

Learning-Based Reliable Integer Ambiguity Resolution for GNSS: A Contrastive Deep Neural Approach

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Problem Definition and Motivation

GNSS Integer Ambiguity Resolution (IAR) for Precise Positioning: Estimation and Validation

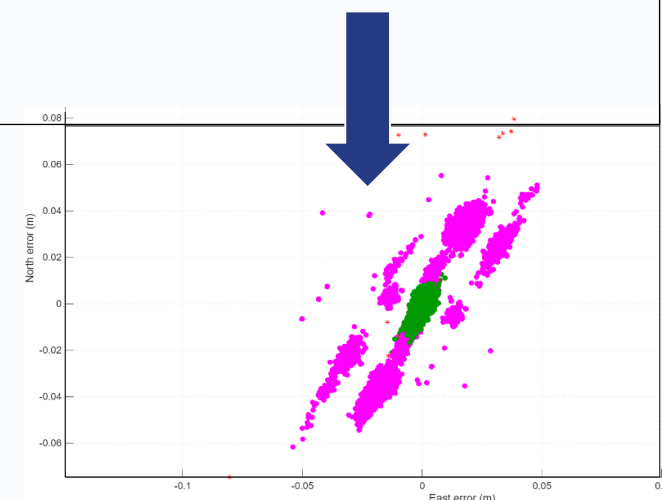
$$\bullet \quad y = Aa + Bb + \eta, \begin{cases} y \in \mathbb{R}^m, a \in \mathbb{Z}^n, b \in \mathbb{R}^p, \eta \in \mathbb{R}^m \sim \mathcal{G}(0, Q_y) \\ A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{m \times p}, Q_y \in \mathbb{R}^{m \times m} \end{cases} \rightarrow \begin{cases} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} A^T \\ B^T \end{bmatrix} Q_y^{-1} [A \ B] Q_y^{-1} y, \hat{a} \in \mathbb{R}^n, \hat{b} \in \mathbb{R}^p \\ \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} \sim \mathcal{G} \left(\begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} Q_{\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}} \end{bmatrix} \right), Q_{\hat{a}} \in \mathbb{R}^{n \times n}, Q_{\hat{b}} \in \mathbb{R}^{p \times p}, Q_{\hat{b}\hat{a}} \in \mathbb{R}^{p \times n} \end{cases}$$

$$\begin{array}{l} \hat{a}, Q_{\hat{a}} \rightarrow \begin{cases} \hat{z} = Z^T \hat{a} \\ Q_{\hat{z}} = Z^T Q_{\hat{a}} Z \end{cases} \rightarrow \begin{array}{l} \text{IAE class} \\ \check{z} = S(\hat{z}) = \begin{cases} \sum_{z \in \mathbb{Z}^n} \omega_z(\hat{z}) z, & \hat{z} \in \Omega \\ [], & \hat{z} \notin \Omega \end{cases} \\ \omega_z(\hat{z}) = \begin{cases} 1, & \hat{z} \in \Omega_z \\ 0, & \text{else} \end{cases}, \quad \Omega_z = \Omega \cap S_z, \\ \mathcal{J}(\hat{z}) = \begin{cases} \{1, \dots, n\}, & \hat{z} \in \Omega \\ \emptyset, & \hat{z} \notin \Omega \end{cases}, \quad S: \mathbb{R}^n \rightarrow \mathbb{Z}^n \cup \mathbb{Z}^0. \end{array} \end{array} \rightarrow \begin{cases} \check{a} = Z^{-T} \check{z}, & \hat{z} \in \Omega \\ \hat{a}, & \hat{z} \notin \Omega \end{cases} \rightarrow \begin{cases} \check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} (\hat{a} - \check{a})^T \\ \hat{b}, \hat{a} \end{cases}$$

$$\bullet \quad \eta = \varepsilon_{\text{noise}} + \delta_{\text{mis}} + \beta_{\text{unmod}} \Rightarrow \hat{a} = a + e_a \Rightarrow \Pr(\check{a}_1 \neq a) > 0$$

Bridge to next slide

Can Artificial Intelligence (AI) be exploited for Reliable Ambiguity Resolution (RAR) in the validation phase of the IAR optimization framework?



Theoretical Gap

❑ Artificial Intelligence (AI) for Reliable Integer Ambiguity Resolution (RAR) in GNSS

H1 – Simultaneous performance superiority

Conditional Success Rate: $P_S = P(\text{correct} \mid \text{fix})$

Fixing Rate (availability): $P_f = P(\text{fix})$

$$H_1: (P_S^{\text{AI}} > P_S) \wedge (P_f^{\text{AI}} > P_f)$$

$$H_0: (P_S^{\text{AI}} \leq P_S) \vee (P_f^{\text{AI}} \leq P_f)$$

Both reliability and fixing availability must improve

Existing AI validation models exhibit a P_S^{AI} - P_f^{AI} availability trade-off

H2 – Recovery beyond the ILS optimum when $\check{a}_1 \neq a$

True integer a rank, $R_{\text{true}} = \text{rank}(\mathbf{a}_{\text{true}}) \in \{2, \dots, k\}$

AI scores candidates $p_1, \dots, p_k$, $\hat{R} = \arg\max_{j \in \{1, \dots, k\}} p_j, p_{\max} > \tau_0$

$$H_2: P_{\text{rec,sub}} > P_{\text{rec,sub}}^0$$

$$H_{2,0}: P_{\text{rec,sub}} \leq P_{\text{rec,sub}}^0$$

$$P_{\text{rec,sub}} = P(\hat{R} = R_{\text{true}}, p_{\max} > \tau_0 \mid R_{\text{true}} \in \{2, \dots, k\})$$

Can AI-based validation recover the true integer candidate?

Existing AI & conventional validation validate only $\check{a}_1$

Why multi-integer candidate ambiguities validation matters?

Top-K ILS integer ambiguity candidates ordered by squared norm:

$$\mathbf{R}_K = \{\check{\mathbf{a}}_k\}_{k=1}^K, L_1 \leq \dots \leq L_K, \check{\mathbf{a}}_1 = \arg\min_{\check{\mathbf{a}} \in \mathbf{R}_K} f_{\text{ILS}}(\hat{\mathbf{a}}), \mathcal{V}_*(\check{\mathbf{a}}_1) \in \{\text{A}, \text{R}\}, * = (\text{conv}, \text{AI}) \quad f_{\text{ILS}}(\hat{\mathbf{a}}) = (\hat{\mathbf{a}} - \mathbf{a})^\top \mathbf{Q}_{\hat{\mathbf{a}}}^{-1} (\hat{\mathbf{a}} - \mathbf{a})$$

Bridge to sub-sequent slides

How should the AI-based Validation model be designed for real time efficient implementation and simultaneously test H1 and H2?

Architecture Design of AI-based Validation Model

Machine Learning (ML) methodological dimensions: learning paradigm, training paradigm, and modeling paradigm.

Learning Paradigm

Generative: learn representations by modelling $p(x)$

Discriminative: learn representations by modelling $p(y|x)$

Contrastive: learn representations by modelling $x^+ \sim p^+(\cdot|x)$ and $x^- \sim p^-(\cdot|x)$

Training Paradigm

Unsupervised: $x \rightarrow$ structure / representation (no target)

Supervised: $x \rightarrow y$ (external target)

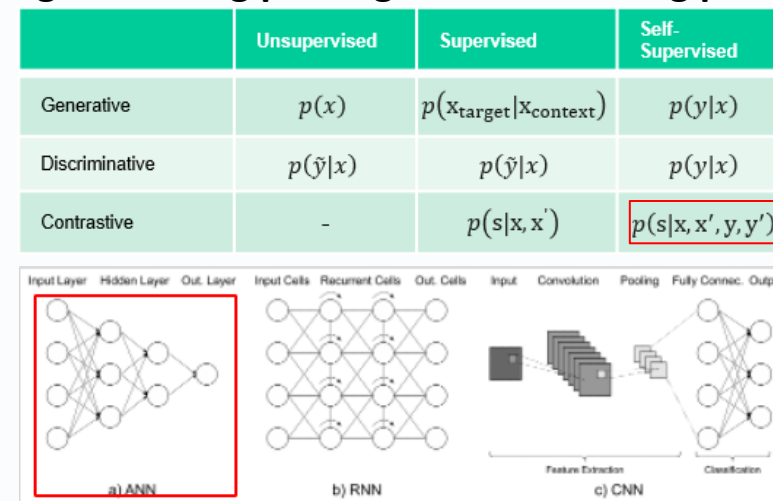
Self-supervised: $x \rightarrow \tilde{y} = T(x)$ (target generated from data)

Model Paradigm

ANN: $\mathbf{h}^{(0)} = \mathbf{x}$, $\mathbf{h}^{(\ell)} = \phi^{(\ell)}(\mathbf{W}^{(\ell)}\mathbf{h}^{(\ell-1)} + \mathbf{b}^{(\ell)})$, $\ell = 1, \dots, L$, $\hat{\mathbf{y}} = \mathbf{h}^{(L)}$.

RNN: $\mathbf{h}_t = \phi(\mathbf{W}_{xh}\mathbf{x}_t + \mathbf{W}_{hh}\mathbf{h}_{t-1} + \mathbf{b}_h)$, $\hat{\mathbf{y}}_t = g(\mathbf{W}_{hy}\mathbf{h}_t + \mathbf{b}_y)$.

CNN: $\mathbf{h}_k^{(\ell)} = \phi^{(\ell)} \sum_{c=1}^{C_{\ell-1}} \mathbf{K}_{k,c}^{(\ell)} * \mathbf{h}_c^{(\ell-1)} + b_k^{(\ell)}$, $k = 1, \dots, C_{\ell}$.



AI model design

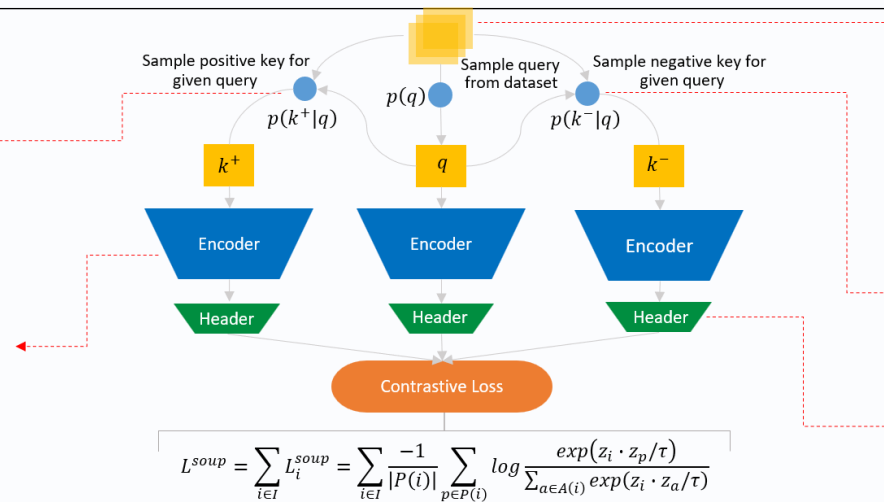
2

Label for **positive sample**:

- 2D coordinates < 5 cm and
- $f_{ILS}(\tilde{a}) = f_{ILS}(a)$

4

Fully Connected Neural Network, which **extracts meaningful representation** from the input features



1

Features vector x from which meaningful **representation** can be extracted for **deciding** whether to **accept the fixed solution** or **reject it**

3

Label for **negative sample**:

- 2D coordinates ≥ 5 cm
- $f_{ILS}(\tilde{a}) \neq f_{ILS}(a)$

5

Used **only during training** to compute contrastive loss, as a Multi-Layer Perception

Input Features

Mathematical construction from ILS candidates to AI input and training datasets, through PPP-RTK Monte Carlo simulations

1 Estimation → Feature Generation → Labels

$$\begin{aligned} \text{PPP-RTK Simulation Engine} \left\{ \begin{array}{l} \alpha, \hat{\alpha}, Q_{\hat{\alpha}} \xrightarrow{\text{LAMBDA}^*} [\check{\alpha}_1, \check{\alpha}_2, \dots, \check{\alpha}_K]^T, \text{ where } \check{\alpha}_K = [\check{\alpha}_{i,1}, \dots, \check{\alpha}_{i,n}]^T, i = 1, \dots, n \quad [\{\check{\alpha}_1, l_{1,1}\}, \dots, \{\check{\alpha}_K, l_{1,K}\}] \quad l_{1,K} = 1, \text{ if } \check{\alpha}_K = a, \text{ else } l_{1,K} = 0 \\ b, \hat{b}, Q_{\hat{b}}, Q_{\hat{b}\hat{a}} \rightarrow \check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} (\hat{a} - \check{\alpha}_K) \rightarrow [\{\Delta\check{b}_1, l_{2,1}\}, \dots, \{\Delta\check{b}_K, l_{1,K}\}] \quad l_K = 1, \text{ if } |\check{b}_K - b| < 5\text{cm} \text{ else } l_K = 0 \end{array} \right. \end{aligned}$$

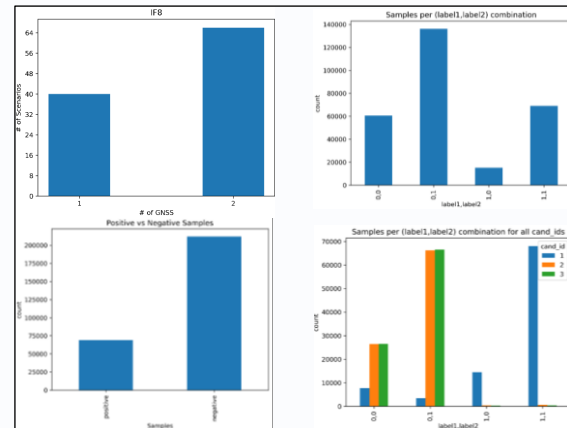
2 Generic feature construction

$$\{x_K^s, y_K^s\} = \left\{ \Phi \left(\tilde{f}_{ILS}(\hat{\alpha}), \frac{R_2}{R_1}, \frac{R_3}{R_1}, R_2 - R_1, R_3 - R_1, \text{ADOP}, \text{HDOP}, \# \text{GNSS}, \#f, \# \hat{a}, \# \check{\alpha} \right), y_K^s \right\}, s = 1, 2, \dots, S, \tilde{f}_{ILS}(\hat{\alpha}) = \frac{R_K - E\{f_{ILS}(\hat{\alpha})\}}{\sqrt{D\{f_{ILS}(\hat{\alpha})\}}}, R_K = \|\hat{a} - \mathbf{a}_K\|_{Q_{\hat{a}}}^2$$

Positive class: $\check{\alpha}_{ILS}' \equiv \check{\alpha} \&\& |\check{b} - b| \leq 5 \text{ cm}$ Negative class: $\check{\alpha}_{ILS}' \not\equiv \check{\alpha} \parallel |\check{b} - b| \geq 5 \text{ cm}$

3 Augmentation (label retain)

IF#	Min	Max	Mean	Median	Std	Q1	Q3
IF1	-8.48	403.85	25.42	2.71	56.02	-3.99	27.80
IF2	1	228431.80	3917.53	1007.43	9089.83	122.44	3921.05
IF3	1	229442.54	3945.78	1027.20	9115.99	128.22	3959.46
IF4	0.00	2099.66	183.92	96.09	252.51	32.06	215.25
IF5	0.00	2099.93	185.92	97.97	253.23	33.50	217.95
IF6	0.01	3.16	0.08	0.04	0.12	0.02	0.07
IF7	0.14	804.46	2.23	1.03	6.17	0.43	3.44
IF8	1	2	1.67	2	0.47	1	2
IF9	2	3	2.56	3	0.5	2	3
IF10	8	48	23.3	21	9.06	18	28
IF11	8	48	23.03	21	9.06	18	28



$$(x, y), x \in \mathbb{R}^d, y \in \mathcal{Y}, \quad \mathcal{A} : \mathbb{R}^d \times \mathcal{Y} \rightarrow \mathbb{R}^d \times \mathcal{Y}, x_K^{s'} = [\mathcal{A}_1(x_1), \dots, \mathcal{A}_d(x_d)]^T$$

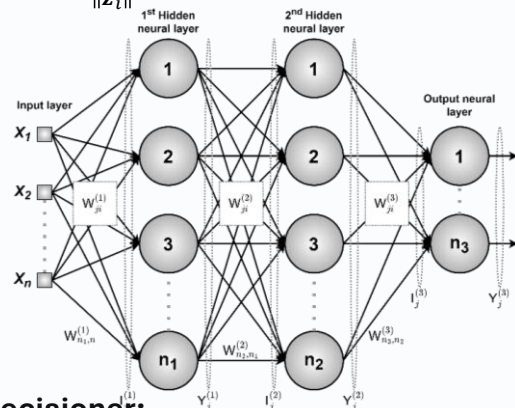
$$\hat{a}' \sim N(a, Q_{\hat{a}}) \left\{ \begin{array}{l} \mathcal{A}_1(\hat{a}', a'_1, Q_{\hat{a}}, n) \rightarrow \begin{cases} f_{ILS}(\hat{a}', a'_1, Q_{\hat{a}}) \\ \frac{f_{ILS}(\hat{a}', a'_1, Q_{\hat{a}}) - n}{\sqrt{2n}}, \text{ if IF}_1 \end{cases} \\ \mathcal{A}_2(\hat{a}', a'_1, a'_k, Q_{\hat{a}}) \rightarrow \begin{cases} f_{ILS}(\hat{a}', a'_1, Q_{\hat{a}}) \\ f_{ILS}(\hat{a}', a'_k, Q_{\hat{a}}) \\ \begin{cases} \frac{R_k}{R_1}, & \text{if IF}_2, \text{IF}_3 \\ R_k - R_1, & \text{if IF}_4, \text{IF}_5 \end{cases} \end{cases}, \text{ else} \end{array} \right.$$

Low-Level Architecture of AI-based Validation Model

Forward Propagation

Contrastive framework:

1. $Enc^{[0]}(X) = X$,
2. $Enc^{[L_E]}(X) = f(W^{[L_E]}Enc^{[L_E-1]}(X) + b^{[L_E]})$ for $L_E = 1, \dots, l_E$
3. $R = Enc(X) \in \mathbb{R}^{D_E \times m}$
4. $\hat{R}_i = \frac{R_i}{\|R_i\|}$, where $i = 1, \dots, m$
5. $Proj^{[0]}(\hat{R}_i) = \hat{R}_i$,
6. $Proj^{[L_H]}(X) = f(W^{[L_H]}Proj^{[L_H-1]}(X) + b^{[L_H]})$ for $L_H = 1, \dots, l_H$
7. $Z = Proj(\hat{R}) \in \mathbb{R}^{D_H \times m}$
8. $\hat{Z}_i = \frac{Z_i}{\|Z_i\|}$, where $i = 1, \dots, m$



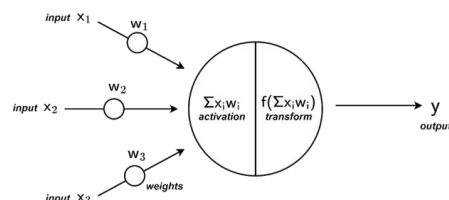
Decisioner:

1. $MLP^{[0]}(Y) = \hat{r}$
2. $MLP^{[L]}(Y) = f(W^{[L]}MLP^{[L-1]}(Y) + b^{[L]})$ for $L = 1, \dots, l_E$
3. $R = MLP(X) \in \mathbb{R}$

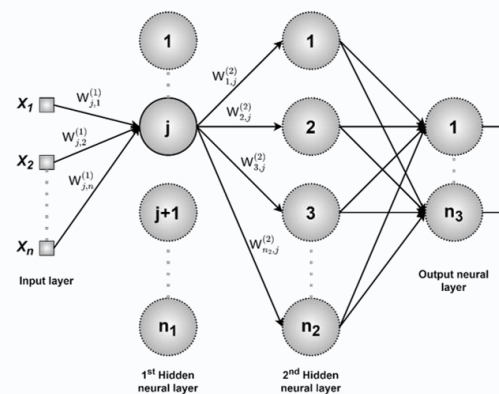
Back Propagation

Contrastive framework:

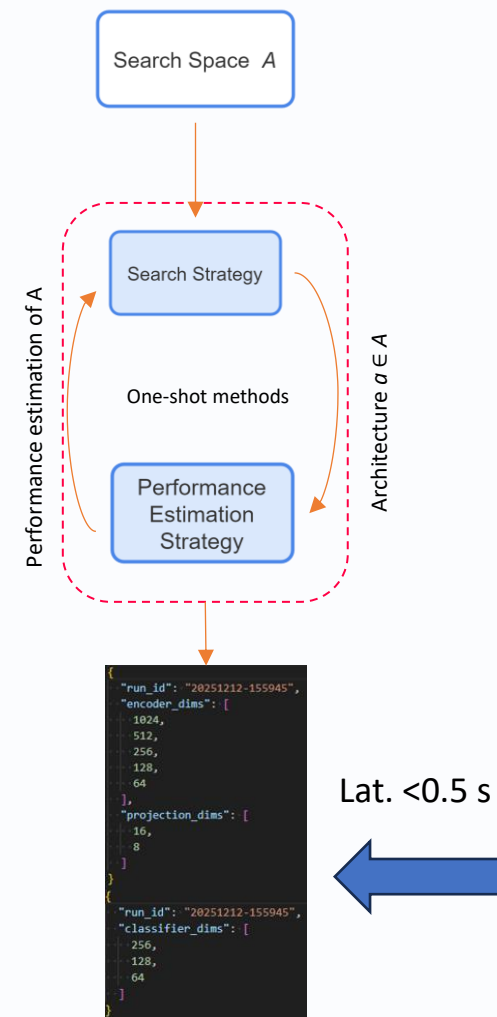
1. $\frac{\partial \mathcal{L}}{\partial Y^{[n_H]}} = ColumnProj\left(\frac{\partial \mathcal{L}}{\partial Z}, Y^{[n_H]}\right)$
2. $s^{[L]} = \left(\frac{\partial \mathcal{L}}{\partial Y^{[n_H]}}\right) \odot f'(I^{[n_H]})$
3. $s^{[L]} = (W^{[L]})^T \cdot s^{[L]} \odot f'(I^{[L]})$, $\frac{\partial \mathcal{L}}{\partial W^{[L]}} = s^{[L]} \cdot (Y^{[L-1]})^T$, $\frac{\partial \mathcal{L}}{\partial b^{[L]}} = \delta^{[L]} \cdot 1_n$



- **ReLU:** $f(x) = \max(0, x)$, Proj, MLP
- **ELU:** $f(x) = \begin{cases} x, & \text{if } x > 0 \\ a \exp(x) - 1, & \text{if } x_{ji} \leq 0 \end{cases}$, Enc
- **Tanh:** $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, MLP
- **Sigmoid:** $f(x) = \frac{1}{1 + \exp^{-x}}$, MLP



1. $s^{[L]} = \left(\frac{\partial \mathcal{L}}{\partial Y^{[n_H]}}\right) \odot f'(I^{[n_H]})$
2. $s^{[L]} = (W^{[L]})^T \cdot s^{[L]} \odot f'(I^{[L]})$, $L < l$, $\frac{\partial \mathcal{L}}{\partial W^{[L]}} = s^{[L]} \cdot (Y^{[L-1]})^T$, $\frac{\partial \mathcal{L}}{\partial b^{[L]}} = \delta^{[L]} \cdot 1_n$

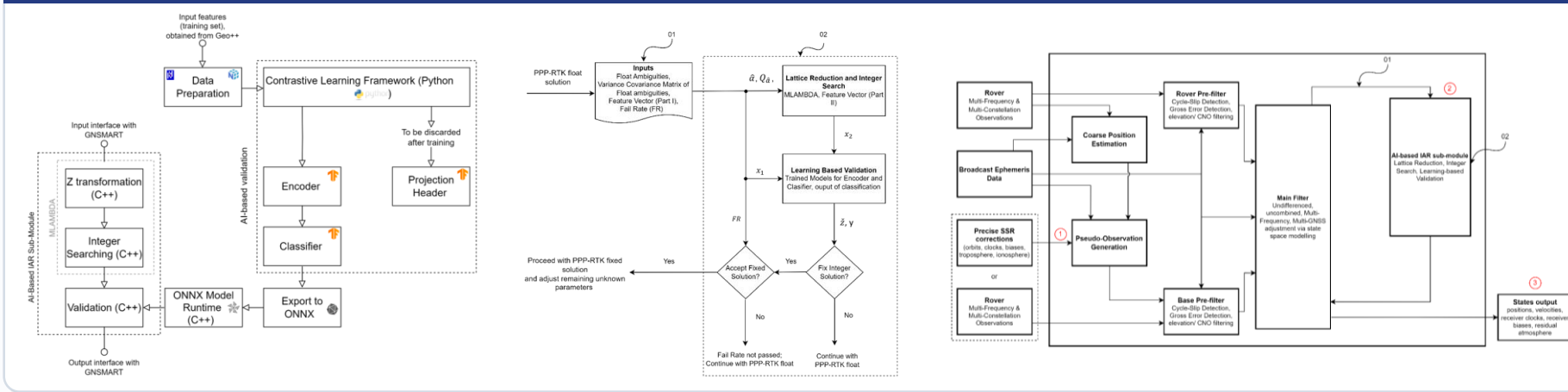


Lat. <0.5 s



Deployment and Data Campaign

Code Deployment and Integration



Experimental campaign for pre-liminary validation

Feature	Attribute
Constellations used for retrieving the reference solution	GPS+GLONASS+GAL+BDS
GNSS system integration and utilization for the pre-liminary validation	GPS+GAL
Measurements	Code and Phase
Frequency-Bands exploited for the pre-liminary validation	GPS : L1C, L2W and L5Q GAL : L1C, L5Q and L7Q
Measurement-precision	3m for code and 4mm for phase
Elevation mask	15°
Measurement weighting	Elevation dependent
Base/Rover(s)	1000/0688 (10 km), 0696 (65 km)
User's environment	Static, open-sky
User's equipment	JAVAD TRE_3 DELTA for 1000 SEPT POLARX5 for 0688 and 0686
Ionospheric delay	Ionosphere-float
Tropospheric- delay	Not estimated
Biases	Corrected

Reliable Reference

Challenging Test Case

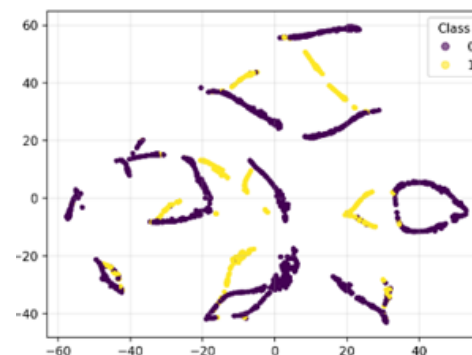
Network	Coordinate Handling	Solution Timespan	AR Algorithm
Uses helper stations	Coordinates Introduced	Considers History	LAMBDA Classical Algorithm with conservative settings
Long Baseline Only	Rover Coord. Estimated	No History Included	Algorithm under Test

Encoder's Training and Interpretation of Results

What is being measured?

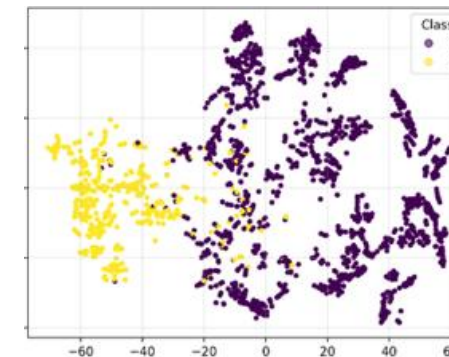
- **LPA**
Linear separability of learned embeddings
- **Cosine similarity**
Positive pairs high; negative pairs low; margin = separation
- **t-SNE**
2D view: raw feature space vs. encoder embedding space
- **Effective rank**
Intrinsic dimensionality: compact but expressive representation

Before learning: raw IF feature space



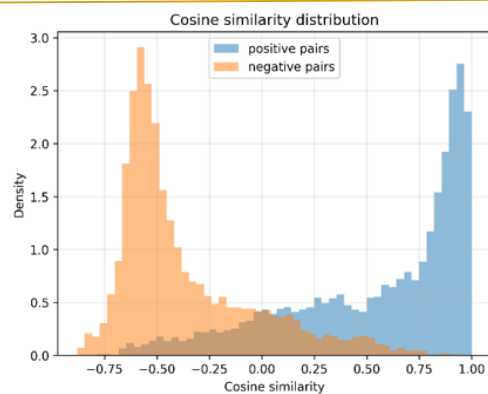
Classes are visually entangled and locally fragmented

After learning: encoder embedding space

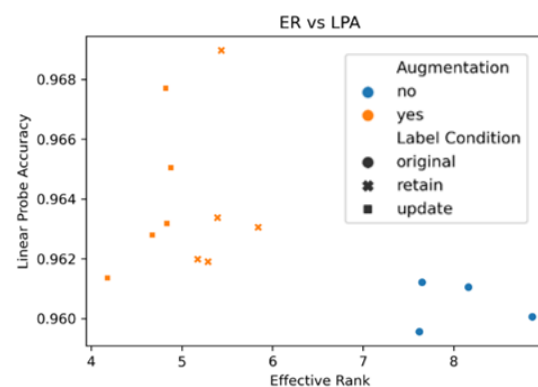


Learned representation shows clearer class separation

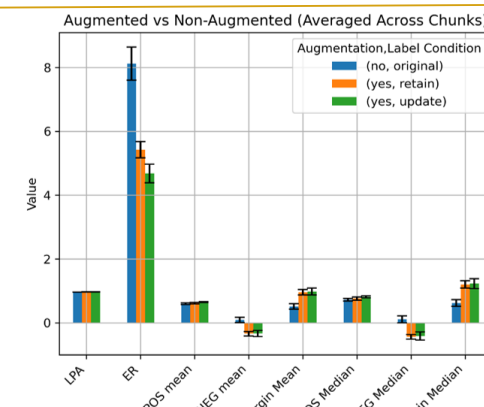
Quantitative diagnostics



Separation emerges as positive pairs move right and negatives move left



Checks whether compact embeddings retain predictive separability

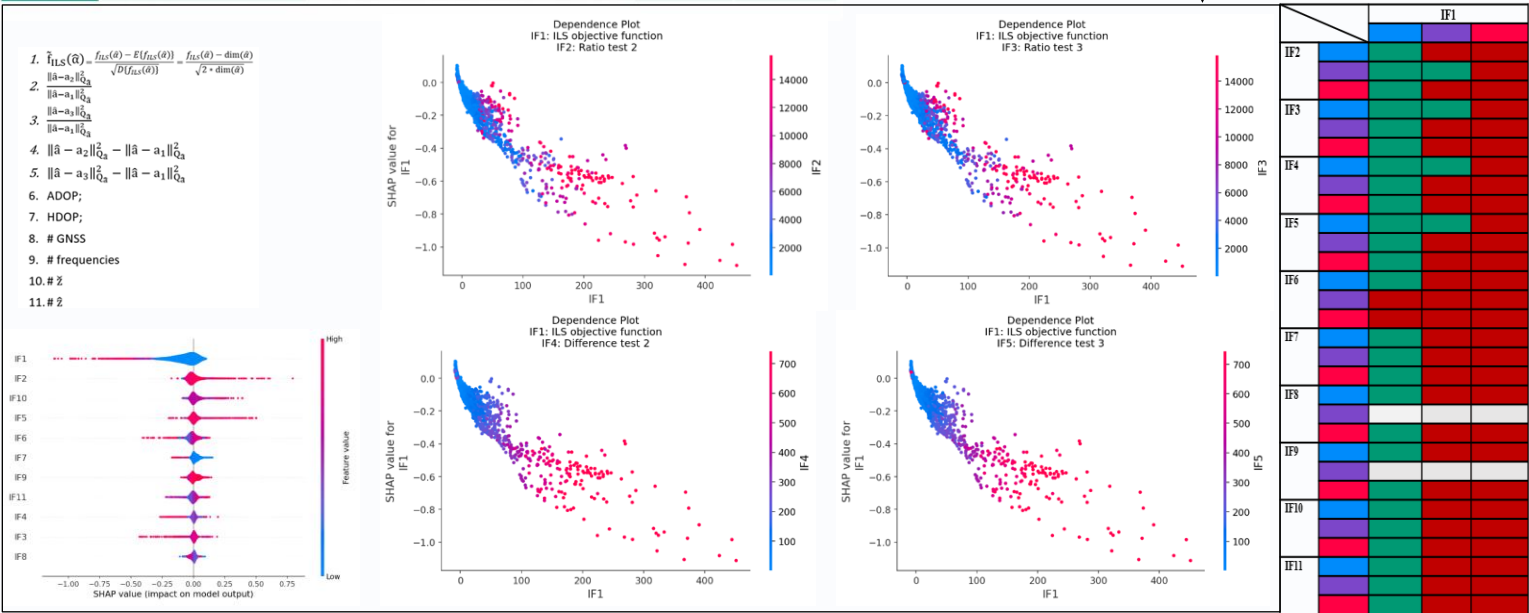
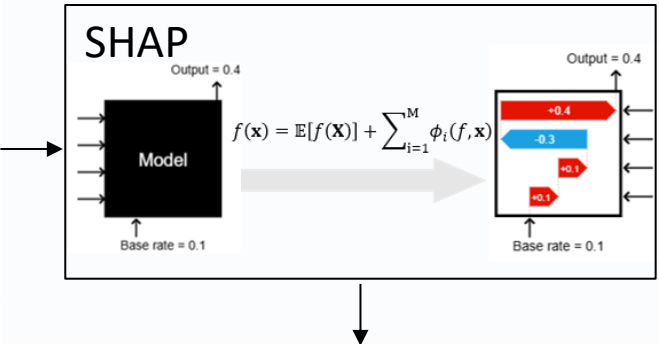


Compares stability of learned statistics under augmentation

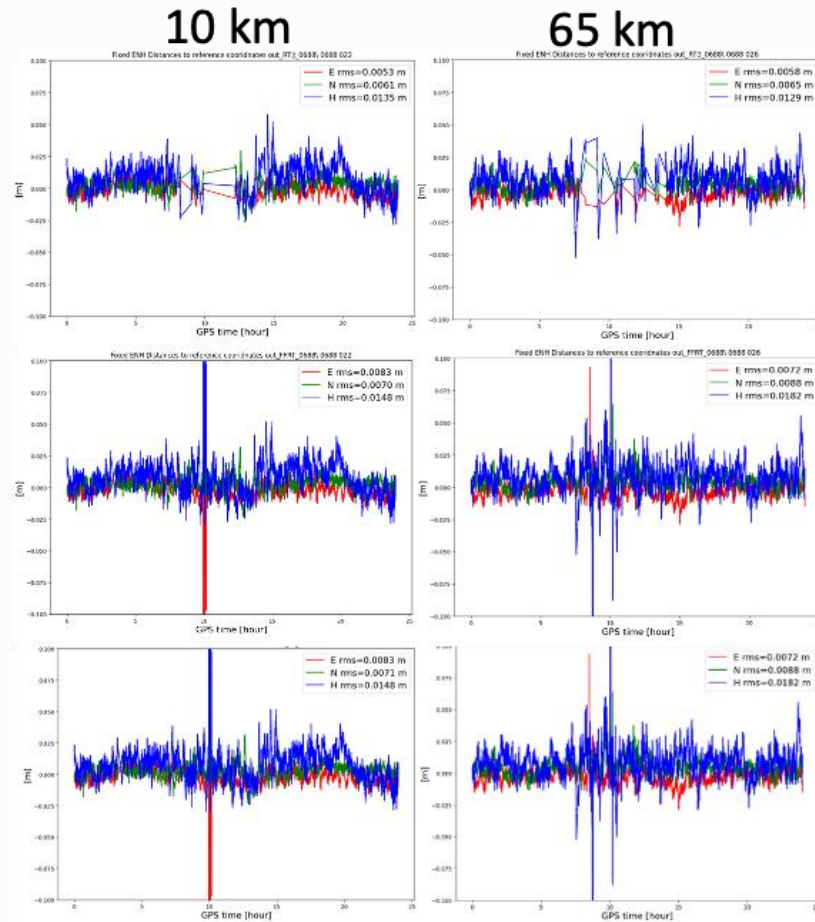
Decisioner's Training, Validation and Interpretation of Results

Metric	Interpretation	Analytical mathematical expression	Score	
			@thresh=0.5	@thresh=0.4303
ROC AUC	• Ability to separate positive vs negative classes; higher is better.	$TPR = \frac{TP}{TP + FN}, \quad FPR = \frac{FP}{FP + TN},$ $AUC_{ROC} = \int TPR d(FPR)$	0.9902	0.9902
PR AUC	• Summarizes precision-recall performance, especially for imbalanced data.	$Precision = \frac{TP}{TP + FP}, \quad Recall = \frac{TP}{TP + FN},$ $AUC_{PR} = \int Precision d(Recall)$	0.9732	0.9732
Macro F1	• Unweighted mean of per-class F1 scores; all classes contribute equally.	$F1_i = \frac{2 P_i R_i}{P_i + R_i}, \quad Macro-F1 = \frac{1}{C} \sum_{i=1}^C F1_i$	0.9440	0.9444
Weighted F1	• Support-weighted mean of per-class F1 scores; accounts for class imbalance.	$Weighted-F1 = \sum_i w_i F1_i, \quad w_i = \frac{n_i}{\sum_j n_j}$	0.9574	0.9576

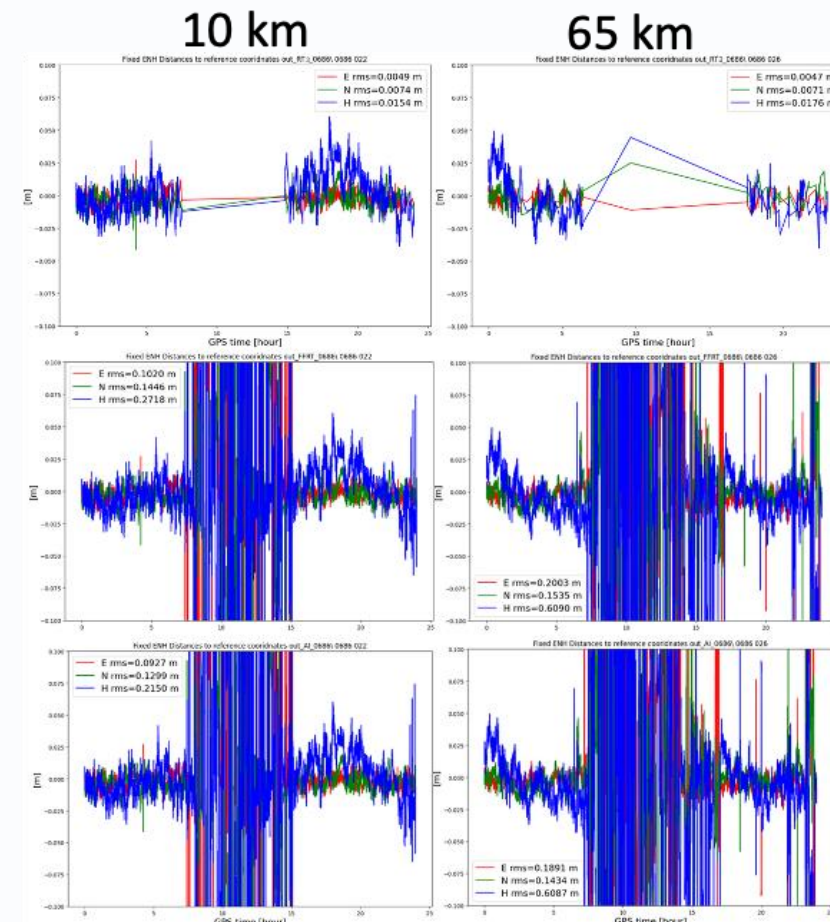
Decisioner's Validation Results and Interpretation



Precise Positioning - Full Ambiguity Resolution (FAR)



	022		026	
	Conditional SR	Availability	Conditional SR	Availability
RT ($\mu = 0.33$)	98.09%	80.08%	100.00%	74.04%
FFRT ($p_r = 0.001$)	87.19%	99.99%	99.57%	99.99%
AI (thres. = 0.5)	87.14%	99.54%	99.57%	99.69%



	DoY: 022		DoY: 026	
	Conditional SR	Availability	Conditional SR	Availability
RT ($\mu = 0.33$)	100.00%	53.37%	100.00%	18.83%
FFRT ($p_r = 0.001$)	71.44%	99.99%	62.13%	99.99%
AI (thres. = 0.5)	74.02	96.46%	65.41%	94.97%

Precise Positioning - Partial Ambiguity Resolution (PAR)

G-2F

GE-2F

GE-3F

10 km

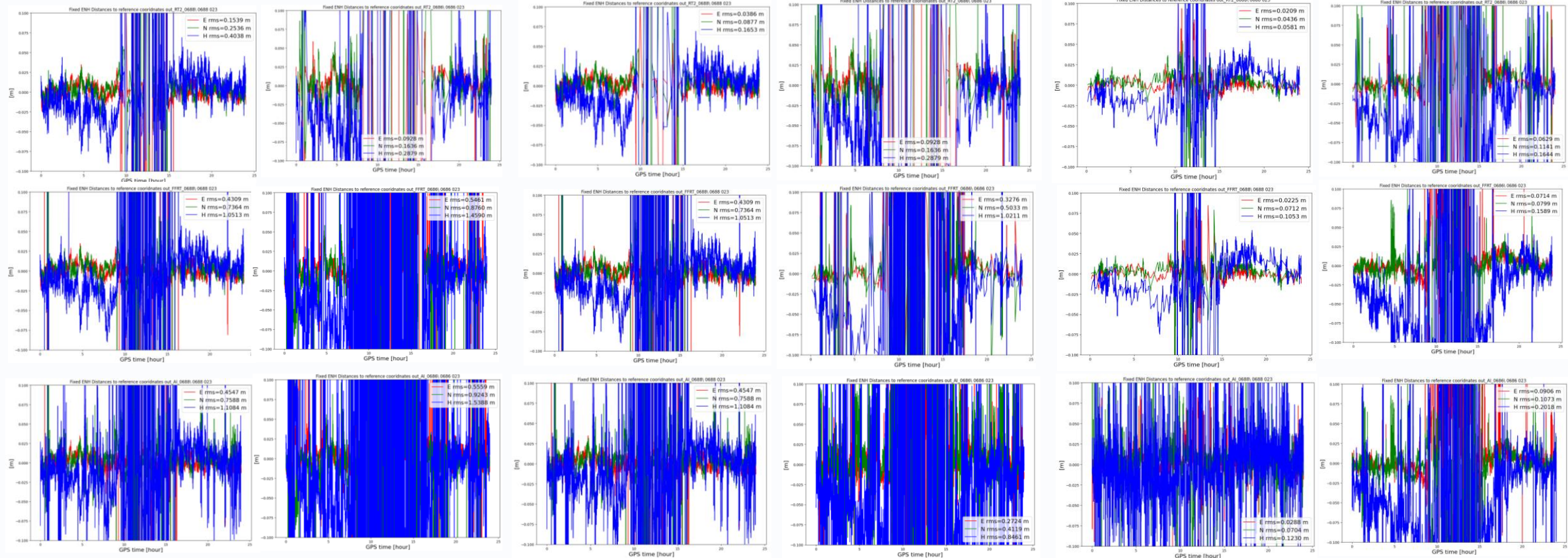
65 km

10 km

65 km

10 km

65 km



	Conditional SR	Availability	Conditional SR	Availability
RT ($\mu = 0.5$)	85.37	82.08	73.10	37.36
FFRT ($p_f = 0.001$)	77.25	98.15	41.91	95.02
AI ($thres. = 0.5$)	80.57	98.32	44.10	99.90

	10 km		65 km	
	Conditional SR	Availability	Conditional SR	Availability
RT ($\mu = 0.5$)	99.57	99.99	73.10	37.37
FFRT ($p_f = 0.001$)	99.57	99.99	37.61	47.56
AI ($thres. = 0.5$)	99.57	99.69	39.34	95.68

	10 km		65 km	
	Conditional SR	Availability	Conditional SR	Availability
RT ($\mu = 0.5$)	93.08	44.82	61.55	43.56
FFRT ($p_f = 0.001$)	91.38	46.50	58.60	99.97
AI ($thres. = 0.5$)	83.55	97.70	51.20	97.18

Conclusion and Future Work

MAIN TAKEAWAY
AI-based IAR is reliable and potentially could achieve simultaneous $P_s \uparrow$ and $P_{fix} \uparrow$, but FAR rank-2/rank-3 recovery remains a key challenge

Topic	Ambiguity Resolution		Feature robustness	Positioning
	FAR	RAR		
CONCLUSION	Trade-off $P_s \uparrow, P_{fix} \downarrow$	Is feasible $P_s \uparrow, P_{fix} \uparrow$	Interpretable model behavior across FAR and PAR	Horizontal positioning accuracy improves
LIMITATION	Rank-2 / 3 recovery remains weak	Rank-2/3 recovery must be boosted	Input-feature normalization may cause domain shift	Current tests do not fully stress kinematic scenarios
NEXT STEP	Fine-tune with real-world datasets		Adapt normalization / feature scaling strategy	Evaluate AI models in challenging kinematic scenarios

Implication: the method is promising for RAR and possibly specific implementations; the next validation step is domain robustness.

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- ☐ The view expressed in such publications can in no way be taken to reflect the official opinion of the European Space Agency.

Thank you for you Attention!!!