

Beyond the Best Candidate in IAR: Solution Performance Bound when the True Integer Ambiguity is not ILS Optimal

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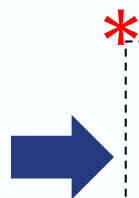
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Integer Ambiguity Resolution Theory: Estimation, Validation & SoTA

$$y = Aa + Bb + \eta, \begin{cases} y \in \mathbb{R}^m, a \in \mathbb{Z}^n, b \in \mathbb{R}^p, \eta \in \mathbb{R}^m \sim \mathcal{G}(0, Q_y) \\ A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{m \times p}, Q_y \in \mathbb{R}^{m \times m} \end{cases} \rightarrow \begin{cases} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \left(\begin{bmatrix} A^T \\ B^T \end{bmatrix} Q_y^{-1} \begin{bmatrix} A & B \end{bmatrix} \right) Q_y^{-1} y, \hat{a} \in \mathbb{R}^n, \hat{b} \in \mathbb{R}^p \\ \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} \sim \mathcal{G} \left(\begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} Q_a & Q_{a\hat{b}} \\ Q_{\hat{b}a} & Q_{\hat{b}} \end{bmatrix} \right), Q_a \in \mathbb{R}^{n \times n}, Q_{\hat{b}} \in \mathbb{R}^{p \times p}, Q_{\hat{b}a} \in \mathbb{R}^{p \times n} \end{cases} \xrightarrow{*} \begin{cases} \check{b} = \hat{b} - Q_{\hat{b}a} Q_a^{-1} (\hat{a} - \check{a}) \\ \check{b}, \hat{a} \end{cases}$$

Conventional
RAR



IAE class

$$\check{z} = S(\hat{z}) = \begin{cases} \sum_{z \in \mathbb{Z}^n} \omega_z(\hat{z}) z, & \hat{z} \in \Omega \\ [], & \hat{z} \notin \Omega \end{cases}$$

$$\omega_z(\hat{z}) = \begin{cases} 1, & \hat{z} \in \Omega_z, \Omega_z = \Omega \cap S_z, \\ 0, & \text{else} \end{cases}$$

$$\mathcal{I}(\hat{z}) = \begin{cases} \{1, \dots, n\}, & \hat{z} \in \Omega \\ \emptyset, & \hat{z} \notin \Omega \end{cases}, S: \mathbb{R}^n \rightarrow \mathbb{Z}^n \cup \mathbb{Z}^0.$$

$$\rightarrow \begin{cases} \check{a} = Z^{-T} \check{z}, & \hat{z} \in \Omega \\ \hat{a}, & \hat{z} \notin \Omega \end{cases}$$



- Established theory!
- Underlying stochastic model assumptions or real-time implementation limitations.



$$\hat{a}, Q_{\hat{a}} \rightarrow \begin{cases} \hat{z} = Z^T \hat{a} \\ Q_{\hat{z}} = Z^T Q_{\hat{a}} Z \end{cases} \rightarrow$$

AI-Based
RAR



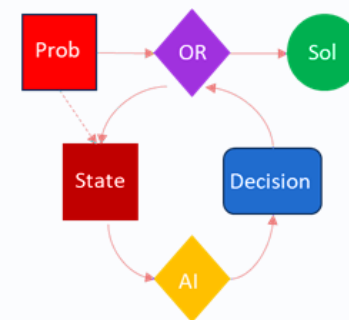
$$\check{z} = S(\hat{z}, \hat{x}) = \begin{cases} \sum_{z \in \mathbb{Z}^n} \omega_z(\hat{z}) z, & \hat{x} \in A_\tau \\ [], & \hat{x} \in R_\tau \end{cases}$$

$$\omega_z(\hat{z}) = \begin{cases} 1, & \hat{z} \in S_z \\ 0, & \hat{z} \notin S_z \end{cases}$$

$$\mathcal{I}(\hat{x}) = \begin{cases} \{1, \dots, n\}, & \hat{x} \in A_\tau \\ \emptyset, & \hat{x} \in R_\tau \end{cases}, S: \mathbb{R}^n \times \mathbb{R}^d \rightarrow \mathbb{Z}^n \cup \mathbb{Z}^0.$$

$$A_\tau = \{\hat{x} \in \mathbb{R}^d: g_\theta(\hat{x}) > \tau\} \quad R_\tau = \mathbb{R}^d \setminus A_\tau$$

$$\rightarrow \begin{cases} \check{a} = Z^{-T} \check{z}, & \hat{x} \in A_\tau \\ \hat{a}, \hat{x} \in R_\tau \end{cases}$$



- Is the claimed superiority on RAR justified?
- Is the AI-model architecture interpretable?



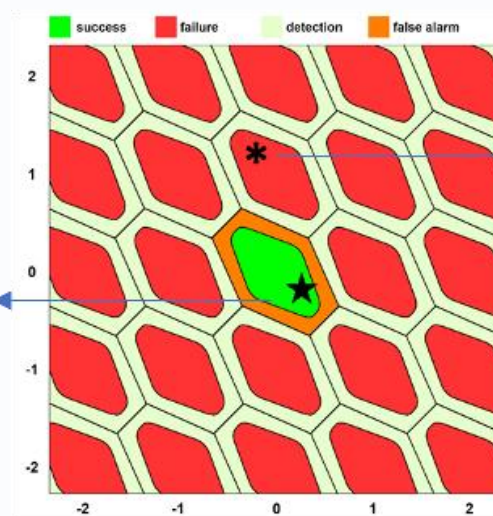
Integer Ambiguity Resolution Theory: Estimation, Validation & SoTA

$$\boldsymbol{\eta} = \boldsymbol{\varepsilon}_{\text{noise}} + \boldsymbol{\delta}_{\text{mis}} + \boldsymbol{\beta}_{\text{unmod}} \Rightarrow \hat{\mathbf{a}} = \mathbf{a} + \mathbf{e}_a \Rightarrow \Pr(\check{\mathbf{a}}_1 \neq \mathbf{a}) > 0$$



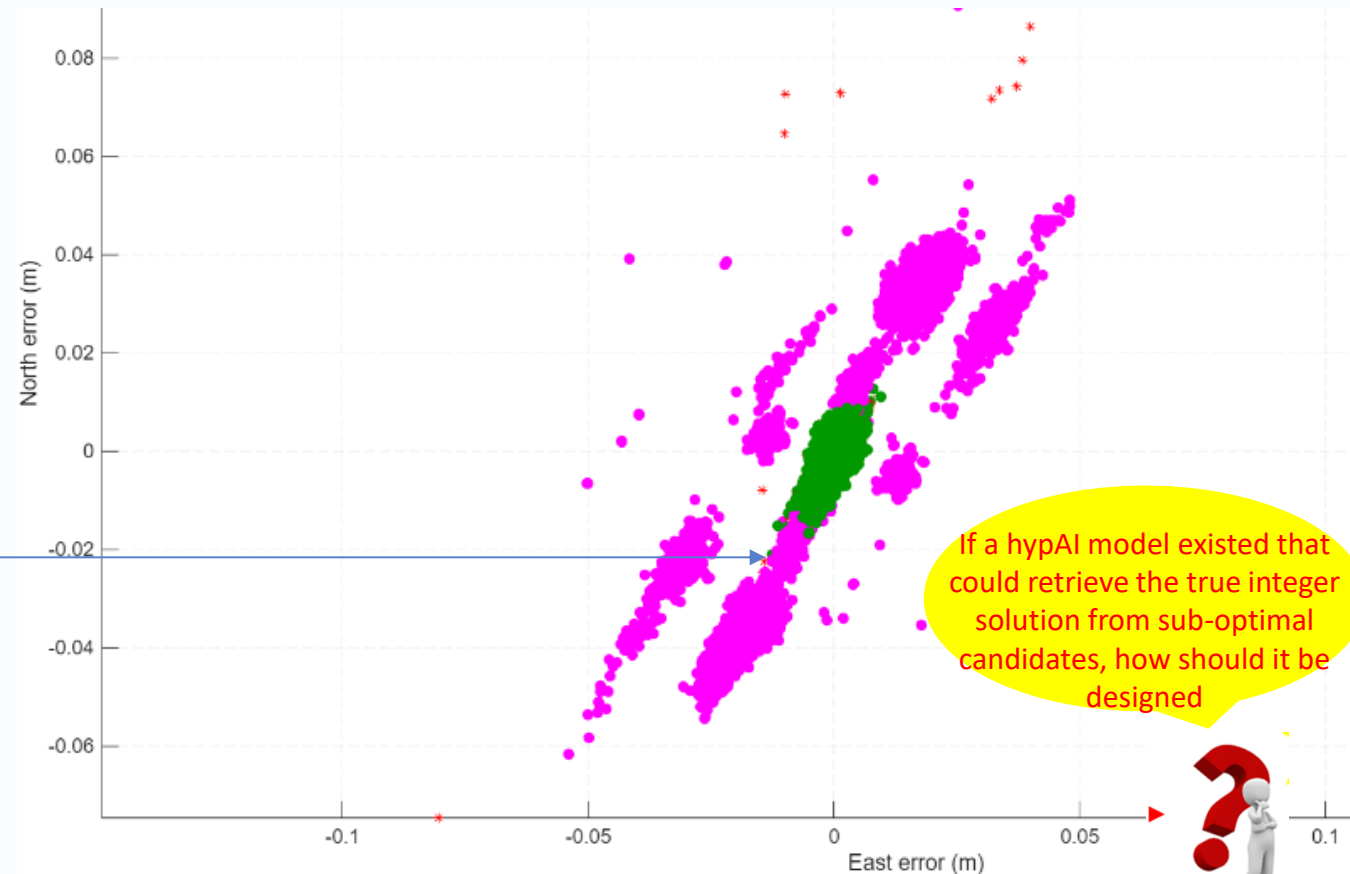
$$\hat{a}_1 \sim N(0, Q_{\hat{a}})$$

$$\check{a}_{ILS,1} \neq a$$



$$\hat{a}_2 \sim N(0, Q_{\hat{a}})$$

$$\check{a}_{ILS,2} \neq a$$



If a hypAI model existed that could retrieve the true integer solution from sub-optimal candidates, how should it be designed



Hypothesis: Multi-Integer Candidate

Physical candidate-list coverage

$$\begin{aligned} R &= \{j \geq 1: \check{\mathbf{a}}_j = \mathbf{a}\}, \\ L &= \{R \leq K\}, \\ O &= R > K, \\ C &= P(L) \end{aligned}$$

Decision alphabet notation

$$\begin{aligned} \hat{y} &= g_\theta(\hat{\mathbf{x}}) \in \{0, 1, \dots, K\}, \\ 0 &= \text{reject}, k = \text{accept } \check{\mathbf{a}}_k \end{aligned}$$

Integer only count

$$\begin{aligned} Y &= R, \text{ if } R \leq K \\ Y &= 0, \text{ if } R > K \end{aligned}$$

Strict simultaneous superiority

$$\begin{aligned} H_1: \exists \theta \in \mathcal{G}: A^*(\theta) > A(\mu) \wedge \text{CSR}^*(\theta) > \text{CSR}(\mu) \\ H_0: \forall \theta \in \mathcal{G}: A^*(\theta) \leq A(\mu) \vee \text{CSR}^*(\theta) \leq \text{CSR}(\mu) \\ * \equiv \text{hypothetical AI} \end{aligned}$$

Sub-optimal truth recovery

$$\begin{aligned} q_{sub}^* &= P(\hat{Y} = R | R \in \{2, \dots, K\}) \\ H_2: q_{sub}^* > 0, \text{ whereas } q_{sub} &= 0 \text{ by definition} \end{aligned}$$

Count form

$$A = F/N, \text{CSR} = CF/F, A^* > A \Leftrightarrow \Delta F > 0, \text{CSR}^* > \text{CSR} \Leftrightarrow \Delta CF > \text{CSR} \cdot \Delta F$$

The “price” of additional fixed epochs is mathematical: the gain in correct fixes must exceed the success-rate-weighted growth of conventional RAR in total fixes.

Deterministic Geometry & MILP Realization

Conditional outcomes

$q = P(\hat{Y} = R L),$	Correct in-list
$r = P(\hat{Y} = 0 L),$	Safe rejection
$w = P(\hat{Y} \neq R, \hat{Y} > 0 L),$	Wrong in-list
$b = P(\hat{Y} > 0 O),$	False acceptance
$q + r + w = 1$	

Error allocation quantities

$e = 1 - q,$	In-list error magnitude
$\lambda_{error} = w/e,$	Controls wrong selection
$\rho_{wrong} = b/e,$	Wrong selection out-list
$D = C + (1 - C)\rho_{wrong}$	

Closed form

$e = (1 - v)/D$	q, r, w, b are not grid parameters; they are algebraic consequences.
$q = 1 - e$	
$r = (1 - \lambda_{error})e$	
$w = \lambda_{error}e$	
$b = \rho_{wrong}e$	

Proof Skeleton — Why the Closed Form Is Forced

- Complement rule
- Conditional Probability
- Law of total probability
- Partition identity

- $P(\hat{Y} = Y) = P(\hat{Y} = R, L) + P(\hat{Y} = 0, O)$

- $v = Cq + (1 - C)(1 - b)$

- $q = 1 - e, b = \rho_{wrong}e$

- $v = C(1 - e) + (1 - C)(1 - \rho_{wrong}e)$

- $1 - v = e[C + (1 - C)\rho_{wrong}] = eD$

- $e = (1 - v)/D \Rightarrow q, r, w \text{ and } b, \text{ are unique}$

Deterministic Geometry & MILP Realization

Feature map and deterministic rule

$$\begin{aligned} x_t &= \Phi(\tilde{f}_t(\hat{\mathbf{a}}, \mathbf{Q}_{\hat{\mathbf{a}}}), (f_i/f_1)_{i=2:3}, (f_i - f_1)_{i=2:3}, \text{Auxiliary info}) \\ z_t &= W(x_t - \mu_x) \\ g_\theta: \mathbb{R}^d &\rightarrow \{0, 1, 2, 3\} \\ \hat{Y}_t &= g_\theta(z_t) \end{aligned}$$

Same-class ε -neighborhood graph

$$\begin{aligned} E_\varepsilon &= \{(t, u): Y_t = Y_u \wedge \|z_t - z_u\|_2 \leq \varepsilon\} \\ D_\varepsilon &= \left[\sum_{(i,j) \in E_\varepsilon} \omega_{tu} \mathbb{I}(\hat{Y}_i \neq \hat{Y}_j) \right] / \left[\sum_{(t,u) \in E_\varepsilon} \omega_{tu} \right] \\ D_\varepsilon &\leq \kappa(1 - v) \end{aligned}$$

Implemented construction graph

Bounded same-class k-NN + ε radius

$$\begin{aligned} E_\varepsilon &= \{(t, u): Y_t = Y_u, u \in N_{\text{same}(t)}^k \|z_t - z_u\|_2 \leq \varepsilon\} \\ \omega_{tu} &\propto \exp\{-(\|z_t - z_u\|_2 / \varepsilon)^2\} \end{aligned}$$

Progressive family constraints

$$\begin{aligned} v_2 > v_1 &\Rightarrow E_{\text{error}}(v_2) \subseteq E_{\text{error}}(v_1) \\ v_2 > v_1 &\Rightarrow D_\varepsilon(v_2) \leq D_\varepsilon(v_1) \end{aligned}$$

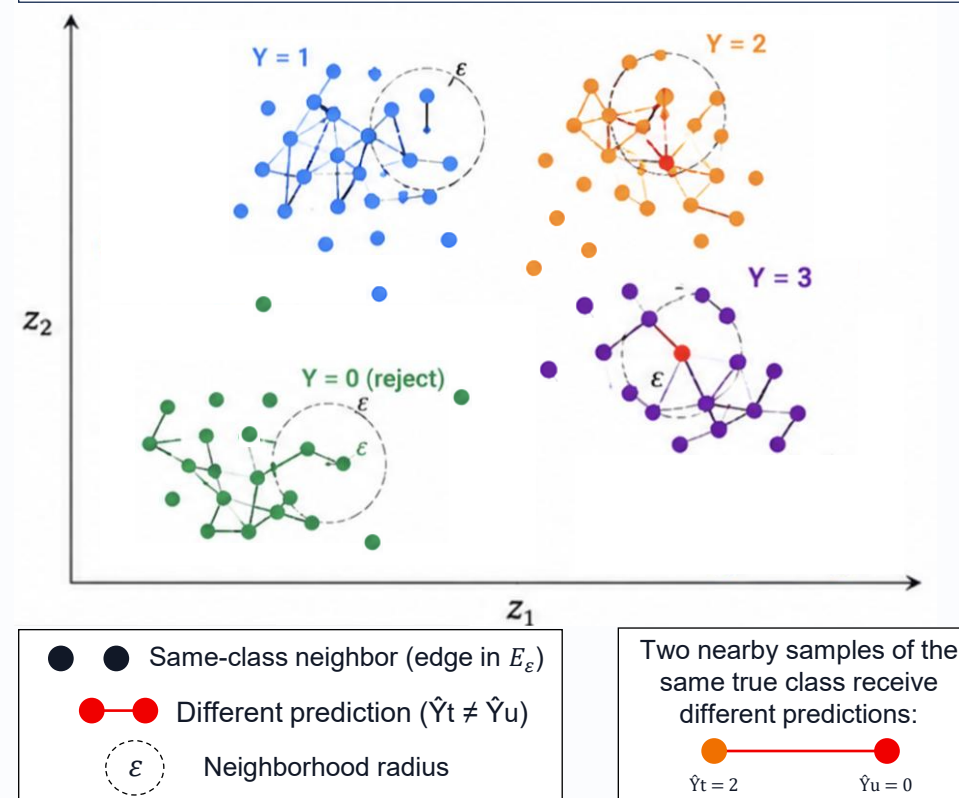
Non-circular transfer

Y_t is used only to construct the training graph.

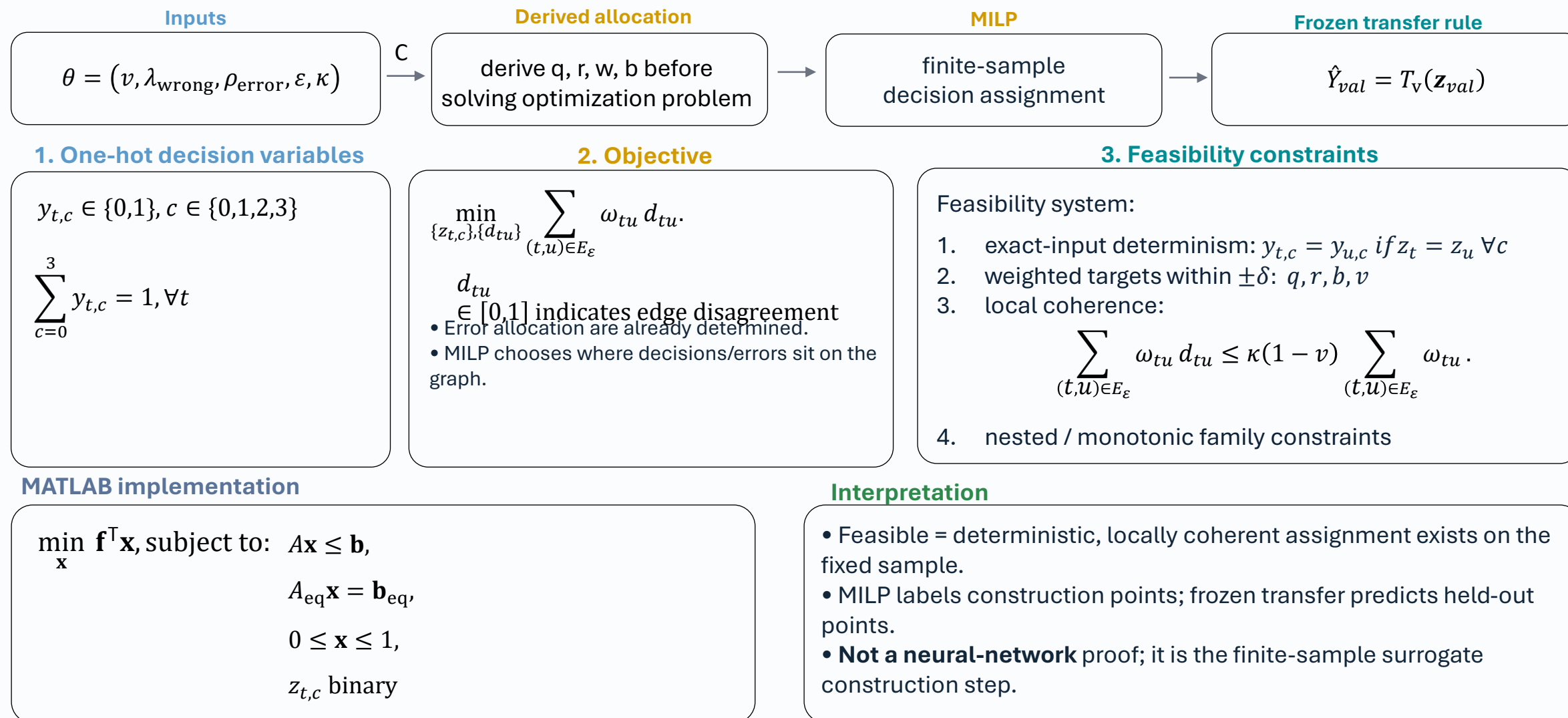
Held-out prediction uses z only; truth is revealed after prediction

Local consistency intuition

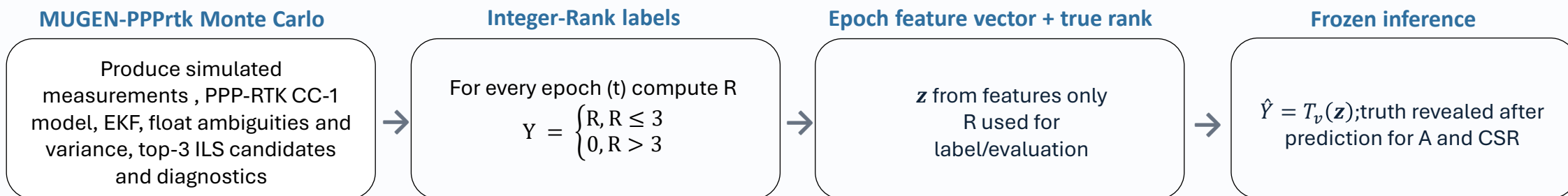
- Edges join statistically close samples only within the same true class.
- The surrogate should make consistent decisions for neighboring same-class samples.
- Only a small fraction of neighboring pairs are allowed to disagree.



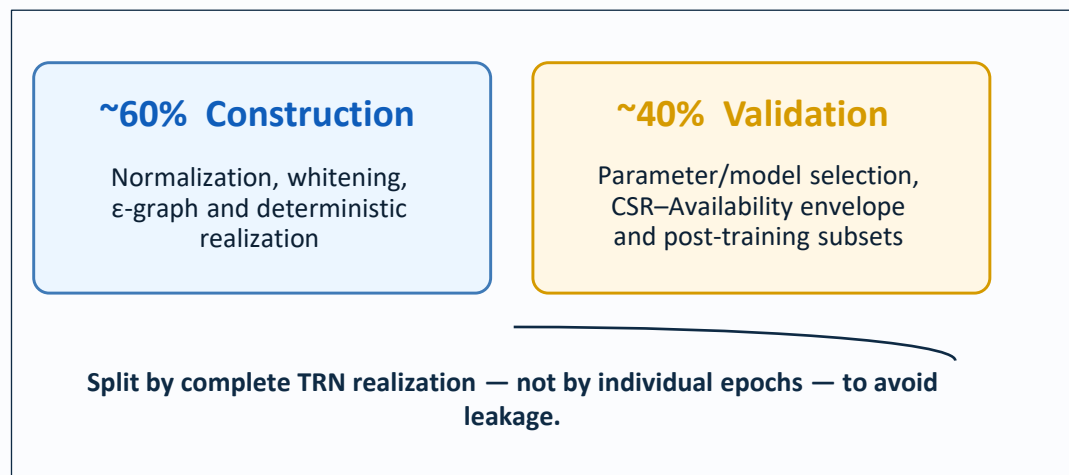
Deterministic Geometry & MILP Realization



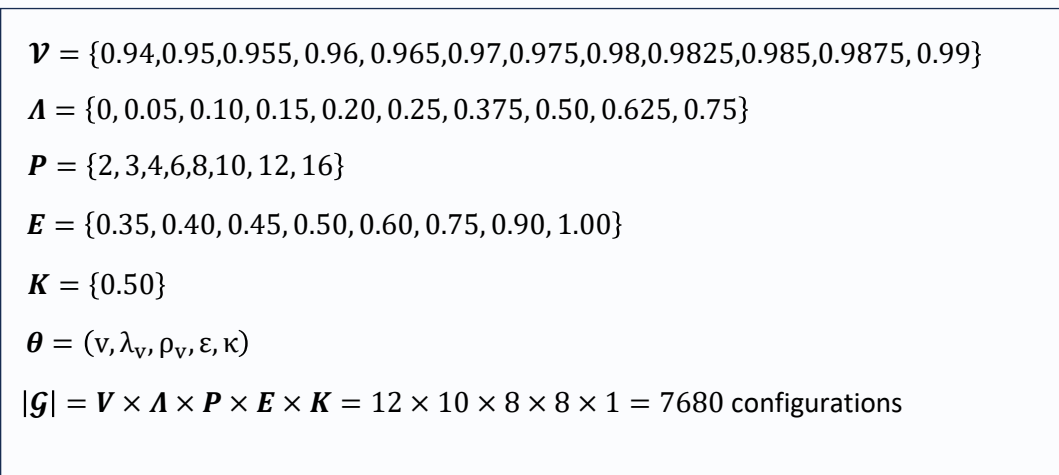
Data Campaign and Parametric Search



TRN-based partitioning by complete realization



Search Grid



Configured / selected

Data derived

Algebraically derived

Evaluation outputs

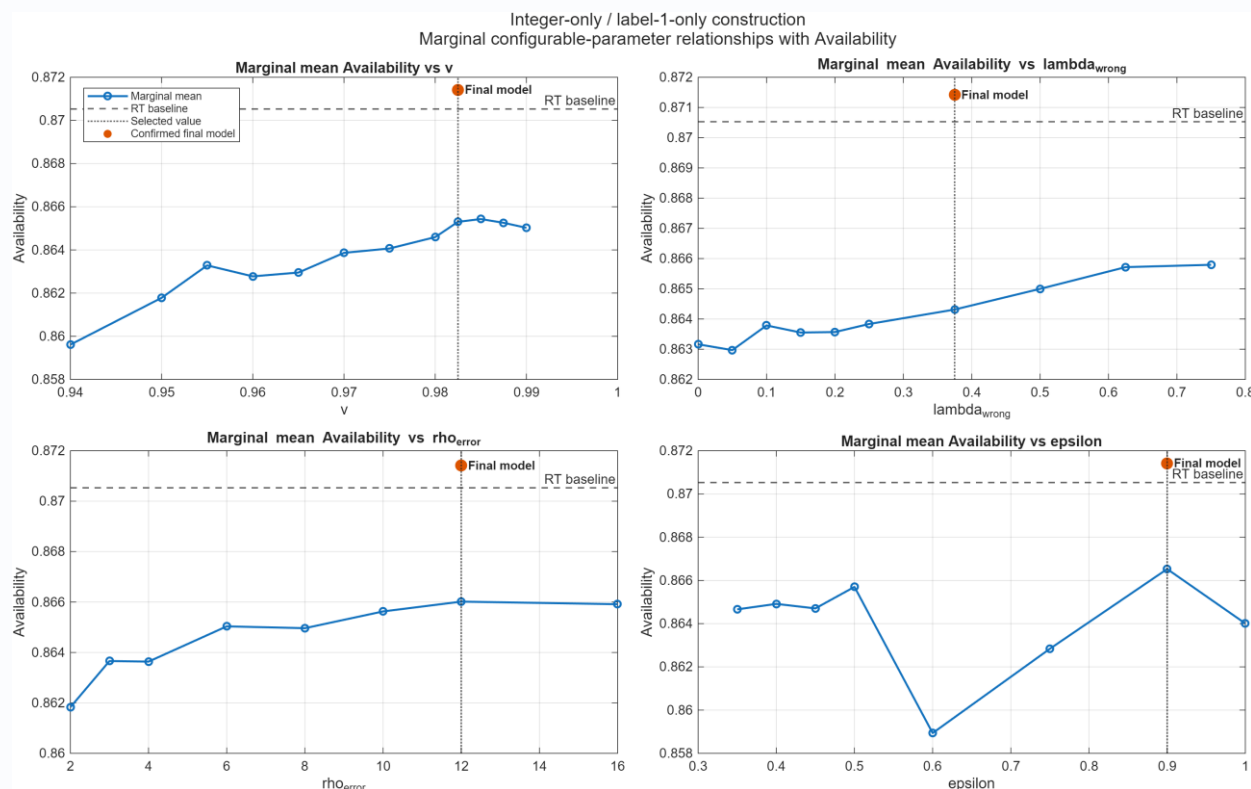
$(v, \lambda_{\text{wrong}}, \rho_{\text{wrong}}, \epsilon, \kappa)$

C

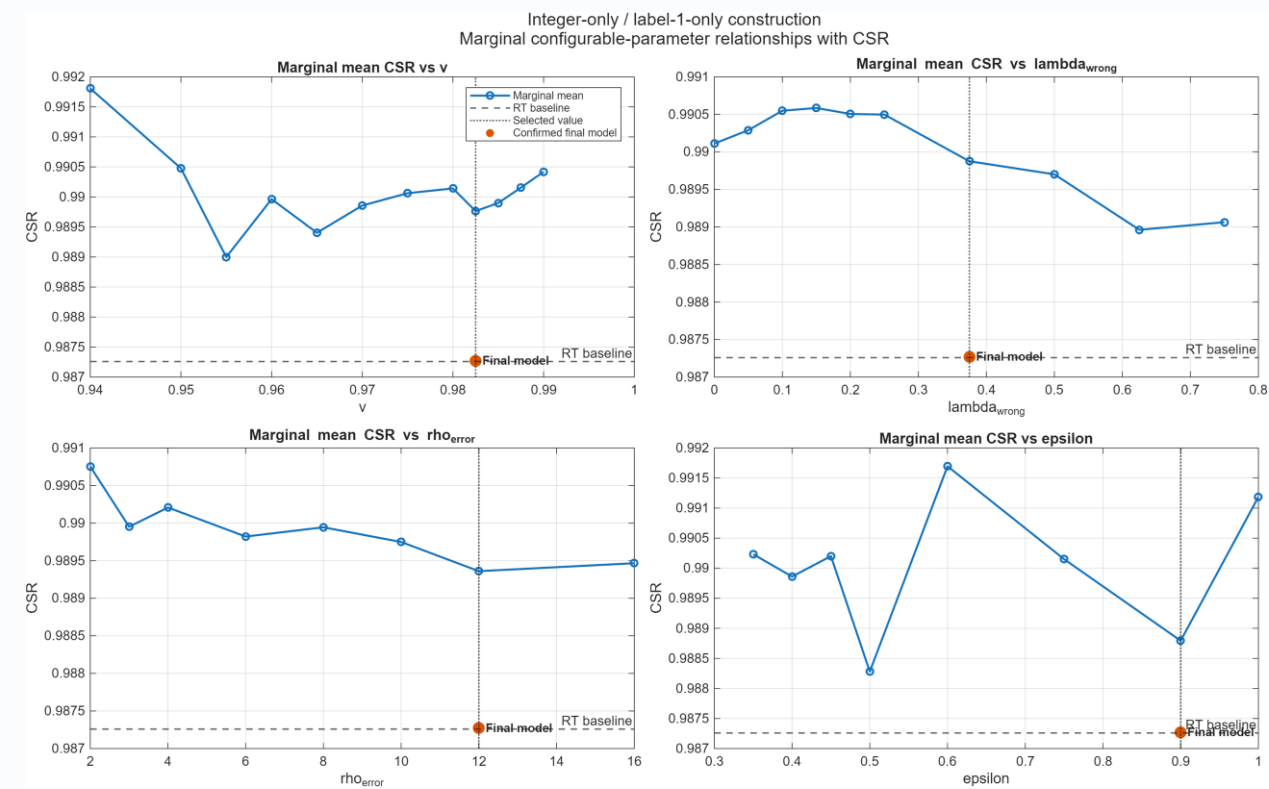
(q, r, w, b)

Integer-only search sweeps configurable parameters only; all IAR metrics (**A**, **CSR**) are derived from the deterministic realization.

Results



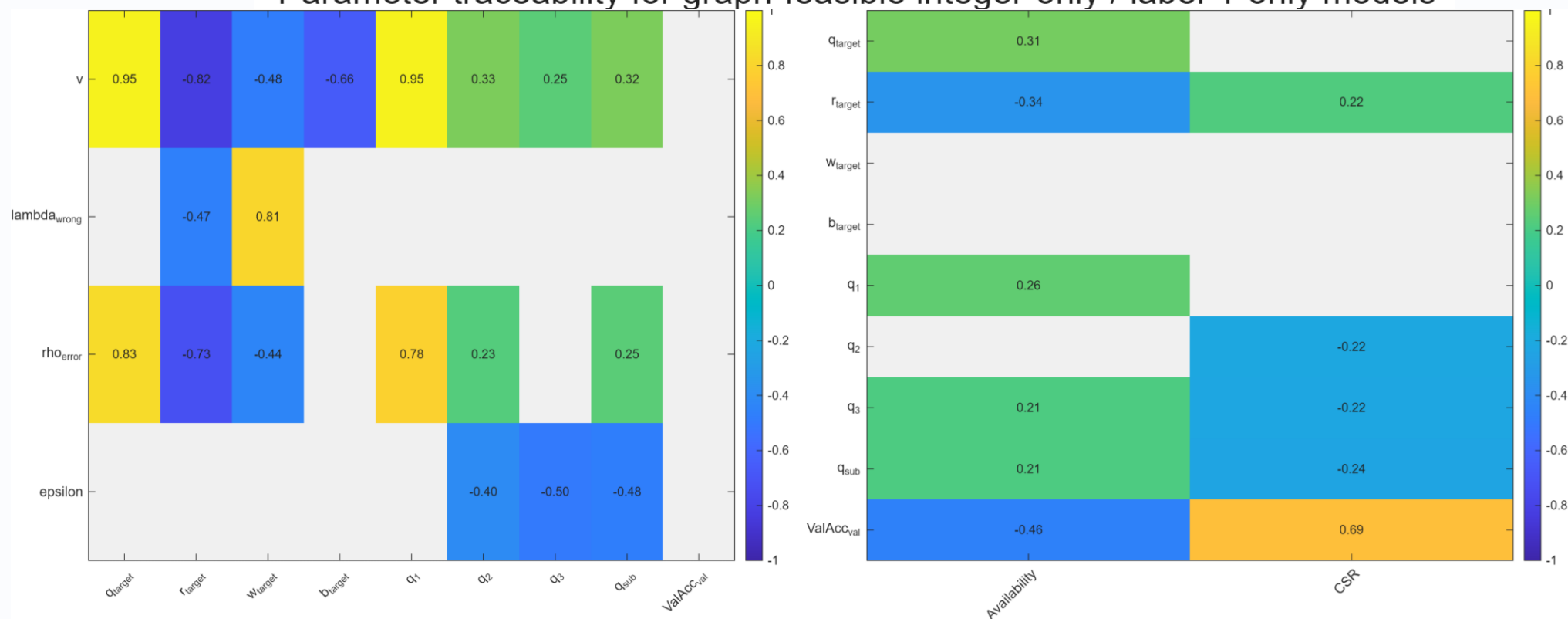
- v , λ_{wrong} , and ρ_{wrong} generally increase marginal Availability.
- ϵ has a non-monotonic effect on Availability.
- Marginal means remain below the RT reference.
- Final $A^* > A$ is achieved by the joint parameter configuration + deterministic refinement



- Increasing λ_{wrong} generally decreases CSR.
- Increasing ρ_{wrong} also tends to reduce CSR.
- v and ϵ show non-monotonic CSR behavior.
- Final CSR improvement is very small, so Availability gains must be achieved while carefully preserving reliability.

Results

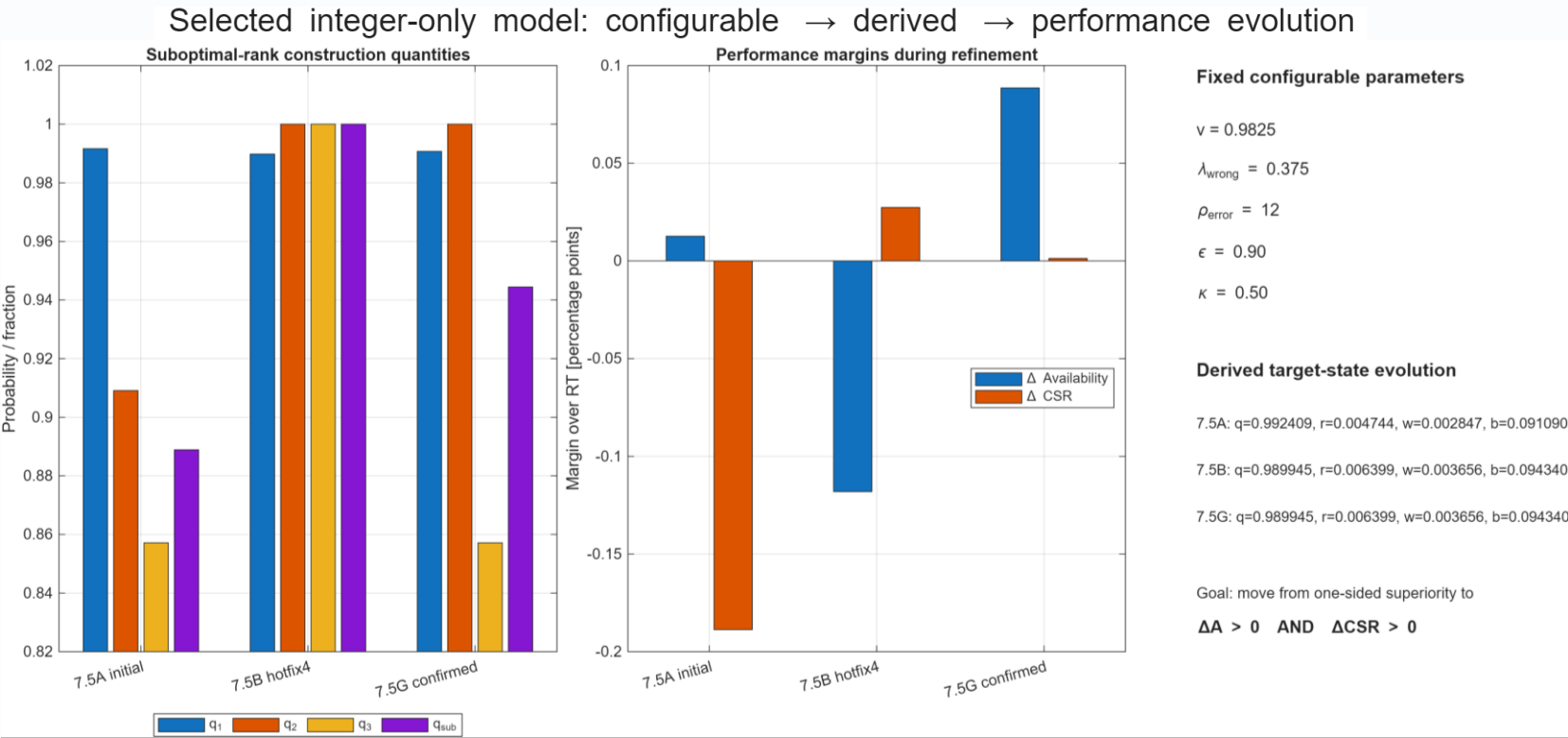
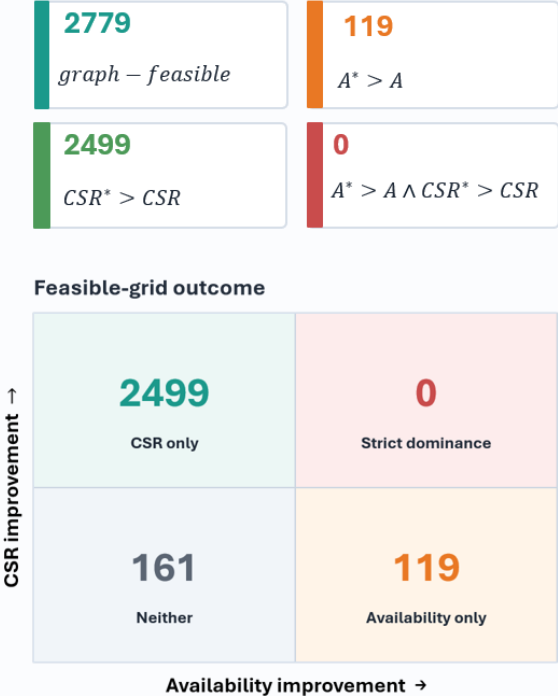
Parameter traceability for graph-feasible integer-only / label-1-only models



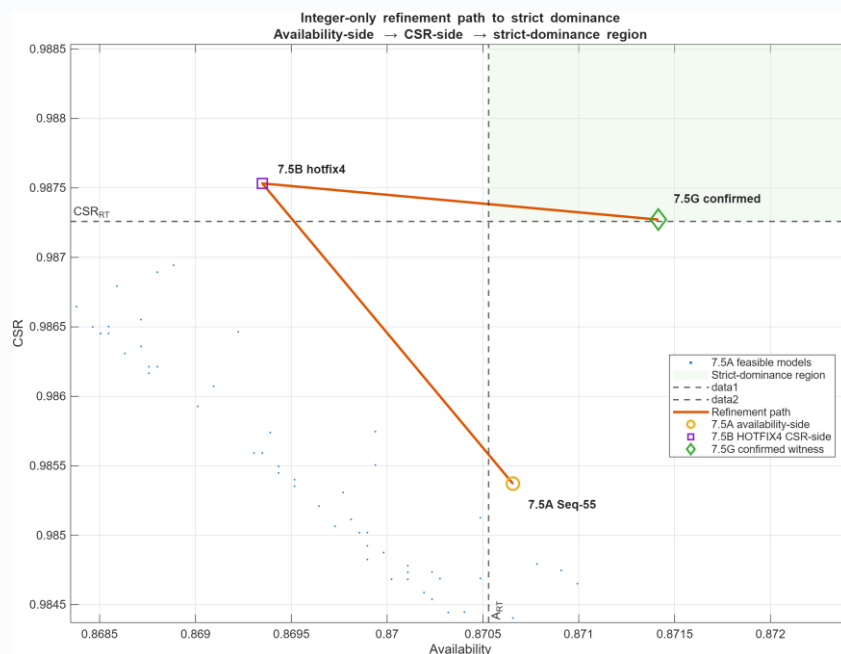
- Higher v generally gives higher q and fewer errors.
- λ_{wrong} shifts errors from rejection to wrong selection.
- ρ_{error} changes the in-list / out-of-list error balance.
- ϵ mainly affects rank-2 / rank-3 recovery.

- Higher q generally improves Availability.
- More rejection r reduces Availability but can protect CSR.
- Better q_2, q_3, q_{sub} helps Availability but may slightly reduce CSR.
- Higher VallAcc mainly improves CSR, with a trade-off in A.

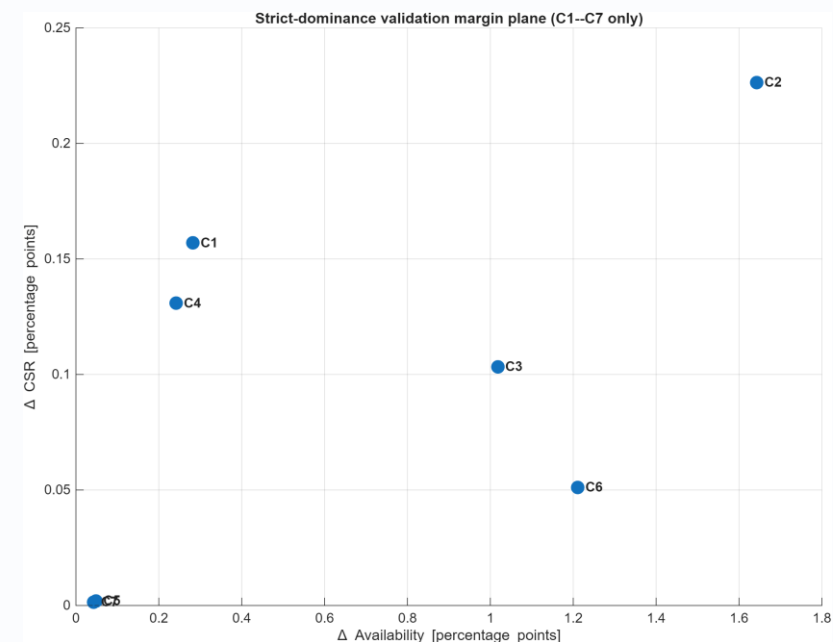
Results



Results



- 7.5A: improves Availability, but CSR remains below the RT reference.
- 7.5B: improves CSR, but sacrifices Availability.
- 7.5G: refinement moves the model into the strict-dominance region.
- The final operating point satisfies:
- $\Delta A > 0 \wedge \Delta CSR > 0$



- Farther upper-right means stronger simultaneous improvement in Availability and CSR.
- C2 is the strongest joint-dominance case.
- C1/C4 provide relatively stronger CSR gains, while C3/C6 emphasize Availability gains.
- C5/C7 lie very close to the boundary and therefore represent weak strict-dominance cases

Conclusions and Future Work

Conclusions

- 1 **Sub-optimal truth is exploitable**
R=2 or R=3 means the truth is not rank-1, not absent from the top-3 list.
- 2 **Strict dominance is achievable**
After refinement hypAI model: $\Delta A = +0.0886$ pp and $\Delta CSR = +0.0013$ pp.
- 3 **Accuracy alone is insufficient**
Similar ValAcc can lead to different rejection (r), wrong selection (w) and false acceptance (b).
- 4 **The graph controls error placement**
Final labeling remains locally coherent: $D_\varepsilon = 0.00704 < 0.00875$.
- 5 **Behavioral blueprint for real AI**
Accept a model (for specific scenarios) only if it passes the IAR gate:
 $\Delta A > 0 \wedge \Delta CSR > 0$.

Future Work

- 1 **Training a multi-integer ILS rank validation model**
Output $\hat{Y} \in \{0,1,2,3\}$: *reject*, $\check{\alpha}_1, \check{\alpha}_2, \check{\alpha}_3$.
- 2 **Using a contrastive-sensitive learning paradigm**
Wrong fixes shall be penalized more than rejections.
- 3 **Reporting rank-specific metrics**
Tracking q and q_{sub} and local disagreement is mandatory.
- 4 **Representing rare rank-2/3 cases**
Identifying methodology for appropriately representation in the training datasets
- 5 **Extending and stress-testing**
Evaluate multi-GNSS, multi-frequency, kinematic and real-time cases.

Thank you for you attention!!!