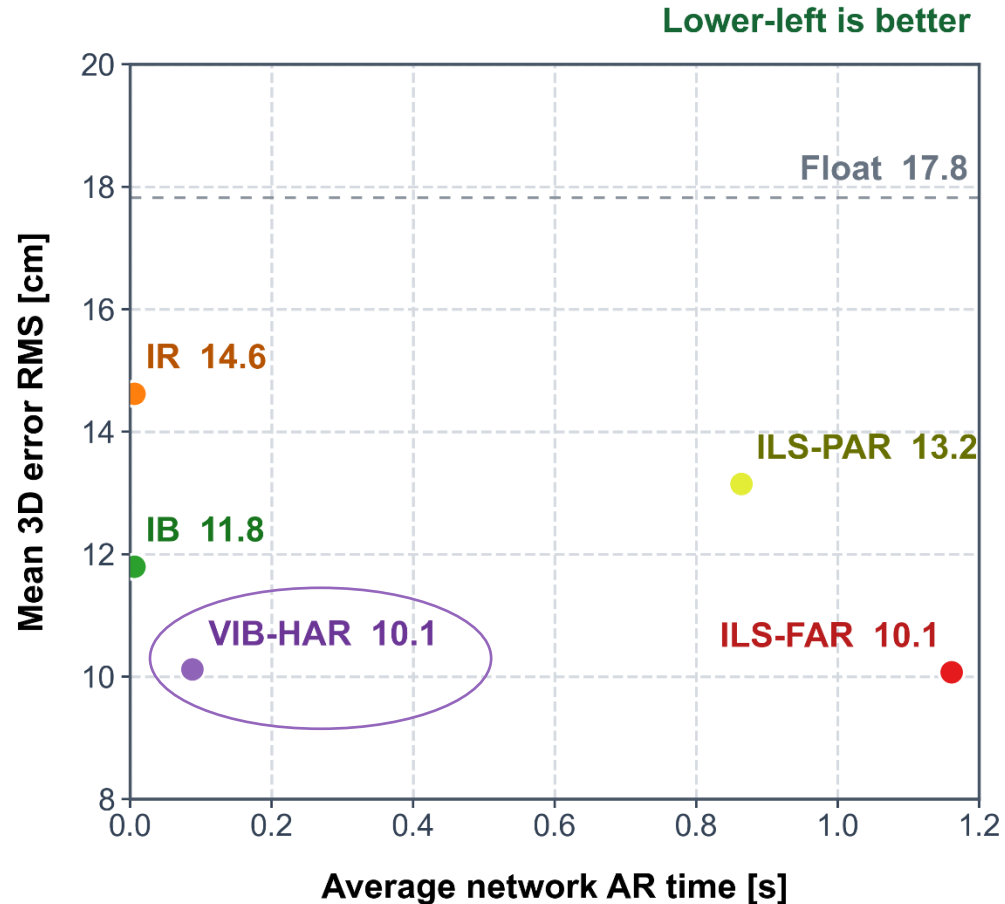
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Conclusions



- Different (classes of) estimator according to ambiguity reliability.
- It exploits moderate-reliability ambiguities that conventional PAR may discard.
- VIB-HAR achieves a trade-off between computational efficiency and solution accuracy



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Thank you!

Yishi Wang
wangyishi@apm.ac.cn

