

A STUDY ON PPP ADVANCES BY INTEGRATING EXISTING GNSS AND LEO-PNT : A STUDY ON CONVERGENCE TIME



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MOTIVATION

Why LEO-PNT?

GNSS works...but

- Weak signals at Earth's surface.
- Limited indoors/urban environments.
- A system optimized for performance: GNSS alone cannot provide resilience everywhere.
- Designed as a single global layer.
- Vulnerable to interference.

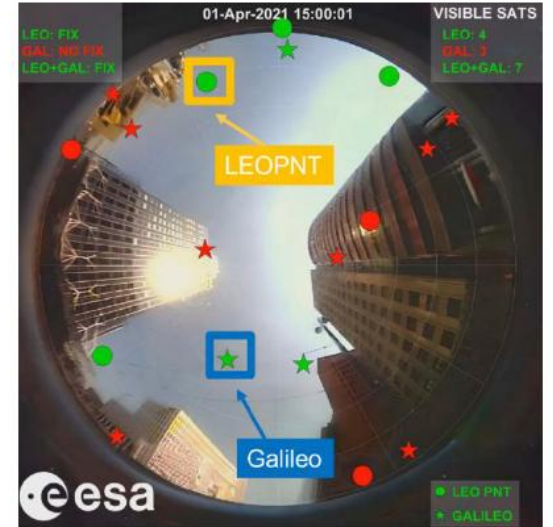


LEO-PNT changes the physics of resilience

- Faster geometry variation
- Stronger signals
- New frequencies
- Global coverage
- GNSS



Source:
FrontierSI(2026), State of the Market Report,
LEO PNT -2025 Edition,
available at frontiersi.com.au



ESA simulation showing Galileo augmented by a 60-satellite LEO PNT constellation

MOTIVATION

LEO – PNT Constellations

Examples of existing LEO constellations

Constellation	Satellites / architecture	Altitude [km]	Inclination / orbital structure	Main application
ORBCOMM OG2	17 satellites deployed	~620–750	~47°; multiple orbital planes	IoT / M2M / AIS
Iridium NEXT	66 operational satellites	~780	Near-polar; 6 planes	Global voice and data communications
OneWeb	648 first-generation satellites	~1200	87.9°; 18 planes	Broadband connectivity
Starlink	Multi-shell mega-constellation	~550 representative shell	53° representative shell; 72 planes	Broadband connectivity
Xona Pulsar	258 satellites	1080	12 inclined orbits (53°) + 6 polar orbits (97°)	Dedicated PNT

Constellation	Status	Role
Celeste (ESA)	In-orbit demonstrator	European LEO-PNT technology demonstration / Galileo complement

Operational LEO constellations demonstrate dense and rapidly moving satellite networks.

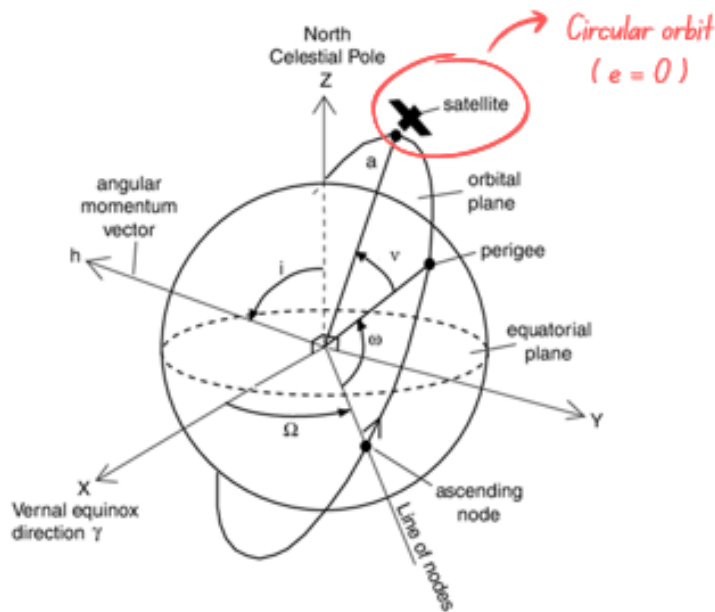


SIMULATED LEO-PNT CONSTELLATION



Simulated according to examples of LEO-PNT concept in the literature.
(In collaboration with ESA)

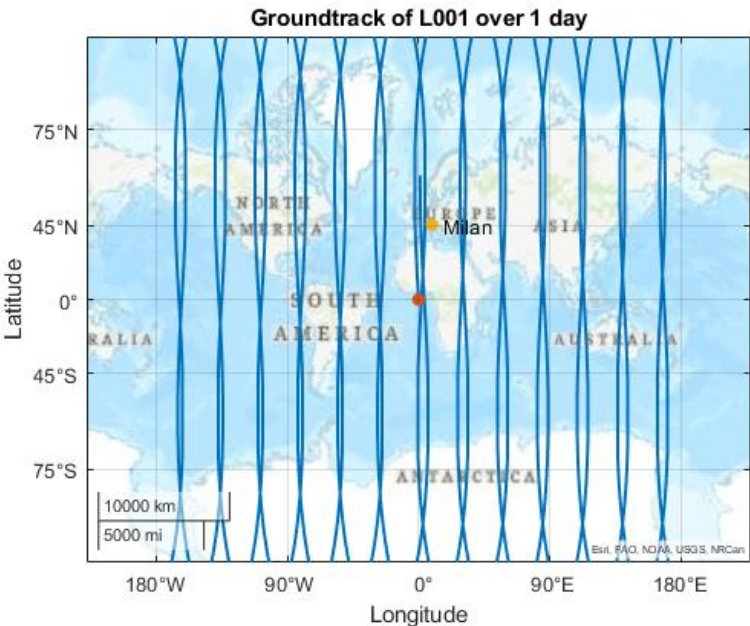
Orbital height 700-1200 km
Speed 7-8 km/s
Orbital period 1.5 - 2 hours



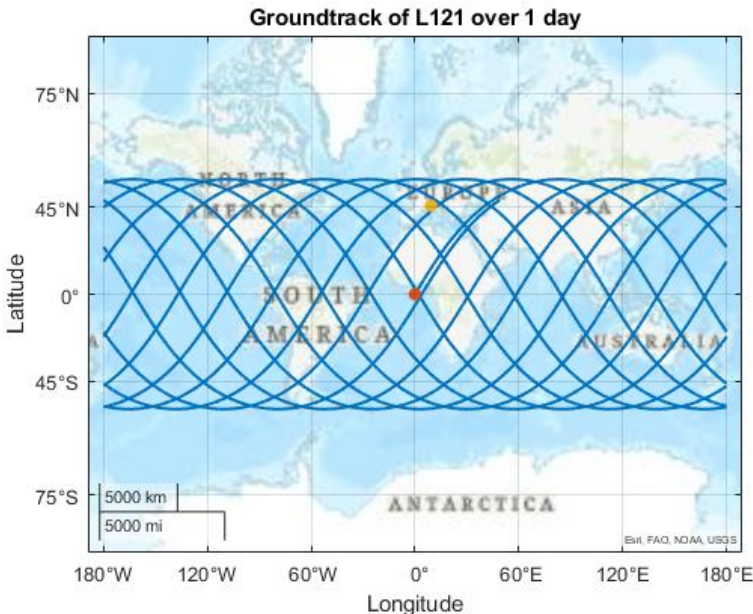
Keplerian Orbital Model

Inclination	#Orbital planes	#Satellites per orbital plane	Orbital Period
89 degrees	8	15	1 h 49 min
55 degrees	11	13	1 h 49 min

LEO-PNT constellation with 2 inclination angles.
Total number of satellites: **263 satellites.**



(a) Polar orbit groundtrack (89°)



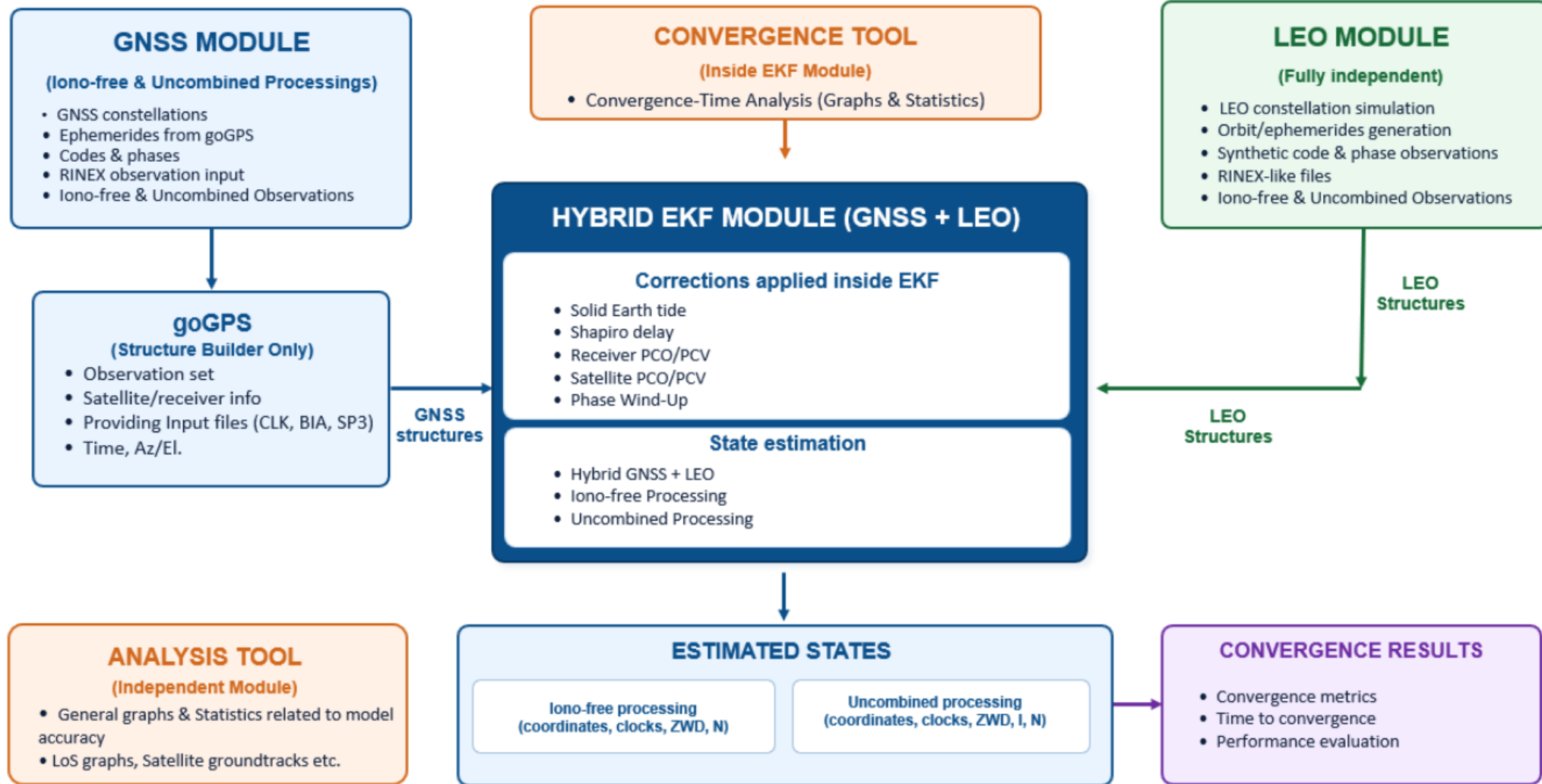
(b) Inclined orbit groundtrack (55°)

Groundtracks for polar and inclined LEO satellites over one day.

PROCESSING FRAMEWORK & ESTIMATION MODEL

ENGINE ARCHITECTURE & WORKFLOW

GNSS + LEO Hybrid EKF Engine Architecture



Key principle: goGPS is used only for GNSS structure building; LEO is fully independent; all EKF corrections are applied inside the EKF module.

Adopted Iono - Free observation equations

GNSS and LEO-PNT Pseudoranges

Iono-Free GNSS Pseudoranges :

For satellite s at epoch t :

$$P_R^{\text{GNSS},s}(t) = \rho_R^s(t, t - \tau_R^s) + T_R^s(t) + c \left(dt_R(t) + d_{R,P}^{s,G}(t) \right) + D_P^s(t) + \nu_P^s(t)$$

Iono-Free LEO-PNT Pseudoranges (Synthetic):

$$P_R^{\text{LEO}}(t) = \rho_R^s(t, t - \tau_R^s) + T_R^s(t) + c \cdot (dt_R(t) + d_R^{\text{LEO}}(t)) + \text{errors}$$

Approximated to the receiver clock
offset estimated from GNSS data

Modelled

- ρ_R^s : Geometric range from satellite s to user UE
- τ_R^s : Signal propagation delay
- $dt_R(t)$: Receiver clock offset
- $d_R^{\text{LEO}}(t)$: LEO satellite clock offset
- $d_{R,P}^G(t)$: GNSS code tracking bias (G stands for constellation)
- $D_P^s(t)$: Deterministic GNSS code corrections
- $\nu(t)$: observation noise
- T_R^s : Tropospheric delay : $T_R^s = \text{ZTD}(t) \cdot M_R^{f_s}(t)$

ZTD(t) : Zenith Tropospheric Delay as provided by GNSS processing

$M_R^{f_s}(t)$: Tropospheric mapping function

In case errors are added: $\text{errors} = \varepsilon_j$ where ε_j is :

$\varepsilon_{orb}^{\text{LEO}}$: Orbit error • –sinusoidal error

$\varepsilon_{clock}^{\text{LEO}}$: Clock residual error • – Gaussian with std 12 cm

$\varepsilon_{obs}^{\text{code}}$: Observation error • –Gaussian error with **std σ = 30 cm**

Adopted Iono - Free observation equations

GNSS and LEO-PNT Carrier-Phase Observations

Iono-Free GNSS Carrier-Phase Observations:

For satellite s at epoch t :

$$\Phi_R^{\text{GNSS},s}(t) = \rho_R^s(t, t - \tau_R^s) + T_R^s(t) + c \left(dt_R(t) + d_{R,\Phi}^{s,G}(t) \right) + K_R^s(t) + D_\Phi^s(t) + \nu_\Phi^s(t)$$

$D_\Phi^s(t)$: Deterministic GNSS phase corrections

Iono-Free LEO-PNT Carrier-Phase Observations (Simulated):

$$\Phi_R^{\text{LEO}}(t) = \rho_s(t) + T_R^s(t) + c \cdot \left(dt_R(t) + d_R^{\text{LEO}}(t) \right) + \varepsilon_{orb}^{\text{LEO}} + \varepsilon_{clock}^{\text{LEO}} + K_R^s(t) + \varepsilon_{obs}^{\text{phase}}$$

K_R^s : Hardware bias of the Iono-Free combination + The so-called ambiguity (not integer),

where:

$$K_R^s = \lambda \eta_R^s + \boxed{cD_R^G}$$

constant $\Rightarrow$ $\left(cdt_{R,P}^G(t) + \underbrace{cdt_{R,\phi}^G - cdt_{R,P}^G}_{cD_R^G} \right)$

- $\varepsilon_{orb}^{\text{LEO}}$: Orbit error • -sinusoidal error
- $\varepsilon_{clock}^{\text{LEO}}$: Clock residual error • - Gaussian with std 12 cm
- $\varepsilon_{obs}^{\text{phase}} \sim N(0, \sigma^2)$: Phase observation error • -Gaussian noise with **std $\sigma = 0.003$ m (3 mm)**

Adopted Iono - Free observation equations

Possible sinusoidal error model in LEO simulated observations

Sinusoidal Error Model in LEO Observations

In LEO pseudorange and carrier-phase observations, we introduce a satellite-dependent sinusoidal error that takes into account for errors in ephemerides and clock offsets.

Mathematical Model :

For each LEO satellite s , the sinusoidal error added to the observations at time t is modelled as:

where: $\varepsilon_j = A \cdot \sin\left(\frac{2\pi t}{T} + \phi_s\right)$

- A : Amplitude of the error in meters.
- T : Period (LEO orbit period = 109.3 min) of oscillation in seconds ($T = 6556$ s).
- ϕ_s : Random phase offset for satellite s : $\phi_s = 2\pi x_s$, $x_s \sim U(0, 1)$.

ESTIMATION MODEL

Weighted LS initialization + sequential EKF

Nonlinear observation model:

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{v}_k \quad \text{where: } \mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_{\text{obs},k})$$

- $\mathbf{y}_k$: vector of observations available at **epoch k**
- $\mathbf{x}_k$: vector of unknown parameters
- $\mathbf{h}_k(\mathbf{x}_k)$: nonlinear observation model
- $\mathbf{v}_k$: measurement-noise covariance matrix
- $\mathbf{C}_{\text{obs},k}(\mathbf{t})$: observation - noise covariance matrix

$$\mathbf{C}_1^+ = \left(\mathbf{N}_1^{(i)}\right)^{-1} \quad \text{Error covariance matrix coming from LS} \\ \text{(This covariance initializes the EKF)}$$

- $\mathbf{N}_1^{(i)}$: normal matrix
- $\delta \mathbf{x}_1^{(i)}$: incremental state correction
- $\hat{\mathbf{x}}_1^+$: converged first-epoch state estimate
- $\mathbf{C}_1^+$: covariance of the initialized state

Processing Logic

When k = 1

Weighted Least
Square / Gauss-
Newton

State iterate becomes: $\mathbf{x}_1^{(i+1)} = \mathbf{x}_1^{(i)} + \delta \mathbf{x}_1^{(i)}$

After convergence: $\hat{\mathbf{x}}_1^+ = \mathbf{x}_1^{(i+1)}$

$\hat{\mathbf{x}}_k^+$ $\mathbf{C}_k^+$

When k > 1

Extended Kalman
Filter

$\hat{\mathbf{x}}_k^+ \mathbf{C}_k^+$: Initial state and covariance passed to the EKF filter.

EKF: PREDICTION & MEASUREMENT UPDATE

Prediction step

$$\hat{\mathbf{x}}_k^- = \mathbf{F}_k \hat{\mathbf{x}}_{k-1}^+$$

$$\mathbf{C}_k^- = \mathbf{F}_k \mathbf{C}_{k-1}^+ \mathbf{F}_k^T + \mathbf{Q}_k$$

$\mathbf{F}$: State transition matrix $\mathbf{F} = \mathbf{I}$.

$\hat{\mathbf{x}}_{k-1}^+$: updated state at epoch $k - 1$

$\hat{\mathbf{x}}_k^-$: predicted state at epoch k

$\mathbf{C}_{k-1}^+$: posterior state covariance

$\mathbf{C}_k^-$: predicted state covariance

$\mathbf{F}_k$: state-transition matrix

$\mathbf{Q}_k$: process-noise covariance matrix

Measurement linearization: The Jacobian matrix of the observation function:

$$\mathbf{A}_k = \left. \frac{\partial \mathbf{h}_k}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_k^-}$$

Innovation covariance: $\mathbf{S}_k = \mathbf{A}_k \mathbf{C}_k^- \mathbf{A}_k^T + \mathbf{C}_{\text{obs},k}$

Kalman Gain: $\mathbf{G}_k = \mathbf{C}_k^- \mathbf{A}_k^T \mathbf{S}_k^{-1}$

State Update: $\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{G}_k \mathbf{v}_k$

Covariance update – Joseph form: $\mathbf{C}_k^+ = (\mathbf{I} - \mathbf{G}_k \mathbf{A}_k) \mathbf{C}_k^- (\mathbf{I} - \mathbf{G}_k \mathbf{A}_k)^T + \mathbf{G}_k \mathbf{C}_{\text{obs},k} \mathbf{G}_k^T$

Although $\mathbf{F}_k = \mathbf{I}$, non-zero process noise allows the corresponding state to evolve between epochs; therefore $\mathbf{F}_k = \mathbf{I}$ alone does not imply a strictly fixed state.

EKF: STATE VECTORS

Ionosphere-Free GNSS & GNSS+ LEO State vectors

IF GNSS – only state vector

$$\mathbf{x}_{\text{GNSS}}^{\text{IF}} = \left[X \ Y \ Z \ dt_{G_1} \ \cdots \ dt_{G_{n_G}} \ ZWD \ K_G^1 \ \cdots \ K_G^{n_A} \right]^T$$

IF GNSS + LEO state vector

$$\mathbf{x}_{\text{GNSS+LEO}}^{\text{IF}} = \left[X \ Y \ Z \ dt_{G_1} \ \cdots \ dt_{G_{n_G}} \ dt_L \ ZWD \ K_G^1 \ \cdots \ K_G^{n_A} \ K_L^1 \ \cdots \ K_L^{n_L} \right]^T$$

- x_R, y_R, z_R : 3D Receiver position
- dt_G : Receiver clock offset relative to GNSS system time (1 clock offset per GNSS constellation)
- dt_L : Receiver clock offset relative to LEO-PNT system time.
- T : Tropospheric delay modeled as a zenithal error in the wet component.
- K : Ambiguities for GNSS carrier-phase and LEO carrier-phase (K_G, K_L)

EKF: STOCHASTIC MODEL

Process-noise and observation-noise covariance

Stochastic conditions on the dynamic model :

$$\mathbf{Q}_k = \text{diag}(\sigma_x^2) \Delta t_k$$

where σ is a vector of standard deviations corresponding to each state component:

- $\sigma_{xyz} = (\varepsilon, \varepsilon, \varepsilon)$: Receiver position assumed constant (quasi - kinematic).
- $\sigma_{\text{clk}, \text{GNSS}}$: GNSS clock, defined per GNSS constellation (*GPS*, *GALILEO*).
- σ_{tropo} : Tropospheric delay noise (Residual wet).
- $\sigma_{\text{clk}, \text{LEO}}$: LEO clock.
- $\sigma_{\text{amb}, \text{GNSS}} = \varepsilon$: Ambiguity noise for GNSS carrier-phase observations.
- $\sigma_{\text{amb}, \text{LEO}} = \varepsilon$: Ambiguity noise for LEO carrier-phase observations.

For: $\Delta t_k = 30 \text{ s}$

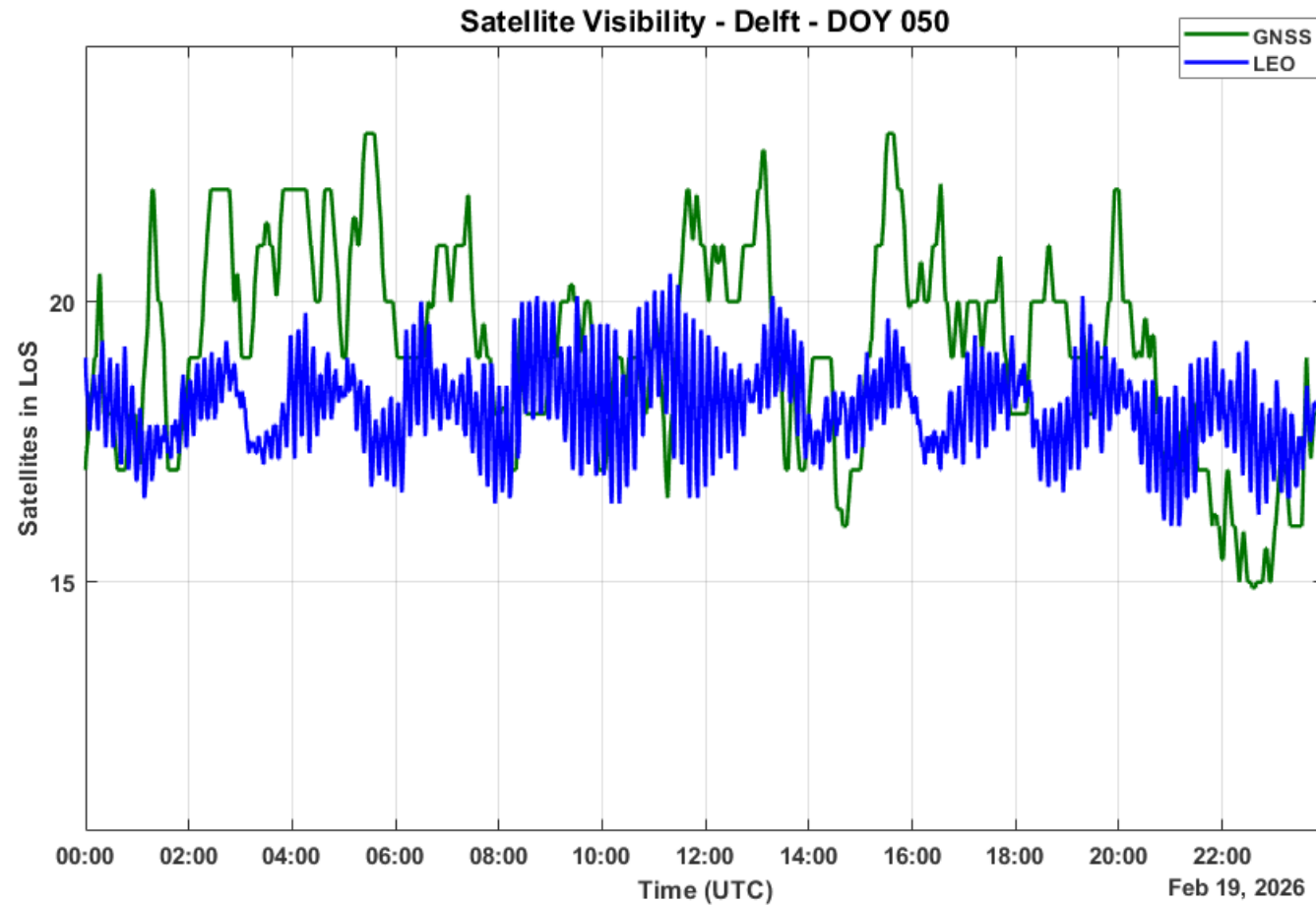
State component	IF system noise
X, Y, Z	1
GNSS clock	1000
LEO clock	1000
ZWD	4×10^{-4}
GNSS ambiguities K_{GNSS}	0
LEO ambiguities K_{LEO}	0

* Units are in $\text{m} / \sqrt{\text{sec}}$

Current hybrid IF

Satellite visibility at 5° cutoff

LoS Visibility GNSS and LEO over 24 hours / Station: Delft DLF1 / DOY: 50 / 19 Feb 2026



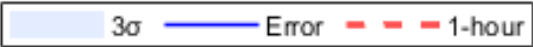
- The LEO constellation provides persistent extra geometry during the whole 24-hour interval.

	min	mean	max
GNSS	14	19	23
LEO-PNT	16	18	21

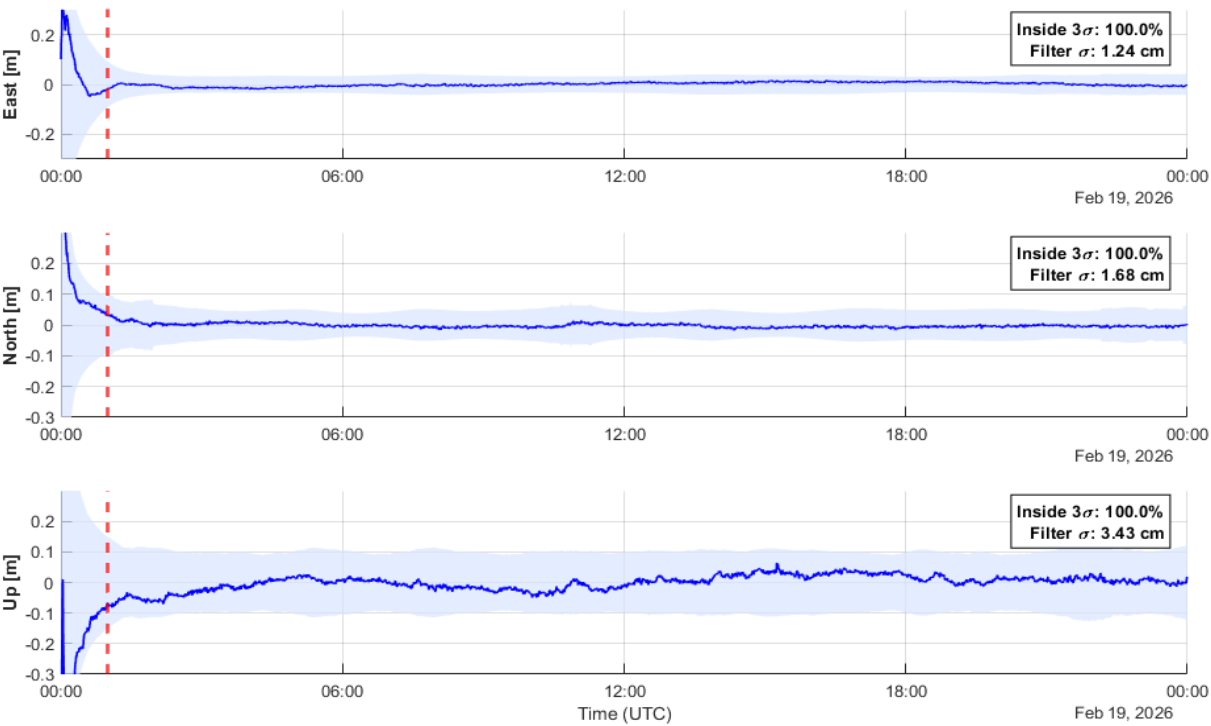
* Only for this experiment GNSS: GPS + Galileo

ENU errors and 3σ confidence

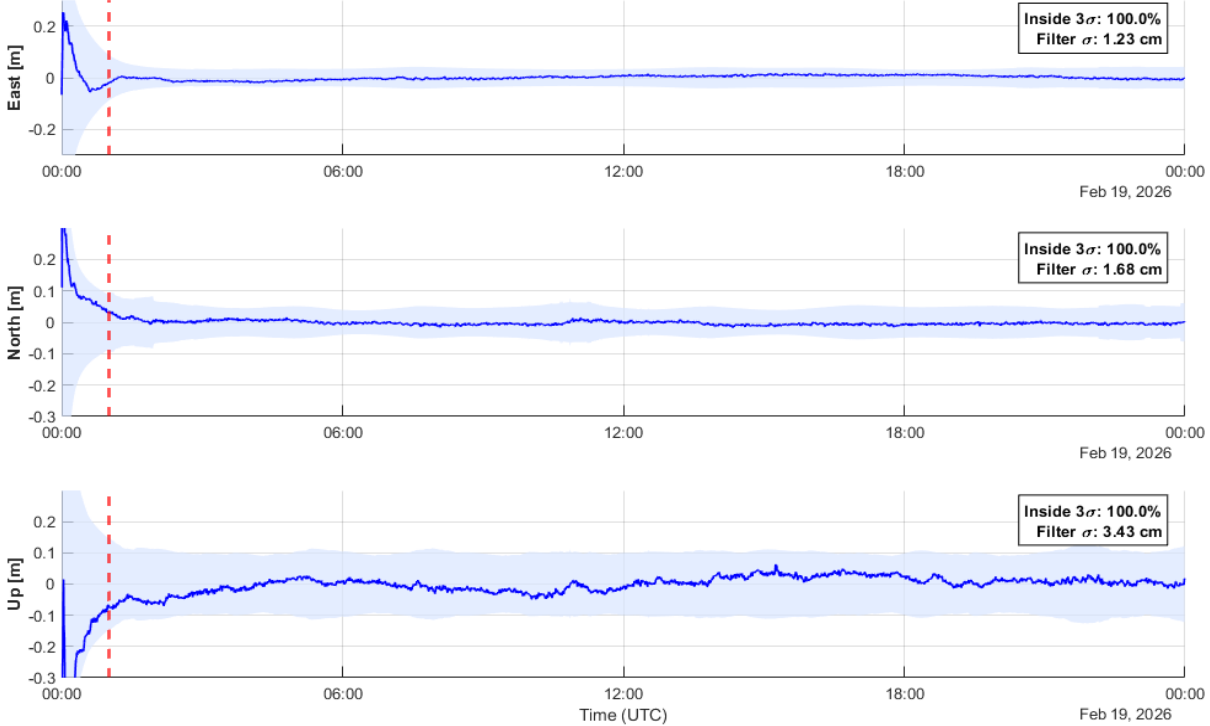
24 hour processing from Delft station / DOY: 50 / Feb 19, 2026



GNSS Only



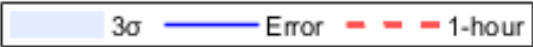
GNSS + LEO codes



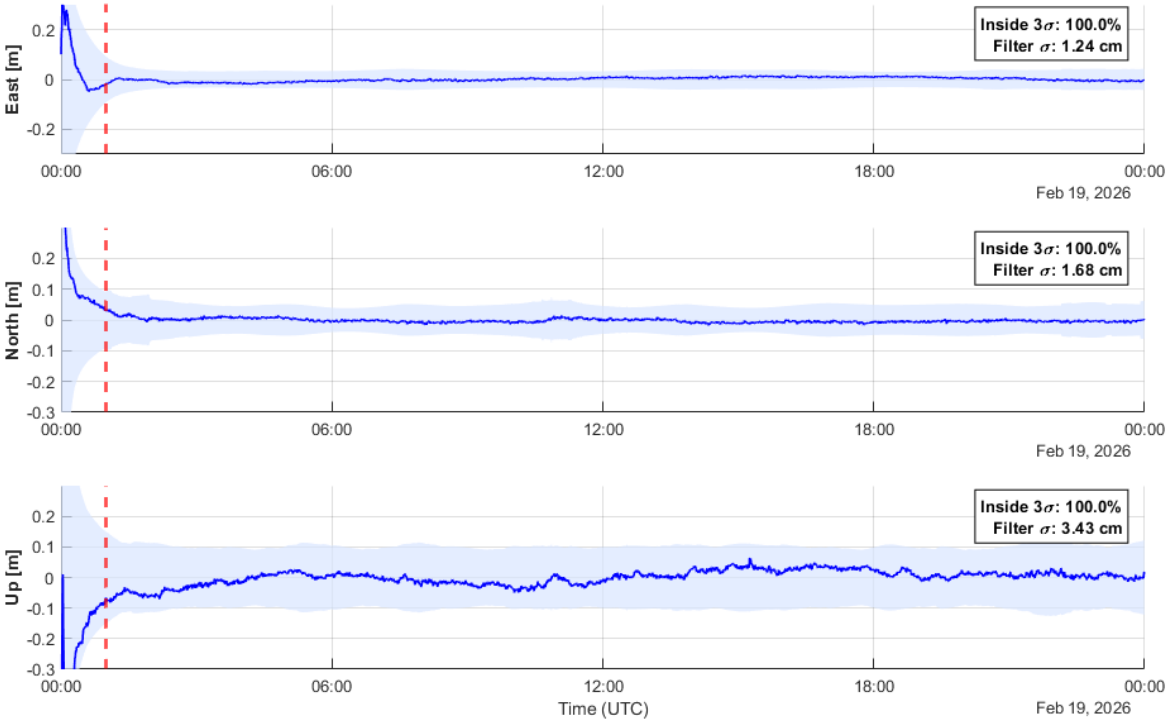
	σ East	σ North	σ Up
GNSS	1.24	1.68	3.43
GNSS +LEO (P)	1.23	1.66	3.43

ENU errors and 3σ confidence

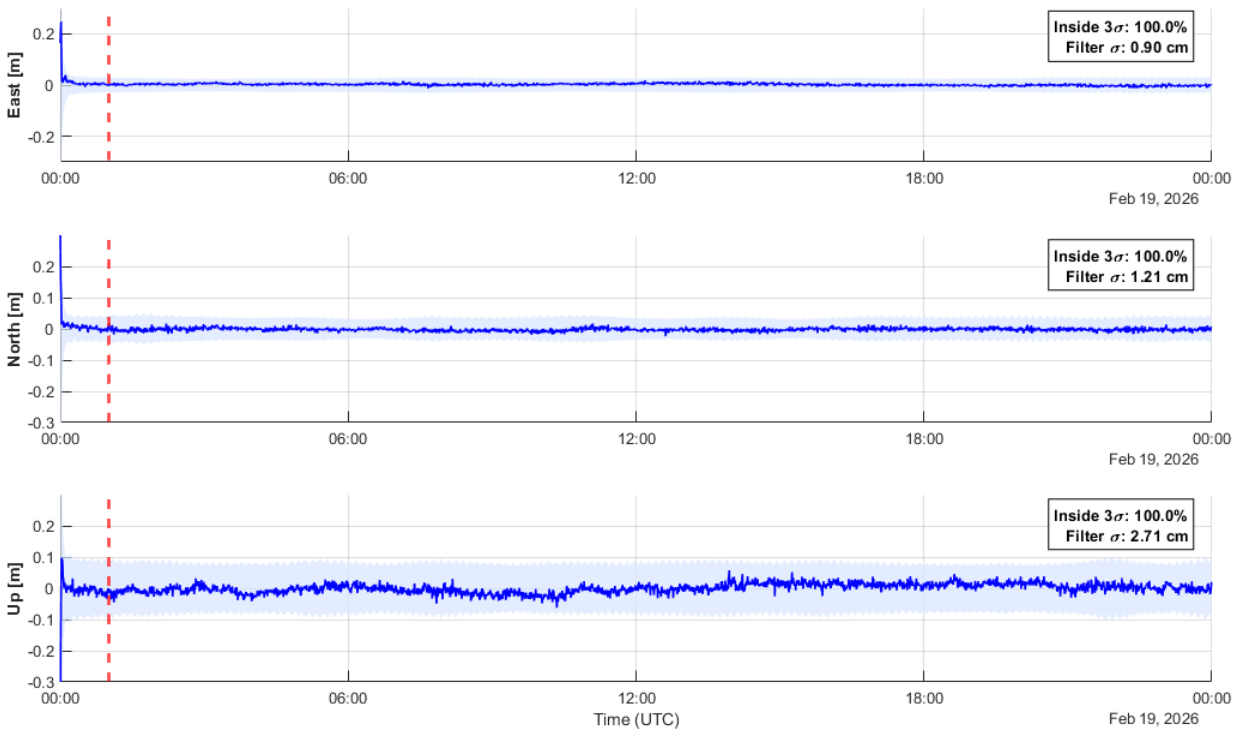
24 hour processing from Delft station / DOY: 50 / Feb 19, 2026



GNSS Only



GNSS + LEO (P + Φ)

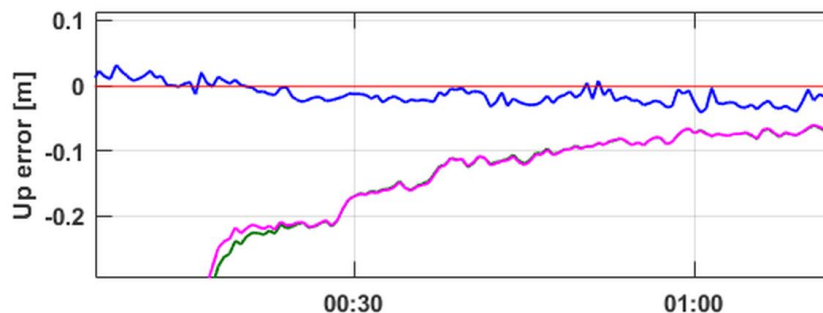
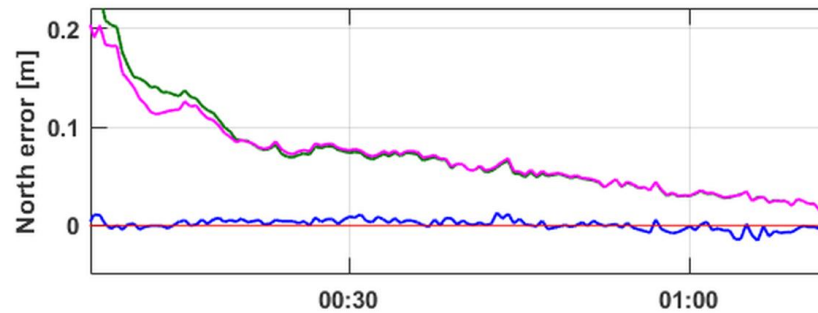
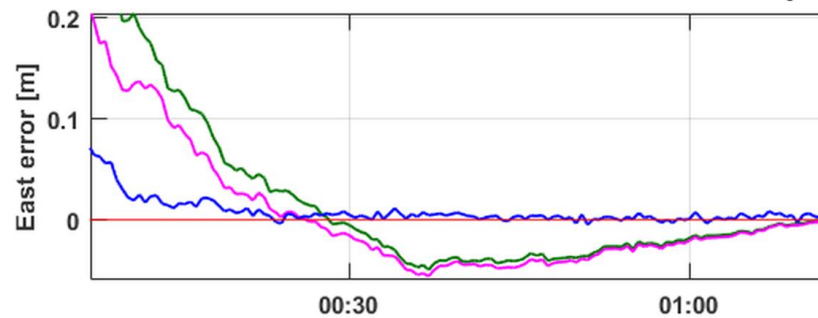
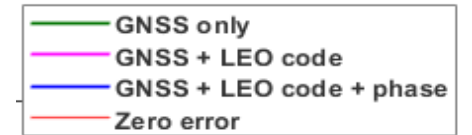


* Units : cm			
	σ East	σ North	σ Up
GNSS	1.24	1.68	3.43
GNSS +LEO(P+ Φ)	0.9	1.21	2.71

All three solutions remain inside the 3σ region; the LEO carrier+phase case shows the smallest reported filter standard deviations.

ENU error comparison - zoomed to 1^o Hour

24 hour processing from Delft station / DOY: 50 / Feb 19, 2026 / GNSS / GNSS + LEO codes / GNSS + LEO codes + phases



Metric [cm]	GNSS	GNSS + LEO (P)	GNSS + LEO (P + Φ)
Bias E	+0.07	+0.06	+0.11
Bias N	-0.21	-0.20	-0.13
Bias U	-0.06	-0.05	+0.01
Std E	0.82	0.81	0.45
Std N	0.66	0.65	0.53
Std U	2.39	2.38	1.51
P50 3D	1.90	1.90	1.31
P95 3D	4.92	4.83	3.01
P99 3D	6.69	6.66	3.84

* Metrics are calculated after the first hour.

- Biases remain very small for all three solutions: $|\text{Bias E,N,U}| < 0.25$ cm
- LEO codes alone have negligible impact on the 24-hour solution.
- LEO carrier phase reduces the vertical scatter from 2.39 cm to 1.51 cm and P95 3D error from 4.92 cm to 3.01 cm.

Moving – window convergence test

Sensitivity test to select the parameters of the convergence detector

Purpose: Identify when the PPP position solution becomes **stable over time**.

For each moving time window W_i , the mean ENU position is computed:

$$\bar{\mathbf{p}}_i = \begin{bmatrix} \bar{E}_i \\ \bar{N}_i \\ \bar{U}_i \end{bmatrix}$$

Three consecutive windows are considered:

$$W_1, W_2, W_3$$

The 3D distance between the mean positions of **every pair of windows** is evaluated:

$$d_{ij} = \|\bar{\mathbf{p}}_i - \bar{\mathbf{p}}_j\|_2 = \sqrt{(\bar{E}_i - \bar{E}_j)^2 + (\bar{N}_i - \bar{N}_j)^2 + (\bar{U}_i - \bar{U}_j)^2}$$

Therefore, d_{12}, d_{13}, d_{23} are computed and the stability condition is:

$$\max(d_{12}, d_{13}, d_{23}) < T$$

where T is the selected 3D stability threshold.

The total time interval covered by the three confirming windows is:

$$T_{\text{span}} = T_{\text{window}} + 2T_{\text{slide}}$$

Configuration	Window length T_{window}	Sliding interval T_{slide}	3D threshold T	Confirmation span T_{span}
1	5 min	2.5 min	1 cm	10 min
2	5 min	2.5 min	2 cm	10 min
3	10 min	2.5 min	2 cm	15 min
4	10 min	5 min	2 cm	20 min

Table: Moving-Window Convergence Test with the tested configurations

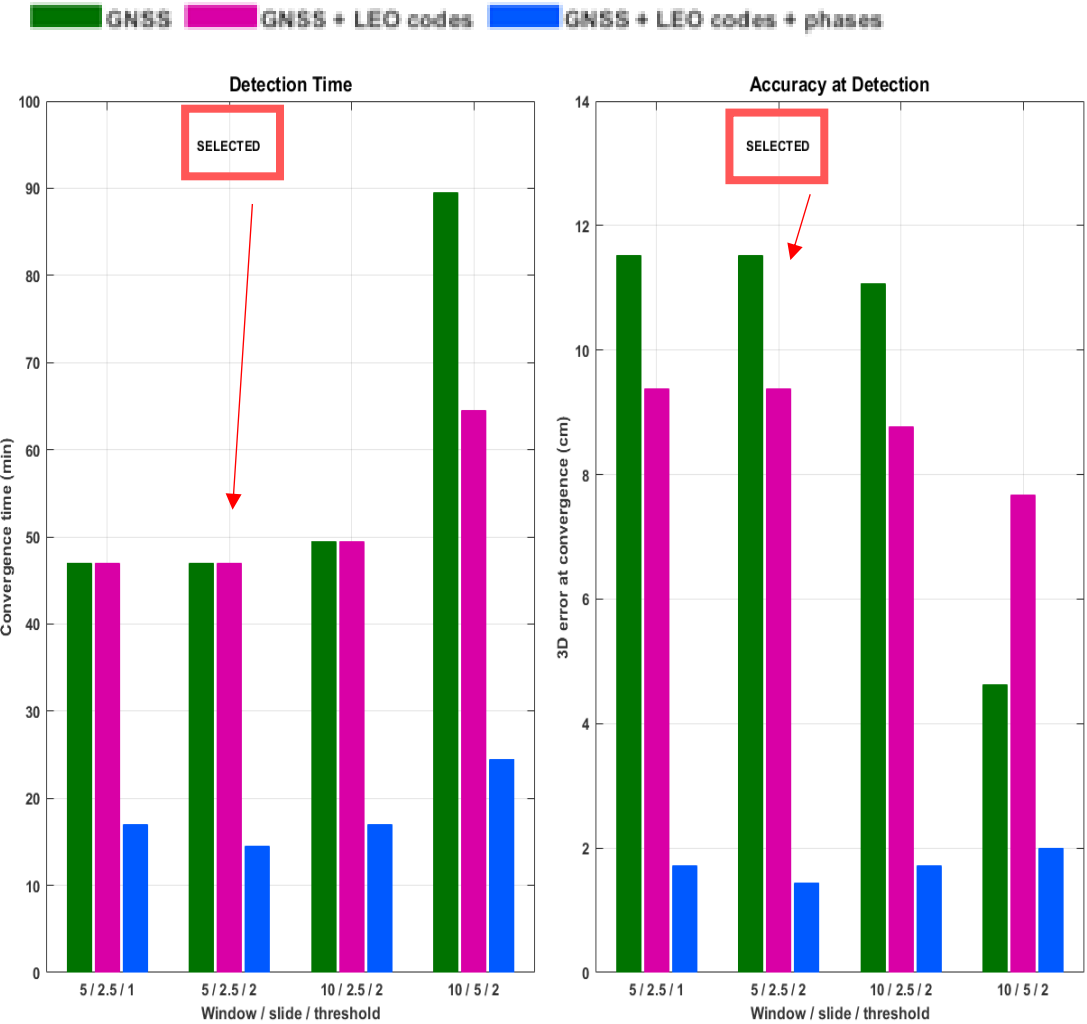
Three consecutive moving windows are required to confirm convergence.

- T_{window} : length of each averaging window.
- T_{slide} : time shift between two consecutive windows.
- T : maximum allowed 3D separation between the window-mean positions.
- T_{span} : total time interval covered by the three windows used to confirm convergence.

Measuring stability of the solution, not absolute accuracy

Moving – window convergence test

Sensitivity test to select the parameters of the convergence detector



The convergence condition is: $d_{max} < T$

Configuration $T_{window}/T_{slide}/T$	Maximum pairwise window-mean separation [cm] $G/C/P$	3D position error at convergence epoch [cm] $G/C/P$	Convergence time [min] $G/C/P$
5 / 2.5 / 2	0.87 / 0.82 / 1.09	11.52 / 9.38 / 1.44	47.0 / 47.0 / 14.5

Table: Moving-Window Convergence Tests with results

G: GNSS,
C: GNSS + LEO codes,
P: GNSS + LEO codes & phases

- Selected configuration: 5/2.5/2**
- Selected as the best compromise between stability and positioning accuracy.
- 5-min window:** sufficient averaging of short-term position variations.
 - 2.5-min slide:** 50% overlap between consecutive windows, providing good temporal resolution.
 - 2-cm threshold:** allows centimetric fluctuations while still requiring the solution to remain stable.
 - 10-min confirmation span:** convergence is confirmed not from a single short interval.

Convergence test on 2-hours sessions

Verifying the **accuracy** of the solution when convergence is declared

Purpose: Evaluate whether the convergence improvement is **repeatable for different session start times and satellite geometries**, rather than being specific to one processing interval.

Test setup:

- The dataset is divided into **22 separate 2-hour sessions**.
- Each session is processed independently, starting with a new **first-epoch LS initialization**, followed by the EKF.
- The same elevation cutoff and stochastic model are used for all sessions.
- Convergence is detected using the selected moving-window criterion:

Selected configuration:

5 / 2.5 / 2

- 5 min window
- 2.5 min slide
- 2 cm 3D threshold

Quantities evaluated for each session

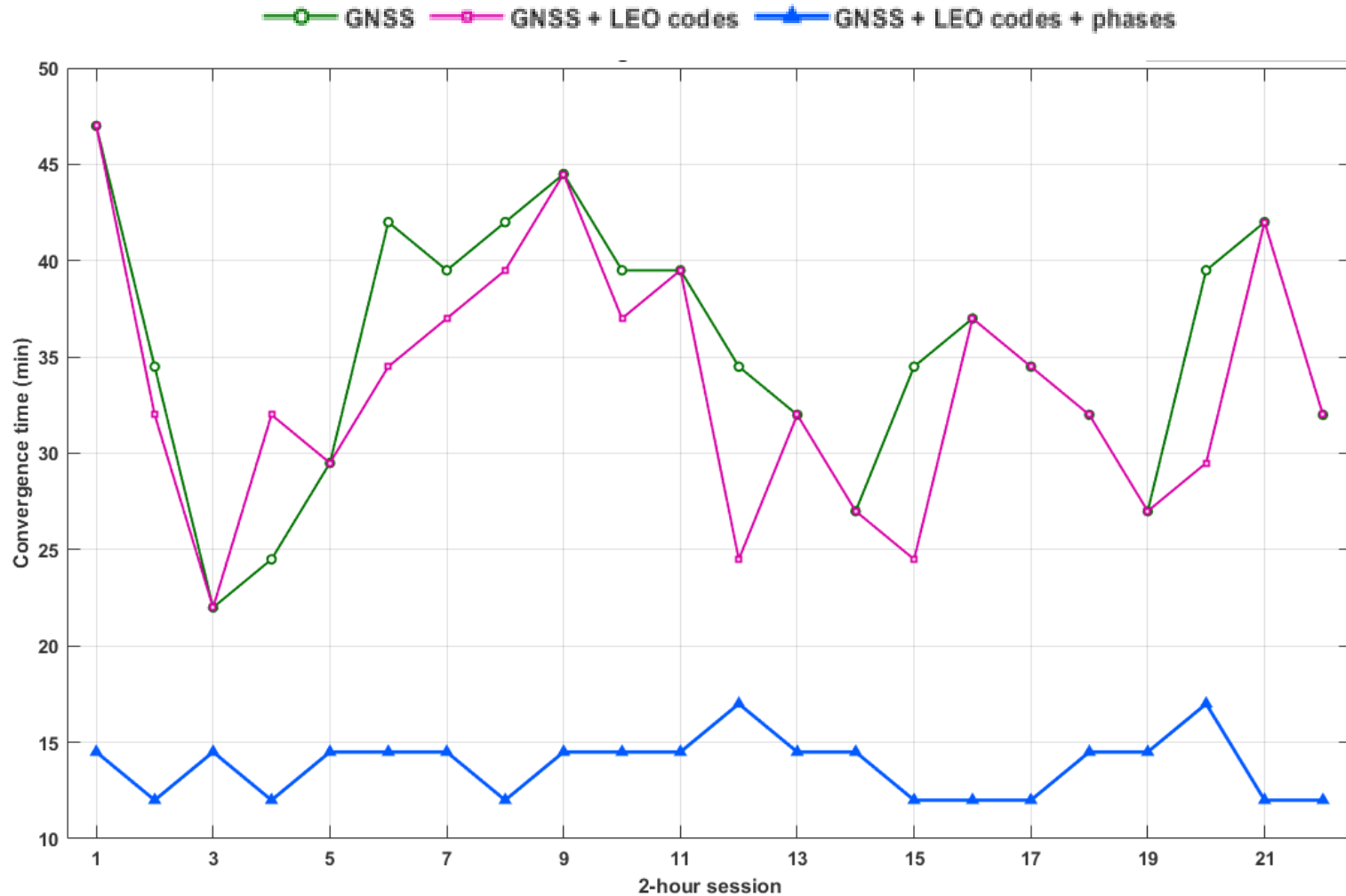
For every 2-hours session, two quantities are extracted:

- t_{conv} : Convergence time: epoch at which the moving-window stability criterion is first satisfied.
- $e_{3D}(t_{conv})$: 3D position error at convergence: positioning accuracy at the detected convergence epoch.

$$e_{3D}(t_{conv}) = \sqrt{e_E^2 + e_N^2 + e_U^2}$$

Convergence test on 2-hours sessions

Verifying the **accuracy** of the solution when convergence is declared



The 22-session analysis verifies that the faster convergence obtained with LEO carrier phases is repeatable across different session start times and satellite geometries, rather than being a result of a single favourable interval.

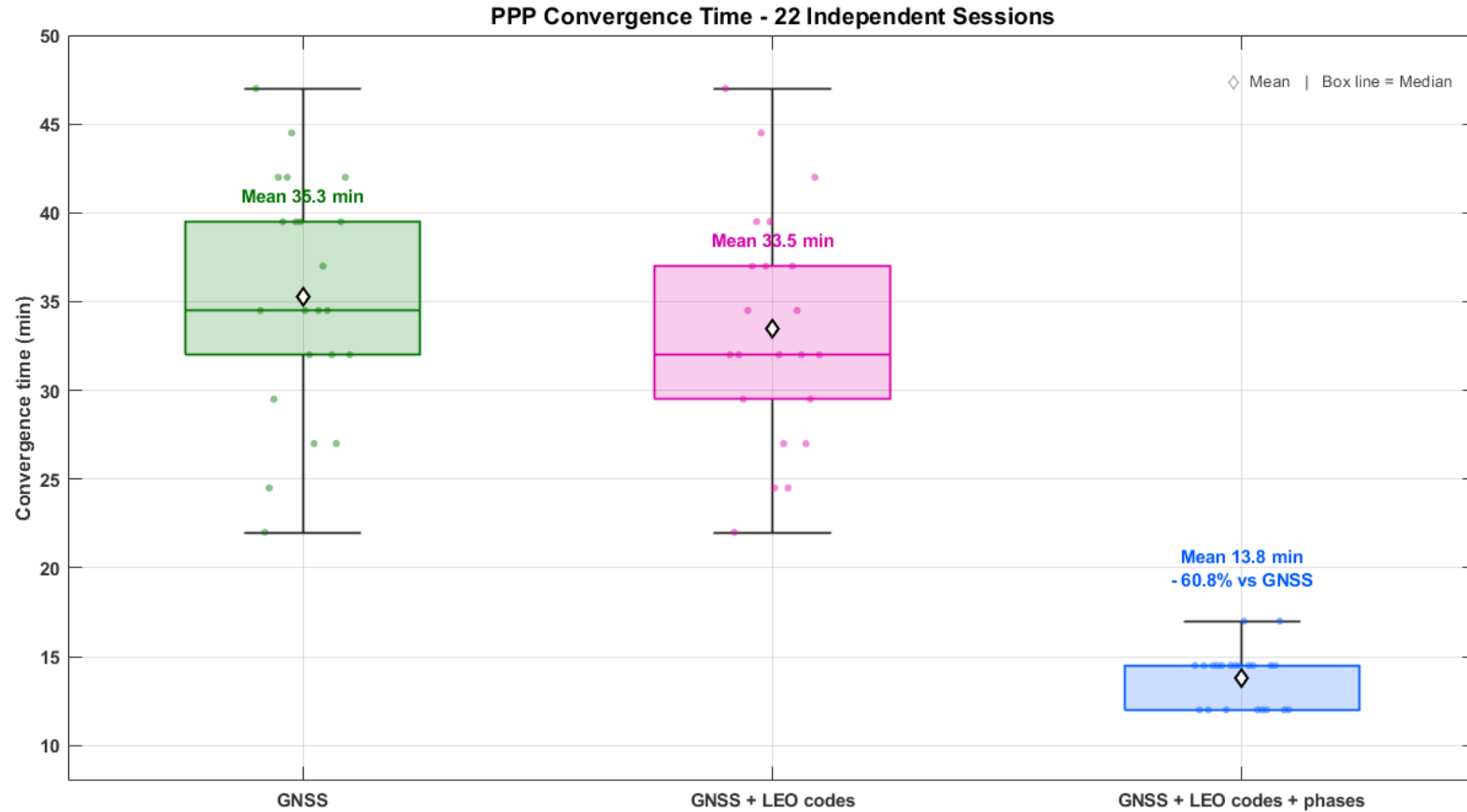
GNSS mean convergence : 35.30 min

GNSS + LEO (code + phase) convergence : 13.82 min

*Mean convergence improvement : **21.48 min***

Convergence test on 2-hours sessions

Verifying the **accuracy** of the solution when convergence is declared



LEO codes alone provide only a limited reduction in convergence time:

35.3 min → 33.48 min

Introducing **LEO carrier-phase observations** produces a large and consistent improvement:

35.30 min → 13.82 min

corresponding to **60.8 %**.

Processing	Mean conv. time (min)	Reduction vs GNSS
GNSS	35.30	-
GNSS + LEO	33.48	5.2 %
GNSS + LEO (P + Φ)	13.82	60.8 %

Final accuracy of the 2-hours sessions

Post – Convergence Positioning Performance

* 22 : 2-hours sessions — statistics evaluated after the first hour

Processing	Std E (cm)	Std N (cm)	Std U (cm)	3D RMS (cm)	P30 of 3D error (cm)	P50 of 3D error (cm)	P95 of 3D error (cm)
GNSS	0.55	0.40	1.05	3.06	2.38	2.83	4.49
GNSS + LEO (P)	0.52	0.40	1.04	2.98	2.32	2.77	4.37
GNSS + LEO (P+Φ)	0.29	0.39	0.95	1.57	1.03	1.34	2.58

Table: Mean statistics over the 22: 2-hours sessions

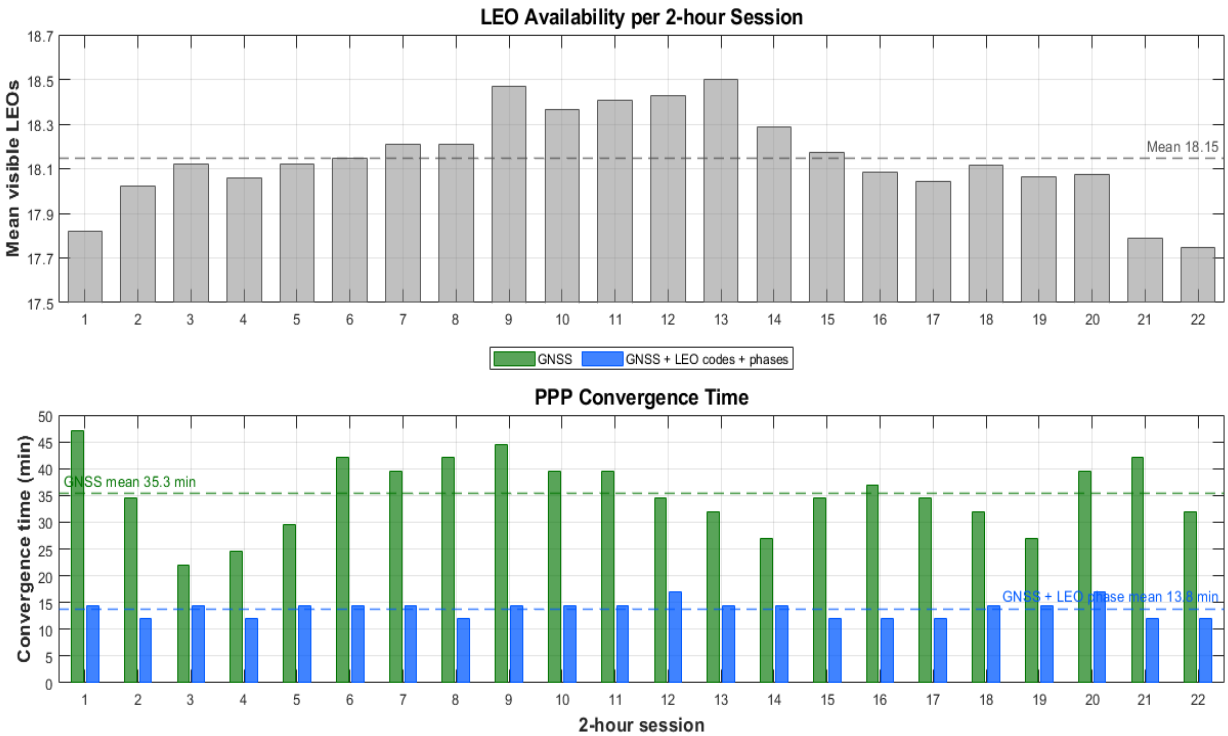
LEO codes alone provide less changes in the post-convergence ENU metrics, while adding LEO carrier phases strongly improves them.

Processing	P30 (%)	P50 (%)	P95 (%)
GNSS + LEO (P+Φ)	56.7	52.7	42.5

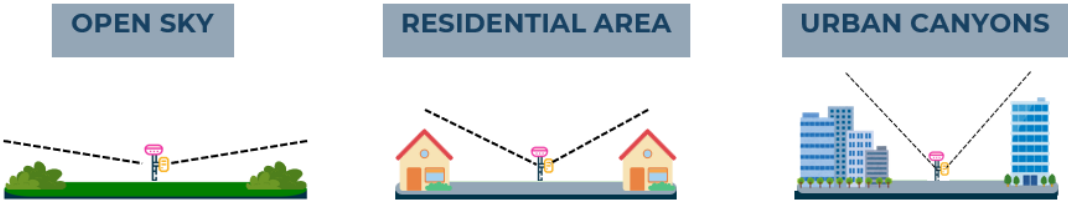
Table: The percentage reduction of percentiles when adding LEO codes and carrier - phase observations, using GNSS only as reference.

Other possible analysis

Possible correlations between the satellite availability and the PDOP



Satellite Count or Geometry ?!



SESSION DURATION

CUTOFF	10 MIN	5 MIN	2 MIN
5°	99.3%	99.6%	99.6%
30°	93.4%	98.9%	99.5%
50°	94.4%	82.1%	1.8%

Study results

Percentage improvement in PDOP values when **integrating LEO satellites with GNSS**, for different session durations and cutoff angles.



“ Improvements in PPP by Integrating GNSS with LEO Satellites: A Geometric Simulation ” Marianna Alghisi, Nikolina Zallemei, Ludovico Biagi

Conclusions & Future Work

General conclusions on 1st preliminary test

- The results confirm that future LEO-PNT signals can act as a complementary layer to GNSS, improving PPP performance especially during the convergence phase.

Benefits of LEO integration

- **LEO carrier-phase integration reduced PPP convergence time by about 61%**, showing a substantial acceleration of solution stabilization.
- **Steady-state 3D positioning performance improved by about 39% in terms of P_{95} error**, indicating a clear reduction of the upper-tail positioning error.
- The improvement is repeatable across different session start times and is not explained only by the number of visible LEO satellites; the rapidly changing LEO geometry is a key factor.

Future work (at TU Delft)

- Extend the framework toward **GNSS + LEO-PNT atmospheric remote sensing**.
- Investigate the use of LEO geometry for **ionospheric parameter estimation** and spatial reconstruction.
- Move from positioning-only analysis toward integrated **PPP + atmospheric sensing** applications.

The next question is therefore no longer only how LEO-PNT can improve positioning, but how this new observational geometry can also be exploited to observe the atmosphere.

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**THANK YOU FOR
YOUR ATTENTION !**

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