

The Dual Estimation Problem in Geodetic Regression: Comparing Adjustment, Bayesian Inference, and Machine Learning for TLS Uncertainty Modeling

From Explicit Models to Probabilistic Machine Learning

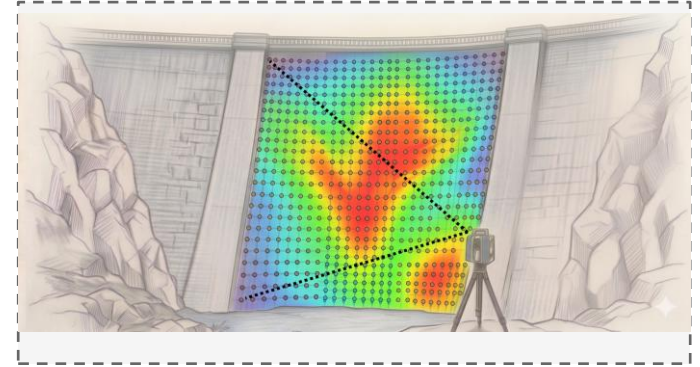
H. Alkhatib · J. M. Hartmann · I. Neumann

Geodetic Institute, Leibniz University Hannover
XI Hotine–Marussi Symposium 2026 — Session: Machine and Deep Learning in Geodesy

Deformation — or measurement uncertainty?

- ▶ Two epochs produce two point clouds.
- ▶ A visible difference can have two explanations:
 - ▶ the object has changed
 - ▶ the measurement process has changed
- ▶ Without an adequate uncertainty model, both conclusions can be wrong.
- ▶ The scientific task is therefore not only to estimate a displacement — it is to decide whether that displacement is distinguishable from measurement uncertainty.

$$\Delta d_{\text{obs}} = \Delta d_{\text{def}} + (\varepsilon_2 - \varepsilon_1)$$



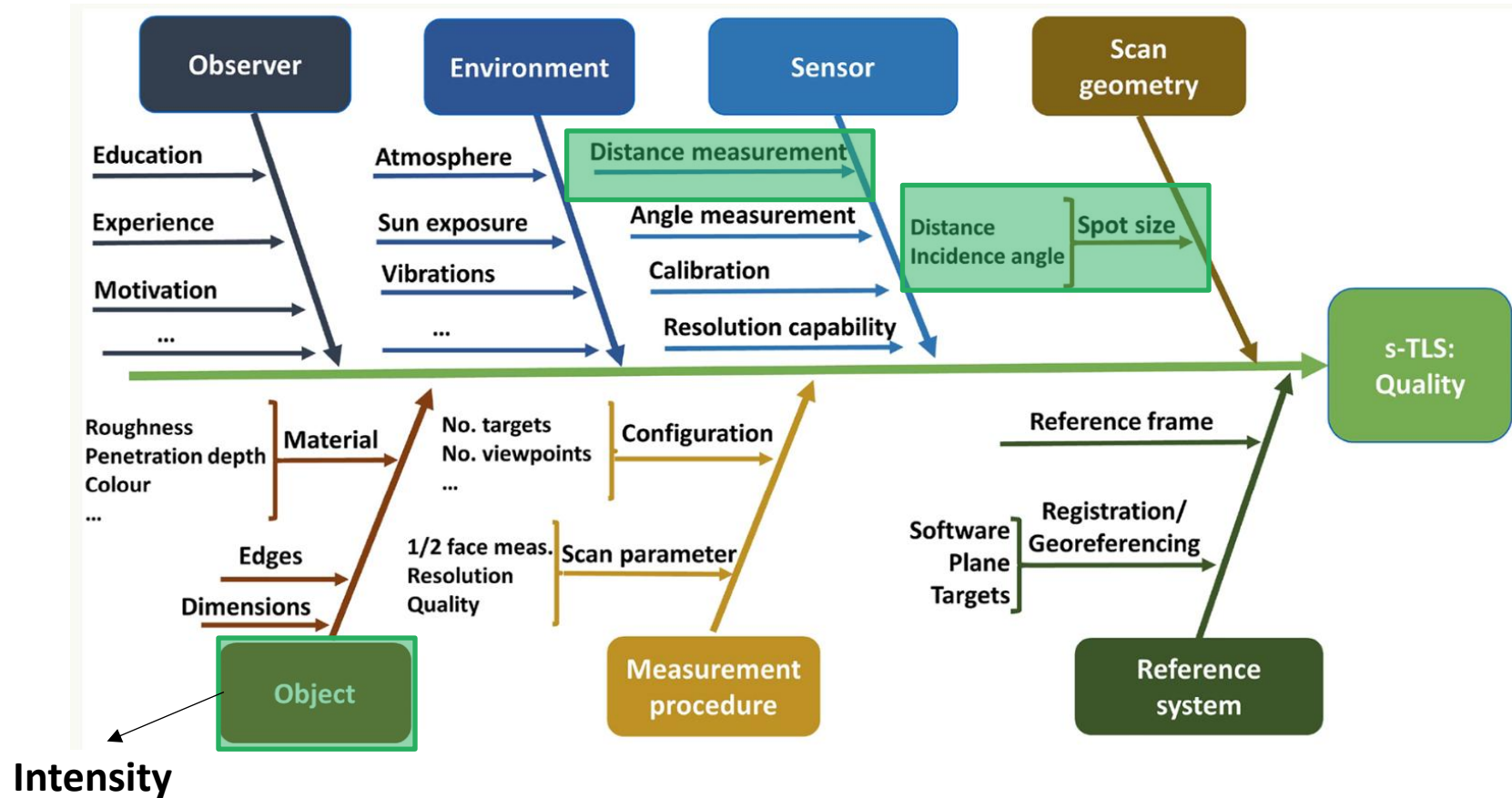
Deformation or Sensor Uncertainty?

Unreliable $\sigma \rightarrow$ **missed detections (safety risk)**
or **costly false alarms**

Type II Error
 σ too large $\rightarrow$
missed deformation

Type I Error
 σ too small $\rightarrow$
false alarm

TLS uncertainty is shaped by coupled influences



Three modelling routes answer the same uncertainty question

CLASSICAL ADJUSTMENT

functional and stochastic models specified

parameters estimated
LS / GEM / VCE

→ covariance and uncertainty propagation

BAYESIAN INFERENCE

model and priors specified

parameters jointly inferred
MCMC

→ posterior intervals and dependencies

PROBABILISTIC ML

features, family and loss specified

$\mu(\mathbf{x})$ and $\sigma(\mathbf{x})$ learned

→ calibration and transfer tests

The modelling boundary moves — the requirement for assumptions and validation does not disappear.

The choice depends on what can be formulated credibly and what must be learned from data.

The explicit route: prescribe the models, then choose the inference

A short reminder from the model presented at Hotine–Marussi 2022 — a different application, the same structure.

$$\mathbf{L}_t = \mathbf{h}_t(\boldsymbol{\beta}) + \mathbf{E}_t$$

FUNCTIONAL MODEL $\boldsymbol{\beta}$

Prescribed geometry or physical relation
→ nonlinear model $\mathbf{h}_t(\boldsymbol{\beta})$

STOCHASTIC MODEL

Prescribed dependence and error family
→ $\mathbf{A}_j, \boldsymbol{\Psi}, \nu$

$$\boldsymbol{\theta} = [\boldsymbol{\beta}^\top, (\boldsymbol{\psi}^2)^\top, \boldsymbol{\rho}^\top, \mathbf{a}^\top, \nu]^\top$$

The model families are fixed — the algorithms differ in the uncertainty object they compute.

CLASSICAL INFERENCE

Generalized Expectation Maximization
optimises the robust likelihood
→ point estimate + local covariance

BAYESIAN INFERENCE

Metropolis-within-Gibbs MCMC
samples the joint posterior $f_{\boldsymbol{\Theta}|\mathbf{L}}$
→ intervals and dependencies

Both routes use all observations; the Bayesian route additionally requires priors.

This route is powerful when the scientific structure is credible and interpretable.

Reference measurements make TLS uncertainty learnable

- **Place:** Basement GIH
- **Reference:** Laser tracker ($15\mu m + 6\mu m/m$)
- **TLS ZF + Imager 5016** ($\sigma_{angle} = 0.004^\circ$, $\sigma_{distance} = 0.2 - 0.4mm$)

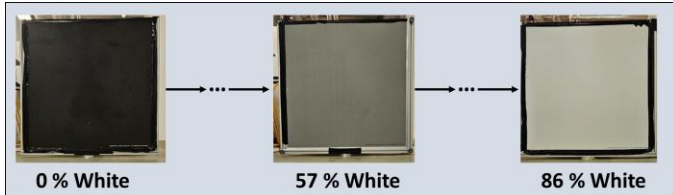


Variation

Reflectivity

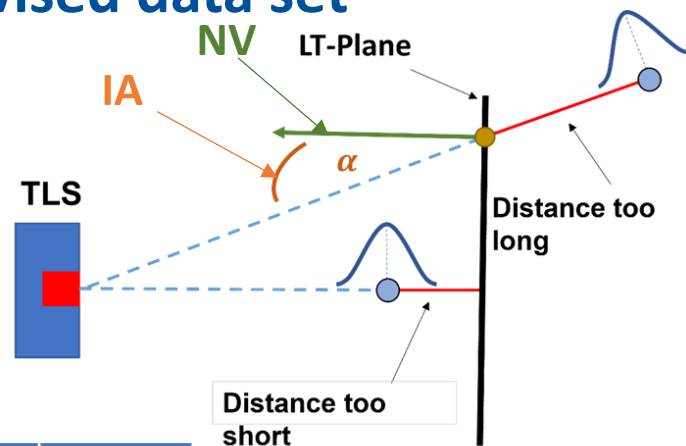
Distance

Incidence angle



From reference measurements to a supervised data set

- Mesh from laser-tracker point cloud; distance deviation by ray-casting
- **label: Δd — negative = measured too short, positive = too long**
- features from the TLS observation itself: intensity, distance, incidence angle
- ≈ 21 mio. points, 4 attributes each



Data set	Reference	TLS	Settings	Date
A (Training)	Leica ATS600 + CCR	Z+F Imager 5016 (Hannover)	Superhigh & Quality +	13.06.25–16.06.25
B (Temporal)	Leica ATS960 + CCR	Z+F Imager 5016 (Hannover)	Superhigh & Quality +	06.05.25–12.05.25
C (Transfer)	Leica ATS600 + CCR	Z+F Imager 5016 (Clausthal)	Superhigh & Quality +	13.06.25–16.06.25

Research Questions:

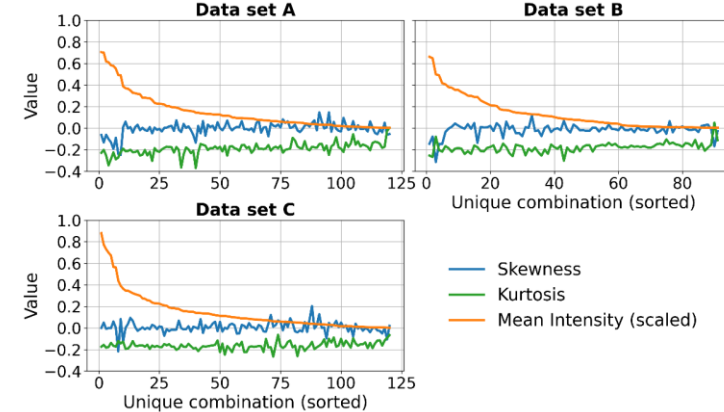
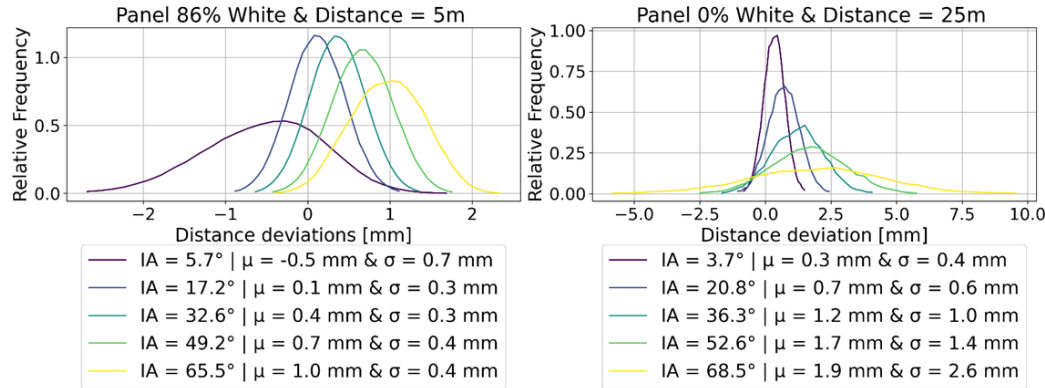
1. Temporal stability?
2. Transferability?

Split by measurement configuration, not randomly — the same panel–distance–angle combination never appears in training and test.

Data analysis – Normal distribution

- Distance deviation distributions are nearly normal distributed
- Skewness and Kurtosis ~ 0 for all combinations
- Bias & Standard deviation depend on IA & Intensity

Data set A



Uncertainty modeling: GUM vs. learned,

GUM:

$$u_c^2 = \underbrace{\sigma_d^2}_{\text{TLS distance}} + \underbrace{\sigma_\alpha^2}_{\text{TLS angular}} + \underbrace{\sigma_{\text{reg}}^2}_{\text{Registration}} + \underbrace{\sigma_{\text{LT}}^2}_{\text{Laser tracker}}$$

- Error propagation
- Manual uncertainty budget
- Each source estimated separately
- Homoscedastic (constant σ)

Our Approach (Machine Learning)

- Joint estimation of μ and σ

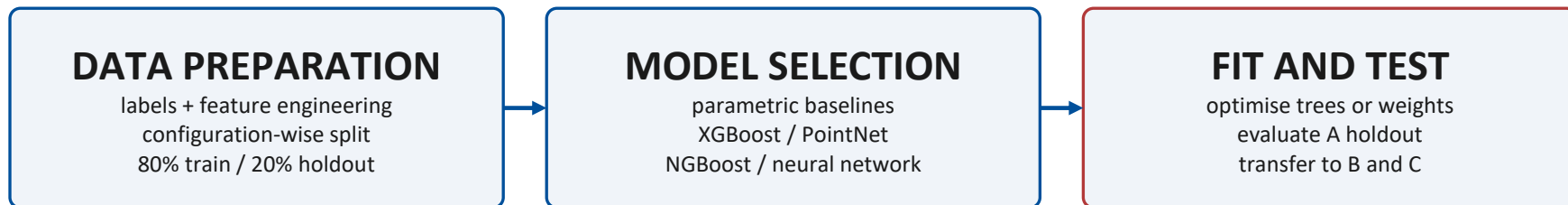
$$\Delta d \mid \mathbf{x} \sim \mathcal{N}(\mu(\mathbf{x}), \sigma^2(\mathbf{x})), \quad \mathbf{x} = (I_{\text{TLS}}, d_{\text{TLS}}, \alpha)$$

- Data driven
- Heteroscedastic: $\sigma = f(\mathbf{x})$

Learning requires a different estimation workflow

$$\mathcal{D}_A = \{(\mathbf{x}_i, y_i)\}_{i=1}^{n_A}$$

The experiment defines the data; the selected model learns the mapping.



Splitting by measurement configuration prevents the same panel–distance–angle combination from leaking into train and test.

The trained mapping may be hard to interpret physically — therefore independent testing becomes part of the model.

Choosing the loss function: MSE vs. GNLL,

Conventional (MSE)

$$MSE = \sum_{i=1}^n (y_i - \mu_i)^2$$

- Only predicts $\hat{\mu}$
- No uncertainty estimates

Probabilistic (GNLL)

- Assume Gaussian distribution

$$GNLL = \sum_{i=1}^n \left[\frac{1}{2} \log(2\pi\sigma_i^2) + \frac{(y_i - \mu_i)^2}{2\sigma_i^2} \right]$$

- Predicts $(\hat{\mu}, \hat{\sigma})$ jointly
- Penalizes overconfidence

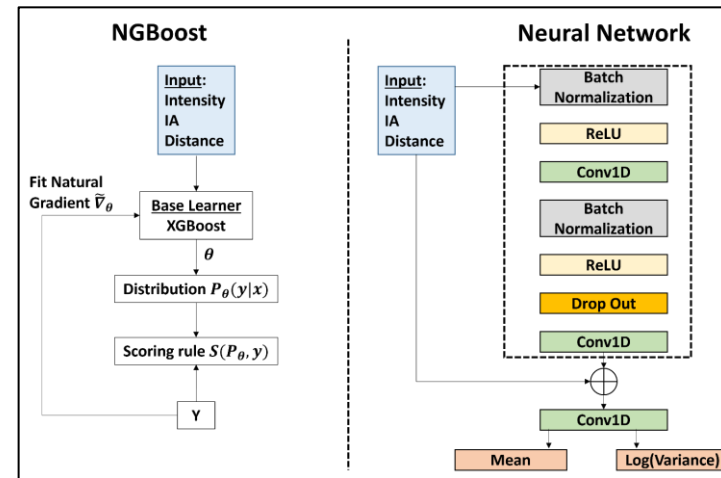
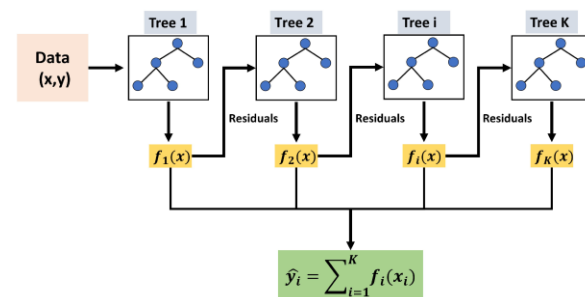
Model Choice and training

Structured data set

- **NGBoost**
 - Natural Gradient descent

$$\theta^{(t+1)} = \theta^{(t)} - \eta \cdot \underbrace{F(\theta)^{-1}}_{\text{Fisher inverse}} \nabla_{\theta} \mathcal{L}$$
$$F(\theta) = \mathbb{E} \left[-\frac{\partial^2 \log p(y | \theta)}{\partial \theta \partial \theta^{\top}} \right] = \begin{bmatrix} \frac{1}{\sigma^2} & 0 \\ 0 & \frac{1}{2\sigma^4} \end{bmatrix}, \quad \theta = (\mu, \sigma^2)$$

- **Neural network**
 - Residual network



Training:

1. Bayesian Hyperparameter tuning
2. Bootstrapping (50 models)

Results – Coverage analysis

Coverage analysis

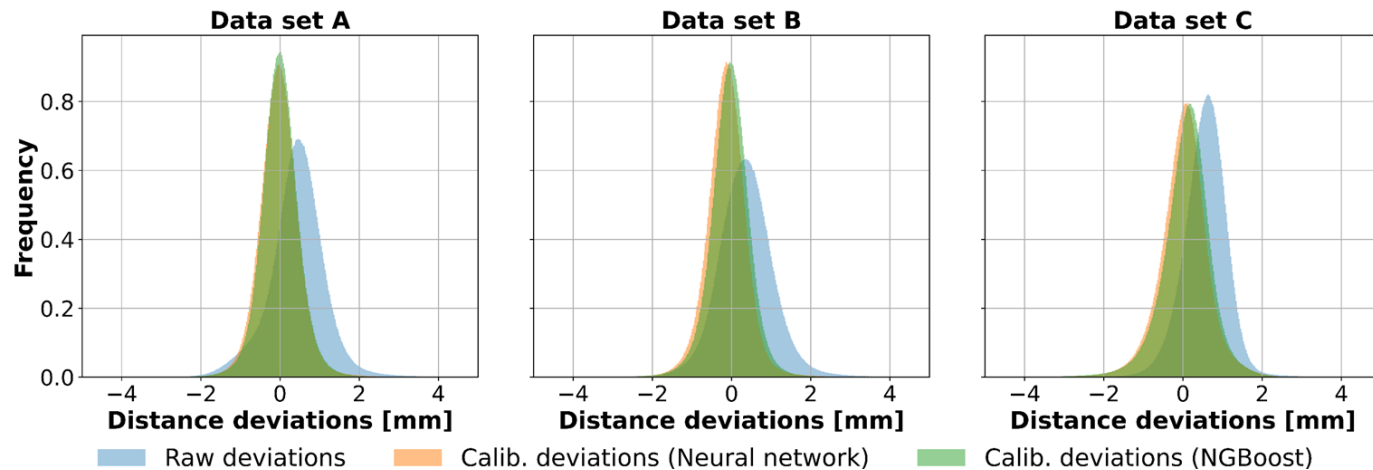
$$C_k = \frac{1}{N} \sum_{i=1}^N \mathbb{I}\left\{|y_i - \hat{\mu}_i| \leq k \cdot \hat{\sigma}_i\right\}, \quad k \in \{1, 2, 3\}$$

Data set	K	Theory [%]	NGBoost [%]	Neural network [%]
A	1	68.3	68.27	65.93
	3	99.7	99.80	99.53
B	1	68.3	67.68	64.89
	3	99.7	99.64	99.64
C	1	68.3	57.98	58.26
	3	99.7	97.67	97.33

Results – Calibration: temporal stability

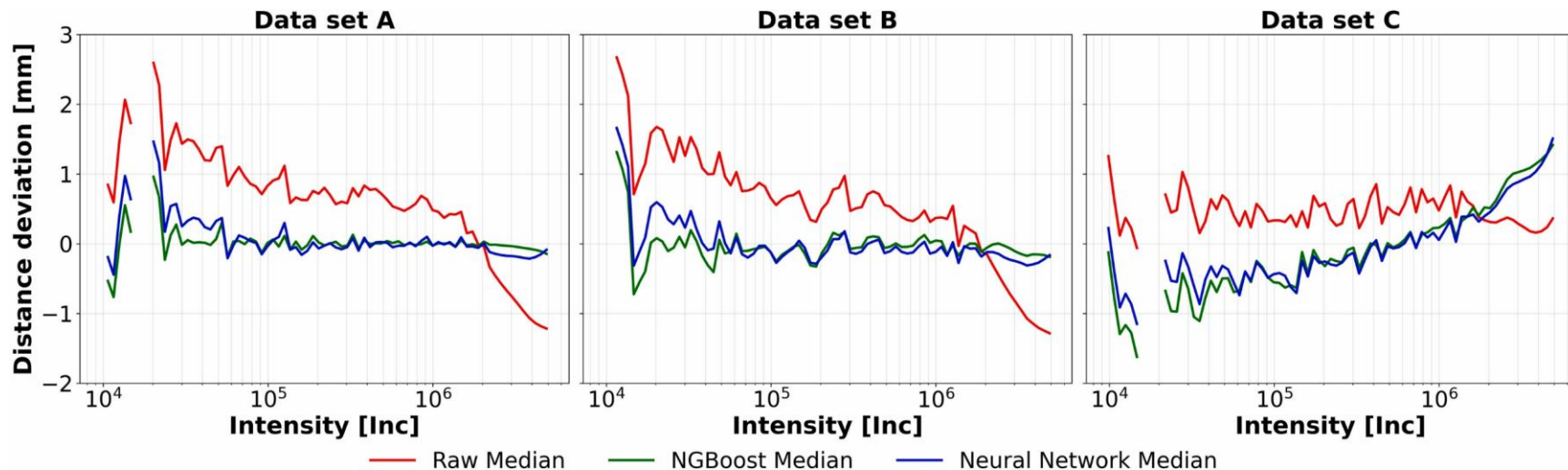
- Calibration of the distance deviation Δd_{cal}
 - $\Delta d_{cal} = \Delta d_{raw} - \Delta d_{pred}$

Data set	Metric	Raw	NGBoost	Neural Net
A	μ [mm]	0.43	0.00	-0.02
	σ [mm]	0.72	0.50	0.52
B	μ [mm]	0.38	-0.03	-0.13
	σ [mm]	0.70	0.52	0.52
C	μ [mm]	0.56	0.08	-0.02
	σ [mm]	0.56	0.64	0.63

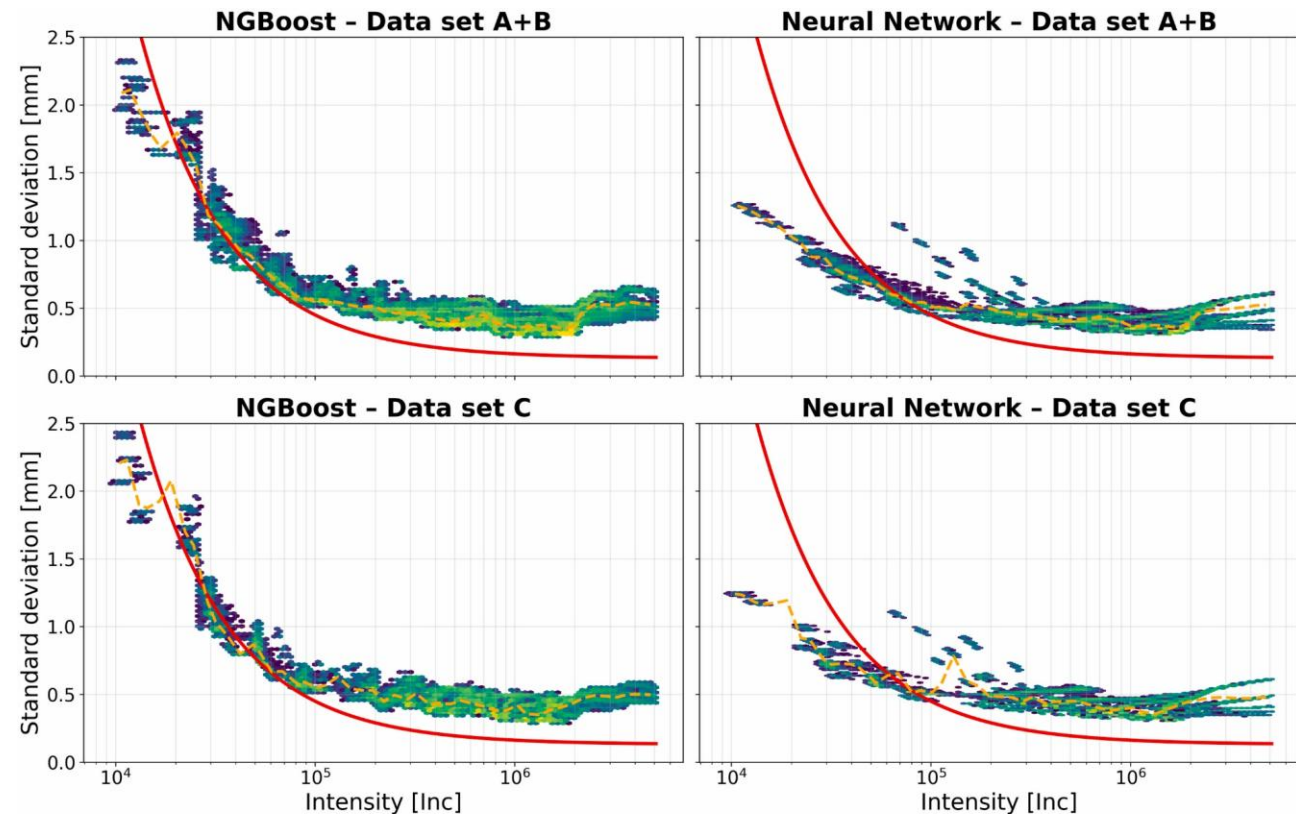


Results – Calibration: transfer fails

- Temporal (A & B)
 - No systematic deviations for intensities $> 10^5$ [Inc]
 - Reduced systematic deviations at low intensities (limited data)
- Transfer (C): → **New systematic deviations due to calibration!**



Results – Aleatoric uncertainty (Precision)



Reference model:

Intensity based model
(Schill et al., 2025)

→ NGBoost good agreement



— Reference: Intensity-based model
- - - Mean standard deviation

Result – Epistemic uncertainty

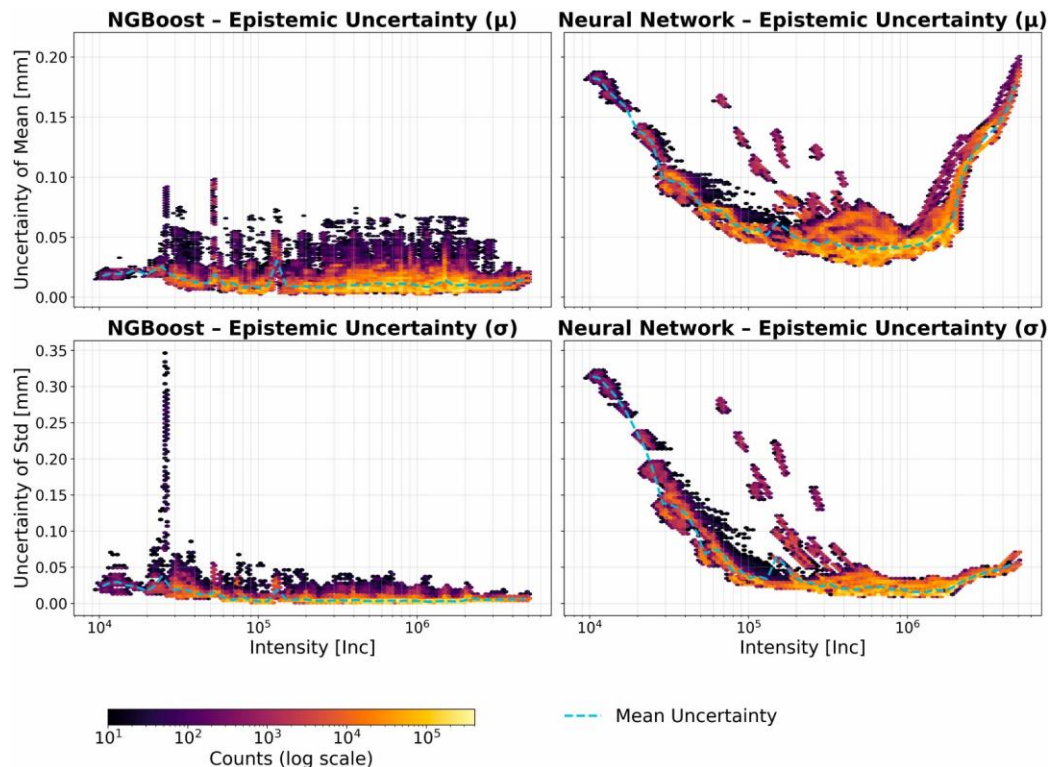
Estimate epistemic uncertainty using Bootstrap (50) models

NGBoost:

- Uncertainty $< 0.05\text{mm}$
- Stable everywhere

Neural Network:

- Uncertainty intensity dependent
- Up to 0.3mm at the boundaries



A calibrated model separates deformation from uncertainty

- ▶ An observed change is interpretable only relative to measurement uncertainty.
- ▶ If the scientific structure is credible, keep it explicit and estimate its parameters with classical or Bayesian inference.
- ▶ If the coupled TLS relation cannot be formulated accurately enough, create traceable reference data and learn a conditional distribution.
- ▶ **Validate not only RMSE, but coverage, sparse regions, time transfer and scanner transfer.**

**Keep credible physics explicit. Learn the missing relation.
Calibrate before deciding deformation.**

Thank you — questions and discussion

References

- ▶ Dorndorf, A., Paffenholz, J.-A., Alkhatib, H. (2025). Empirical Application and Evaluation of an Advanced Bayesian Robust Multivariate Time Series Model in Nonlinear Regression with Vector Autoregressive and t-Distributed Errors. *Contributions to Statistics*, 129–145.
- ▶ Hartmann, J., Ernst, D., Neumann, I., Alkhatib, H. (2024). PointNet-based modeling of systematic distance deviations for improved TLS accuracy. *Journal of Applied Geodesy* 18(4), 613–628. doi:10.1515/jag-2023-0097.
- ▶ Hartmann, J., Neumann, I., Alkhatib, H. (2026). Modeling the distribution of TLS distance related uncertainties. *Measurement* 270, 120908. doi:10.1016/j.measurement.2026.120908.
- ▶ Hartmann, J. M., Heiken, M. L., Alkhatib, H., Neumann, I. (2023). Automatic quality assessment of terrestrial laser scans. *Journal of Applied Geodesy* 17(4), 333–353. doi:10.1515/jag-2022-0030.
- ▶ JCGM 100:2008. Evaluation of measurement data — Guide to the expression of uncertainty in measurement.

Results – Aleatoric uncertainty (Precision)

- NGBoost: Offset of 0.2mm vs. Reference (True TLS precision)

$$u_c^2 = \underbrace{\sigma_d^2}_{\text{TLS distance}} + \underbrace{\sigma_\alpha^2}_{\text{TLS angular}} + \underbrace{\sigma_{\text{reg}}^2}_{\text{Registration}} + \underbrace{\sigma_{\text{LT}}^2}_{\text{Laser tracker}}$$

0.004°

Registration residuals ~0.6mm **NGBoost**

15μm + 6μm/m

Our $\hat{\sigma}$ is a realistic estimate of the set up

