

Bathymetry Prediction From Airborne Gravimetry by Tikhonov Generalized Inversion – Case From Greenland Airborne Quantum Gravimetry

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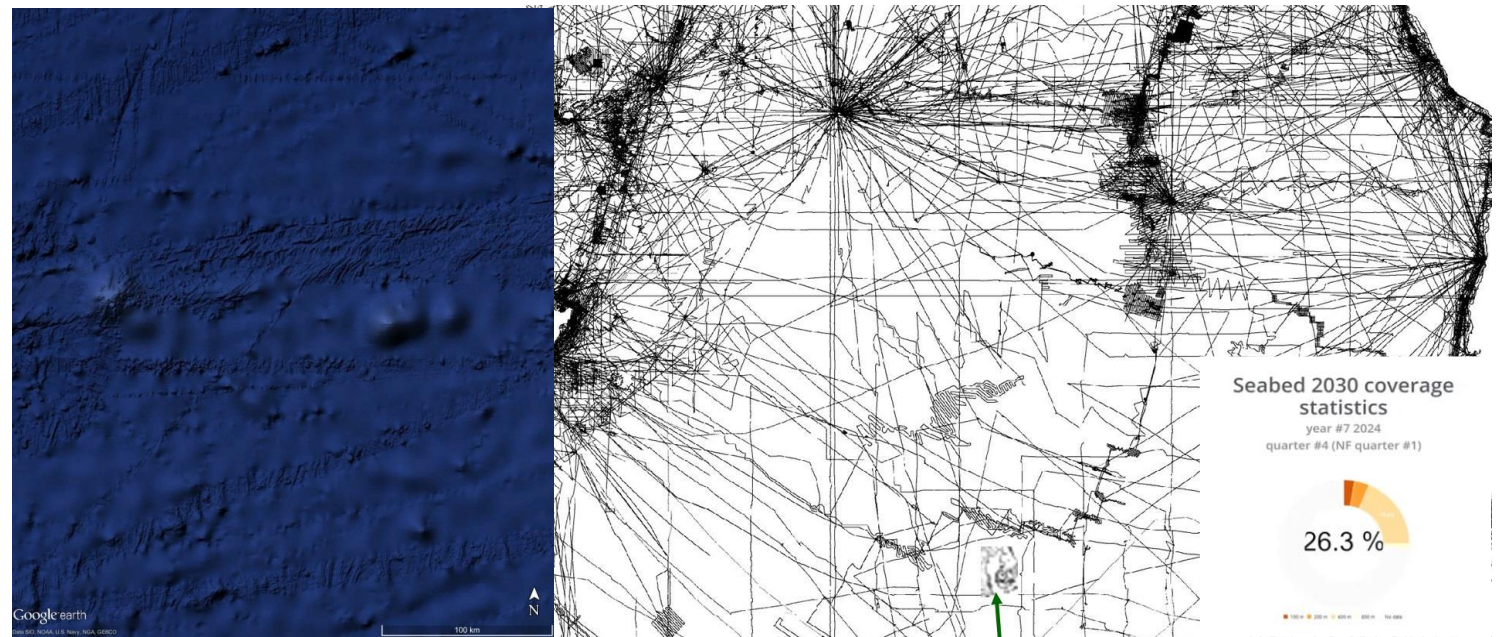
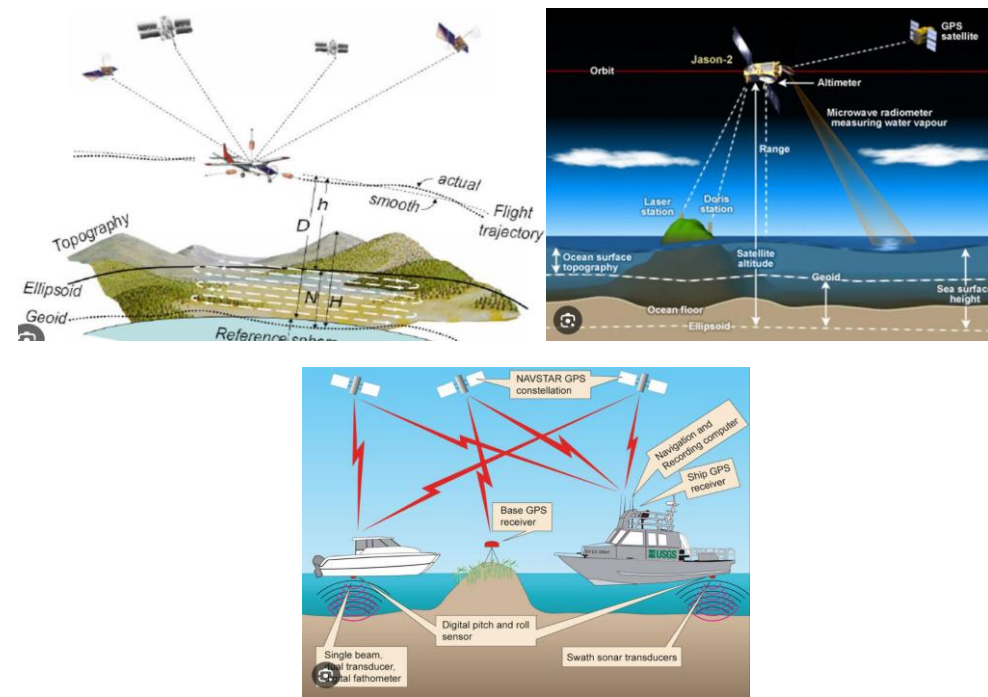
DTU Space, Technical University of Denmark, Lyngby, Denmark

XI Hotine-Marussi, September 7 – 11, 2026

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- Background
- Bathymetry Prediction
- Summary and Future Work

Background Bathymetry



Importance of Bathymetry

Military use

Enforcement in deep-sea fishing areas

Protection and preservation of the marine environment

Marine scientific research

Artificial islands and structural installations

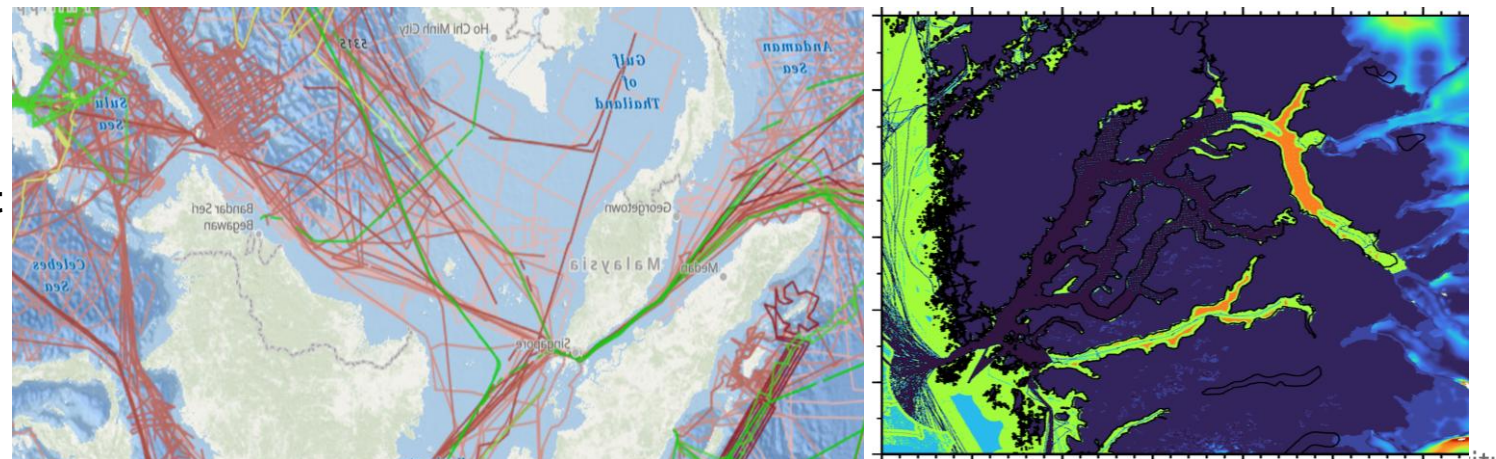
Submarine cables and pipelines

Coastal development and marine cadastre

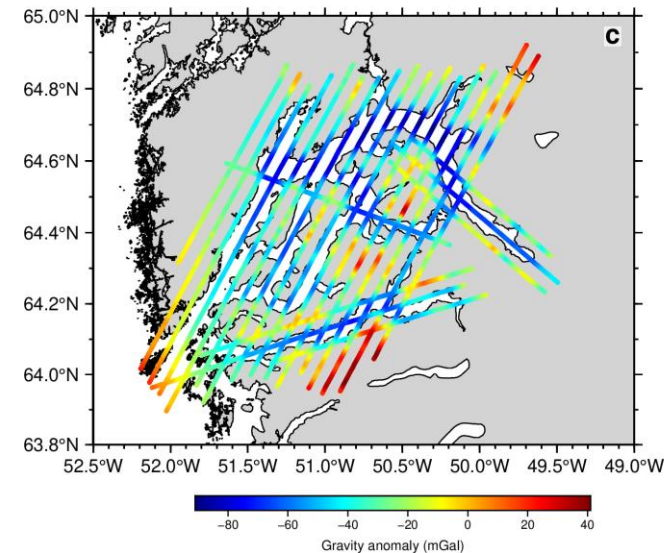
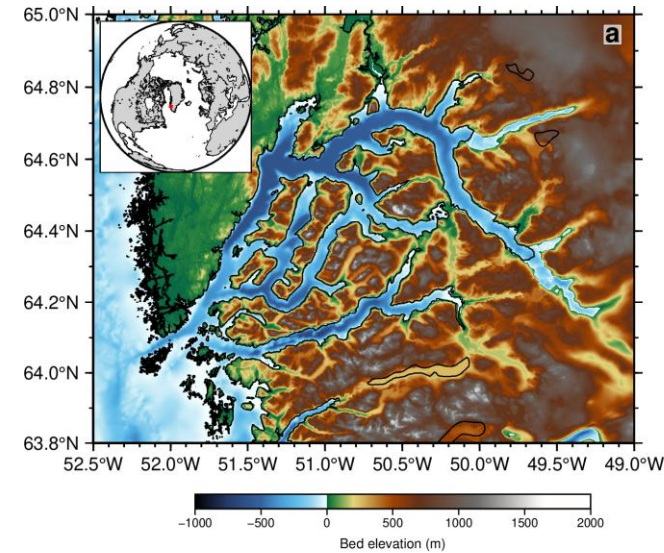
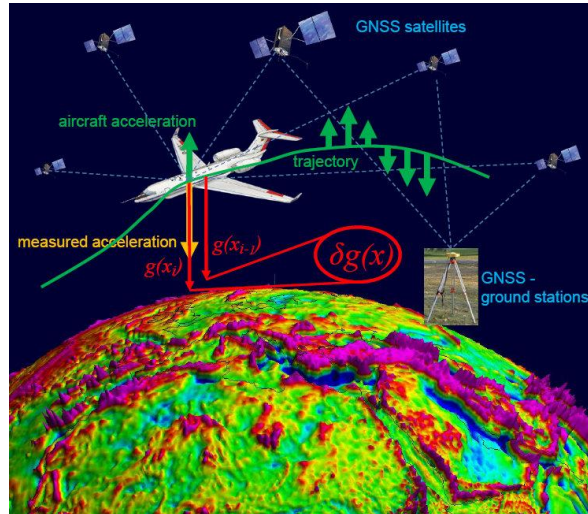
Climate change

Ship Safety

Etcetc.



Background Airborne Quantum Gravimetry

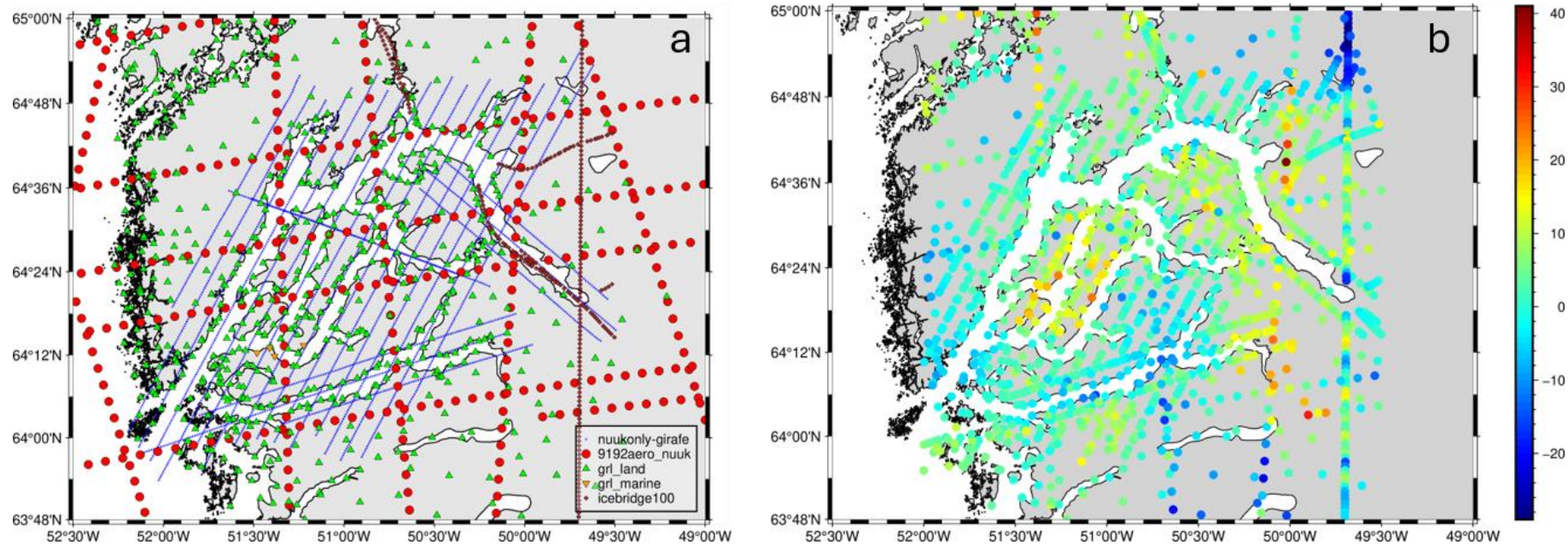


Bidel et al., 2020
Jensen et al., 2025

Airborne quantum gravimetry at Nuuk area, Greenland

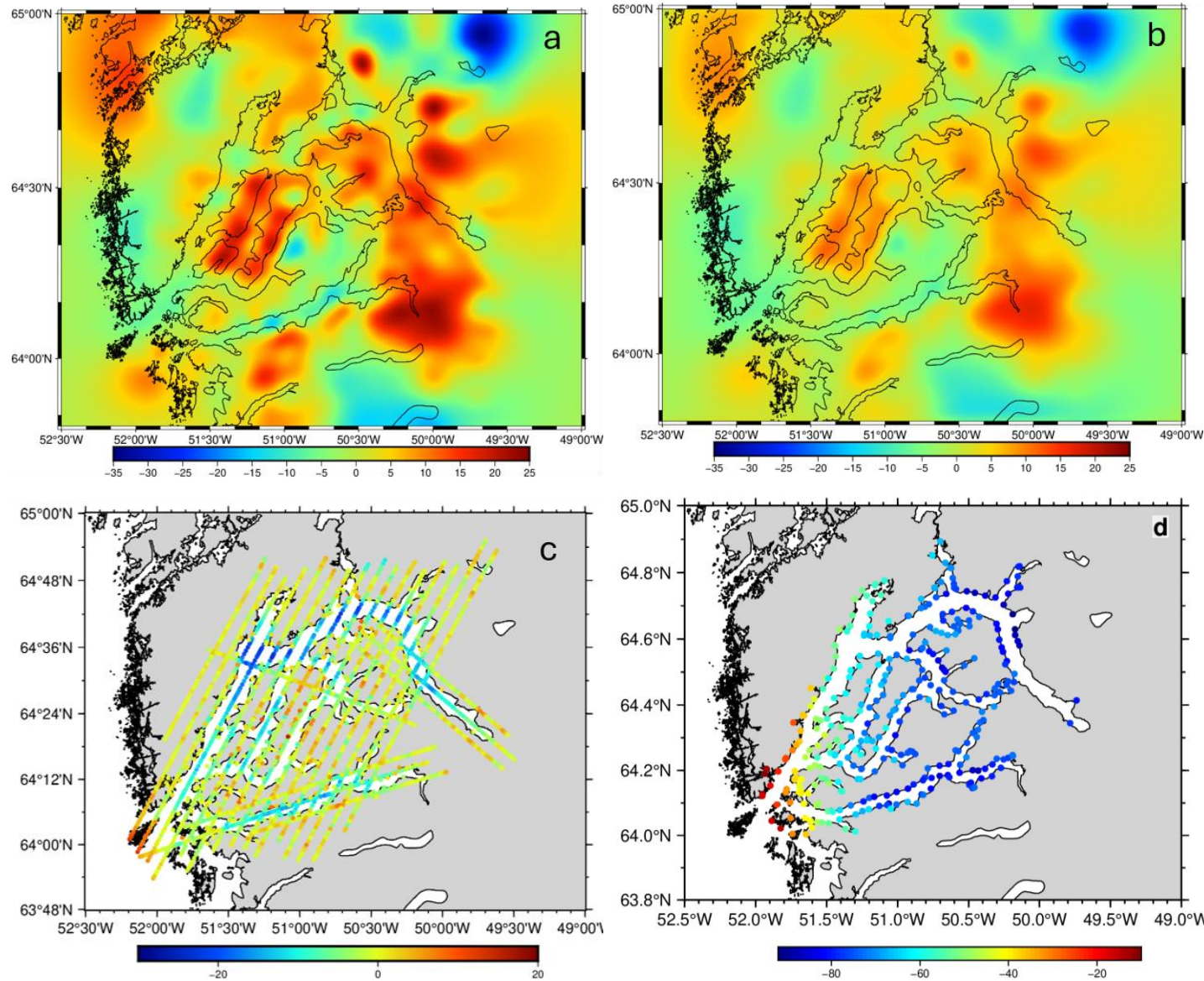
Bathymetry Prediction from airborne gravimetry by Tikhonov Generalized Inversion

Bathymetry Prediction – Data Processing



Regional gravity observations and background-field control data. (a) Gravity observations available in the Nuuk region and used to construct the regional background field, including airborne, terrestrial, marine, and IceBridge data. (b) Background-control gravity values after excluding observations within the fjord network and removing the EGM-calculated gravity contribution and the RTM terrain effect.

Bathymetry Prediction – Data Processing



Background-field processing and residual-gravity construction. (a) Background-control field downward continued to sea level using GRAVSOFT. (b) Background field after 30 km low-pass filtering and upward continuation to 1,800 m, the approximate mean airborne-survey altitude. (c) Airborne and ground gravity anomalies after removal of the constructed background field; these residual anomalies form the gravity observations used in the bathymetric inversion. (d) Ground gravity data along the fjord.

Bathymetry Prediction – Method

The bathymetric inverse problem is solved in the spatial domain using an iterative **Tikhonov generalized inversion** (Tikhonov & Arsenin, 1977). The starting-depth grid **defines vertical rectangular prisms** with a constant rock-seawater density contrast of 1.64 g cm^{-3} . The exact vertical-gravity response of every active prism is evaluated at each observation's original horizontal position and elevation. This formulation follows the general use of regularization to stabilize ill-conditioned geophysical inverse problems (Oldenburg et al., 1993; Tushmalani & Saibi, 2015) and uses rectangular-prism gravity responses of the type described by Nagy et al. (2000).

At each iteration, the full nonlinear prism response is calculated for the current depth model, and the residual vector is

$$\mathbf{r} = \mathbf{d} - \mathbf{g}(\mathbf{m}).$$

The sensitivity matrix is approximated from the response of a 10 m thick plate added at the base of each active prism. The dimensionless depth-adjustment vector minimizes

$$\Phi(\mathbf{q}) = \|\mathbf{A}\mathbf{q} - \mathbf{r}\|_2^2 + \lambda \|\mathbf{q}\|_2^2.$$

In the implementation,

$$\lambda = N_g \kappa,$$

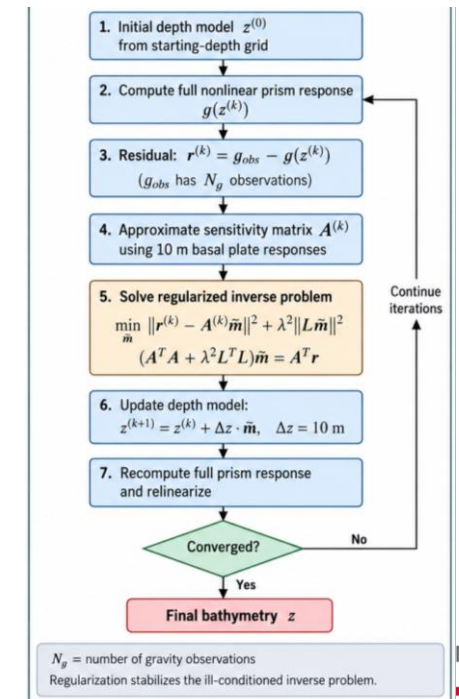
where N_g is the number of gravity observations. The equivalent normal equations are

$$(\mathbf{A}^T \mathbf{A} + \lambda \mathbf{I}) \mathbf{q} = \mathbf{A}^T \mathbf{r}.$$

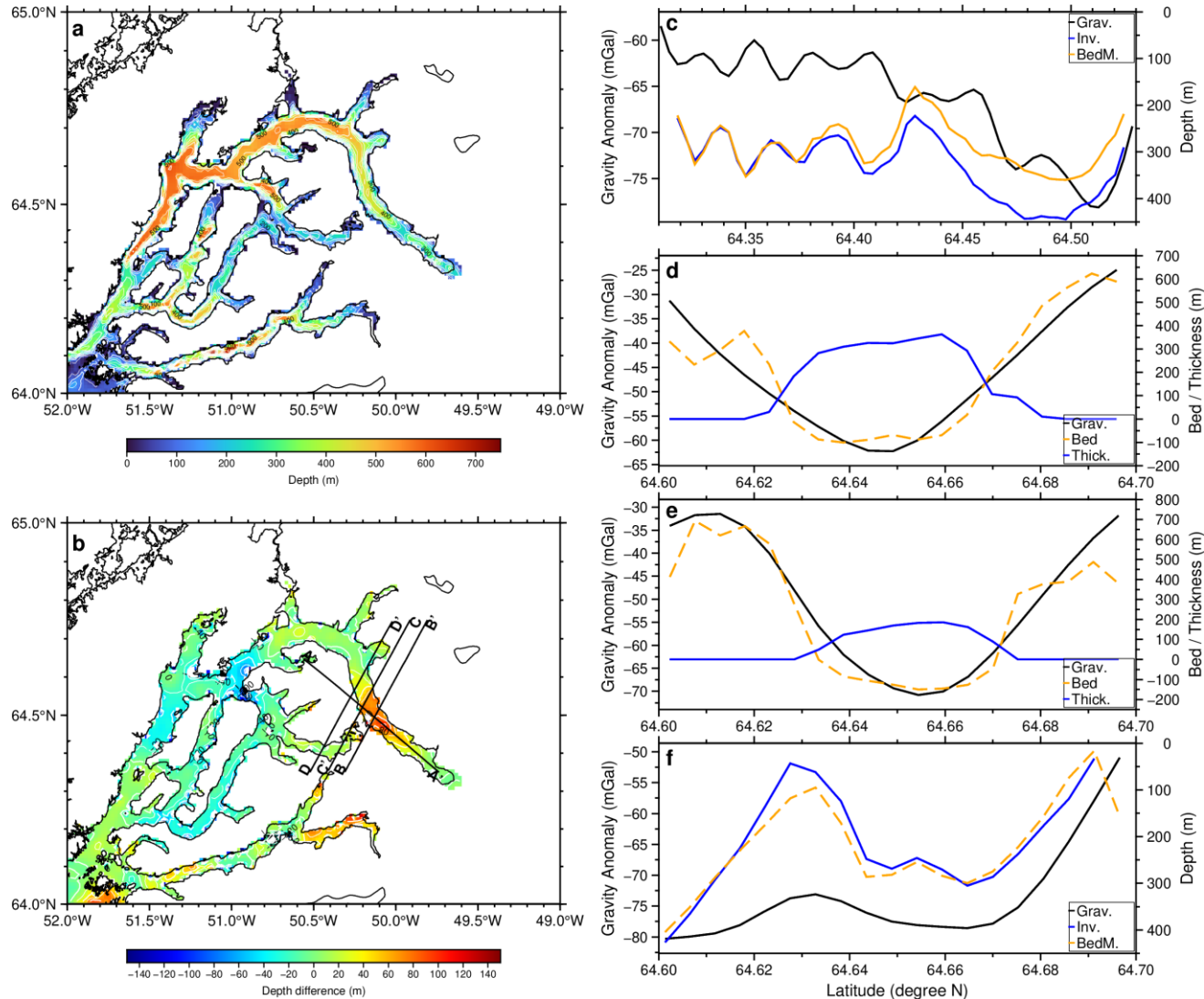
The depth model is updated as

$$\mathbf{m}_{\text{new}} = \mathbf{m}_{\text{old}} - \Delta z \mathbf{q},$$

where $\Delta z = 10 \text{ m}$. The full prism response is then recomputed and the problem is relinearized.

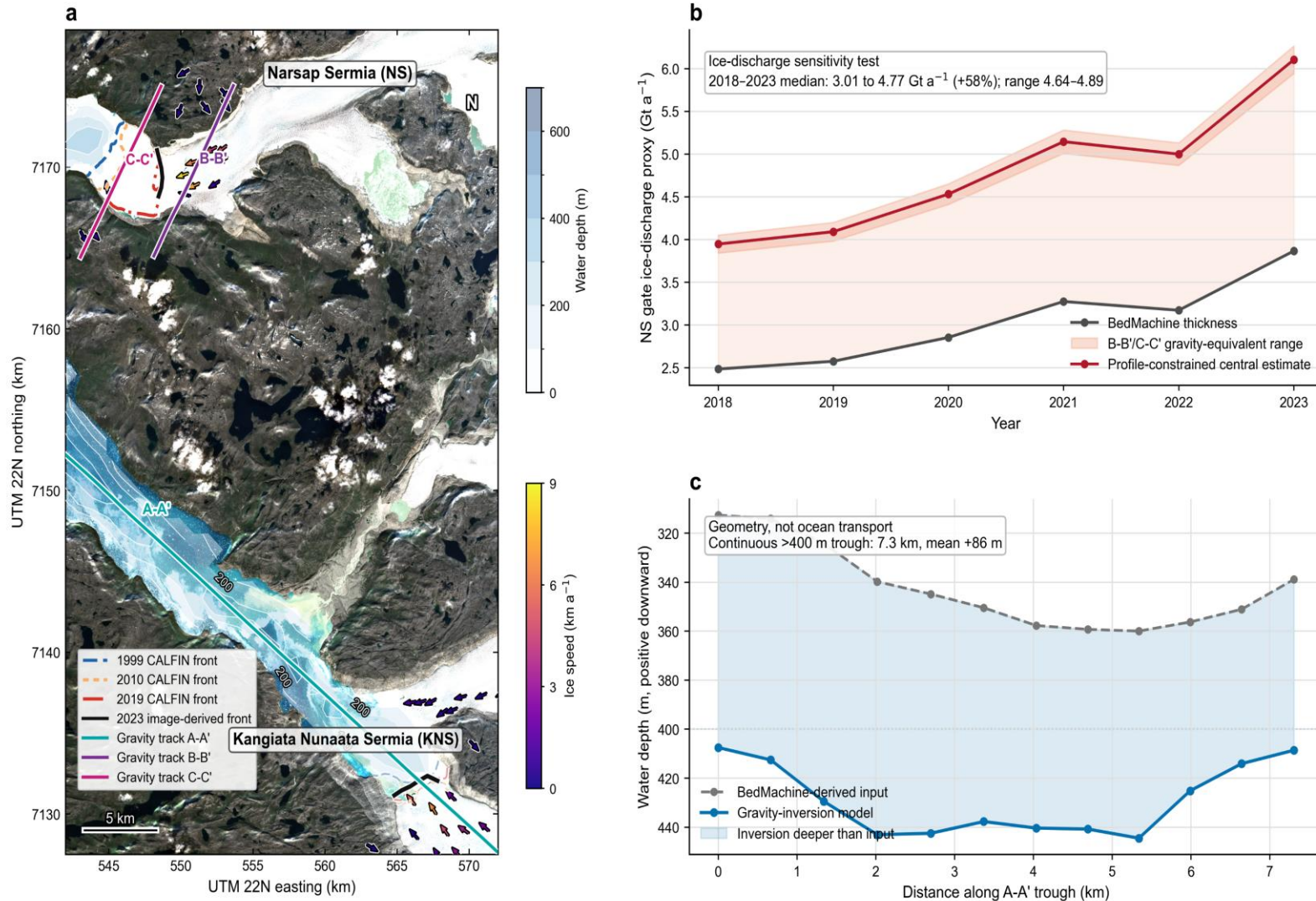


Bathymetry Prediction – Results



Inverted bathymetry and profile comparisons. (a) Gravity-derived fjord bathymetry. (b) Difference between the inversion and interpolated BedMachine starting model, expressed as inversion minus BedMachine; positive values indicate a deeper inversion. (c) A-A' crosses the eastern mid-fjord deep feature and compares observed gravity, inverted depth, and BedMachine depth. (d-f) Survey profiles B-B', C-C', and D-D' compare gravity with bed, ice thickness, or water depth as applicable.

Bathymetry Prediction – Results



- The accuracy of the inverted model is about 50 m.
- The inversion identifies a 7.3 km eastern mid-fjord trough, locally up to 105 m deeper than BedMachine.
- Gravity profiles near Narsap Sermia suggest possible ice-thickness underestimation and a 58% ice-discharge sensitivity

Lu et. al., local bathymetry prediction in Greenland fjords, 2026, ready to submit.

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National Space Institute



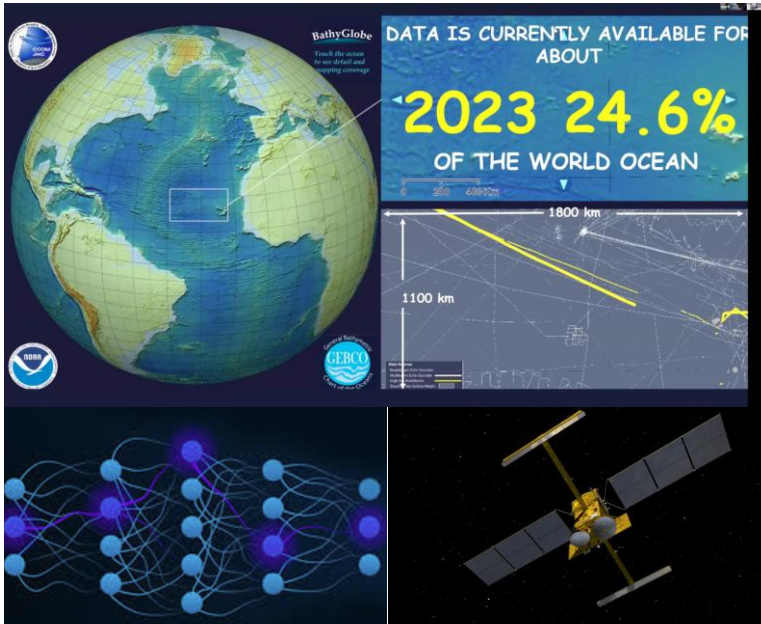
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Bathymetry Prediction by AI

Bathymetry Prediction + AI - Background

What to do with the remaining 75% of the globe
DTU workshop MSS, Gravity and Bathymetry



Bathymetric Prediction Workshop:
Introduction and Data Files
October 18, 2024

Global Bathymetry Prediction

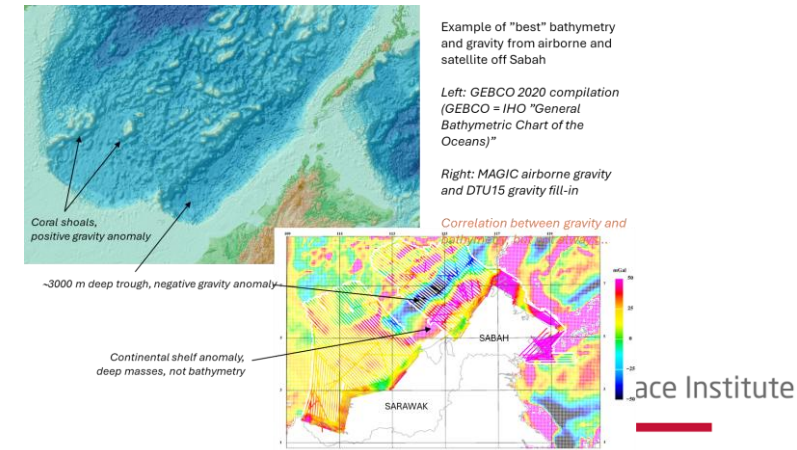
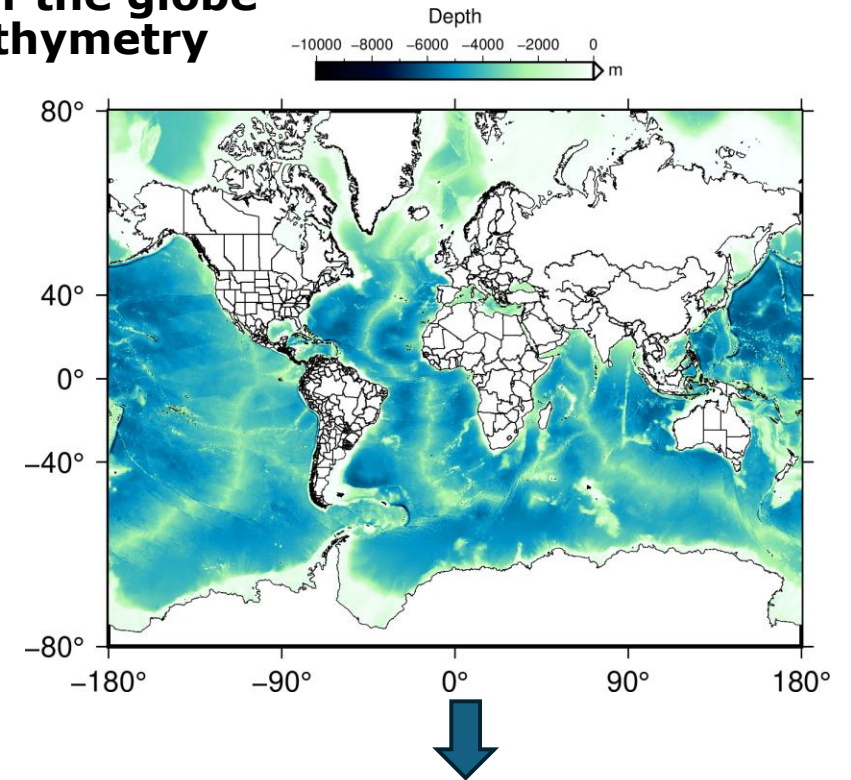
SWOT + AI



Significant improvements (up to 20-30%) compared to traditional inversions (Nettleton method) with only gravity anomalies

New generation of global bathymetry model!

Ongoing future work:
Regional Prediction in **Greenland, Malaysia** and so on

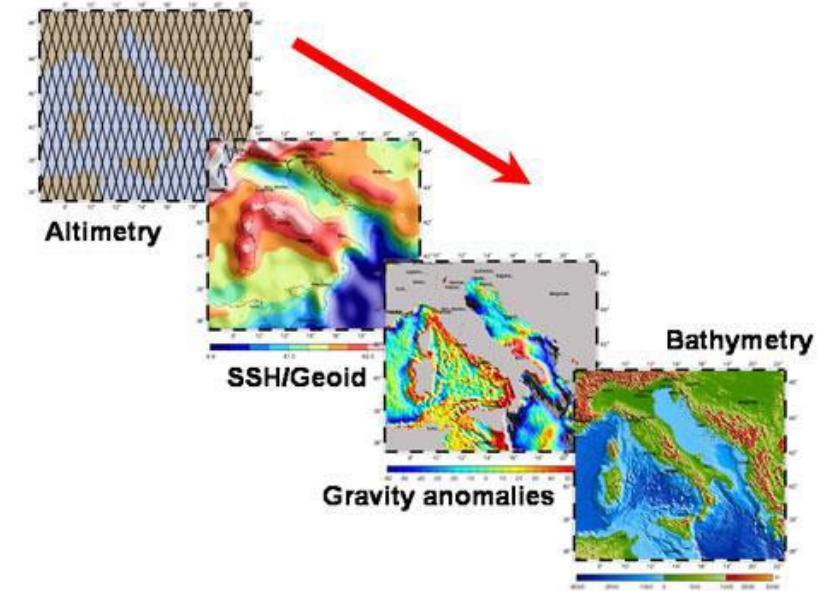


Methods: Nettleton method

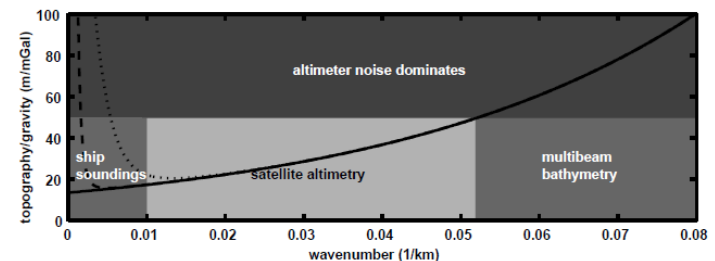
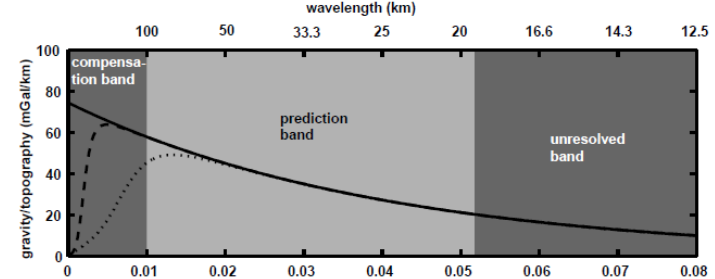
Inverse Nettleton (SIO products)

- Grid available depth soundings.
- Separate into low-pass and high-pass filtered components (~135 km).
- High-pass filter gravity and downward continue to low-pass filtered depths.
- Perform a robust linear regression of high-pass topography and high-pass, downward-continued gravity in small regions.
- **Multiply gravity by topography/gravity slope to predict topography in pass band.**
- Add original low-pass filtered depth.
- Force agreement with soundings.

Non-linear problem
Multi gravity signals

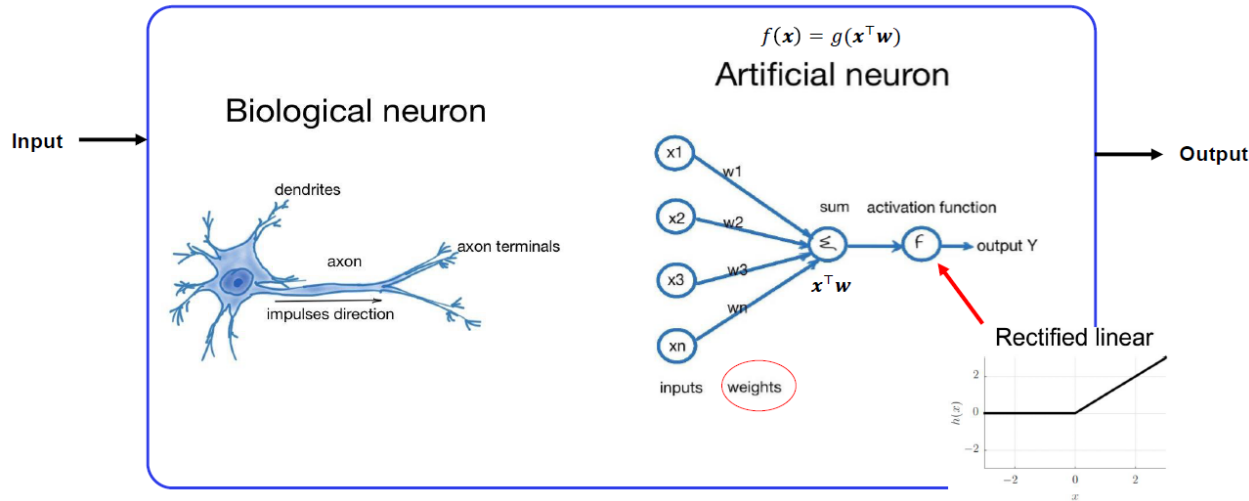


upward continuation and isostatic compensation
wavenumber domain

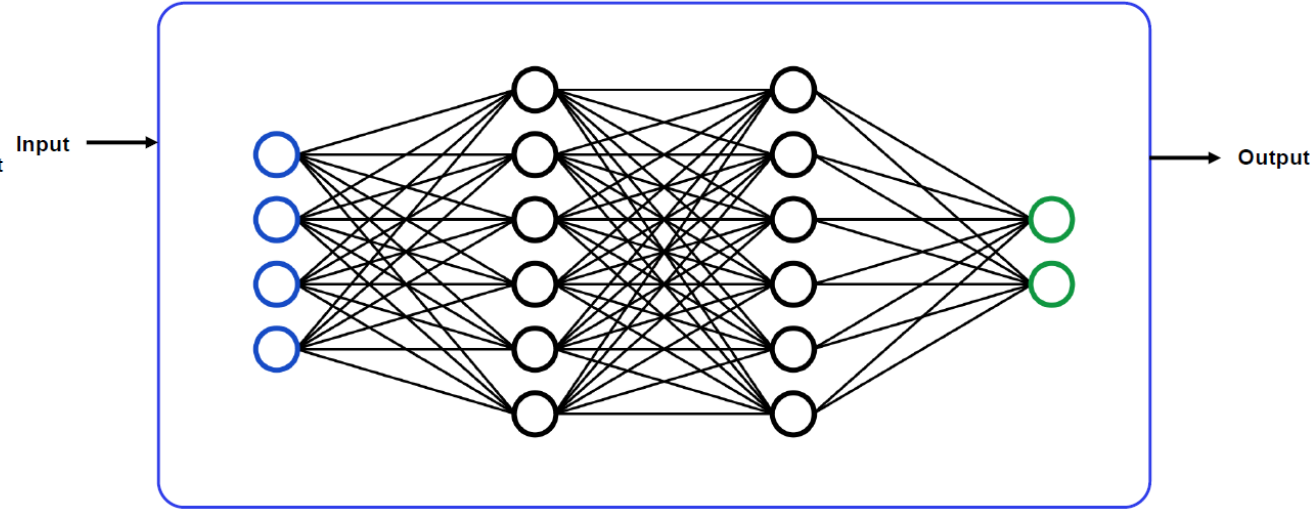


Methods: AI methods

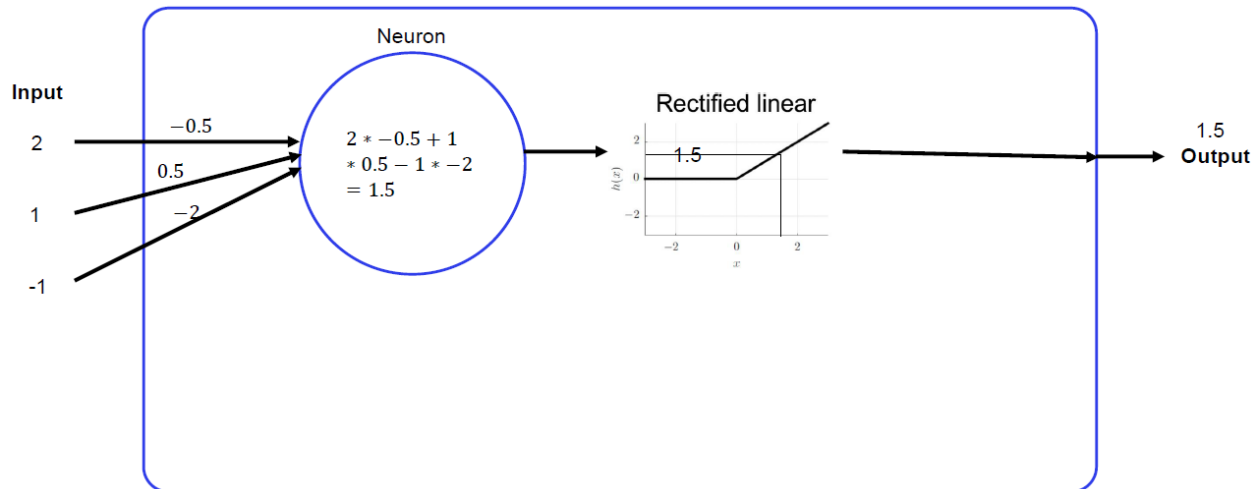
Neural network – the most general model



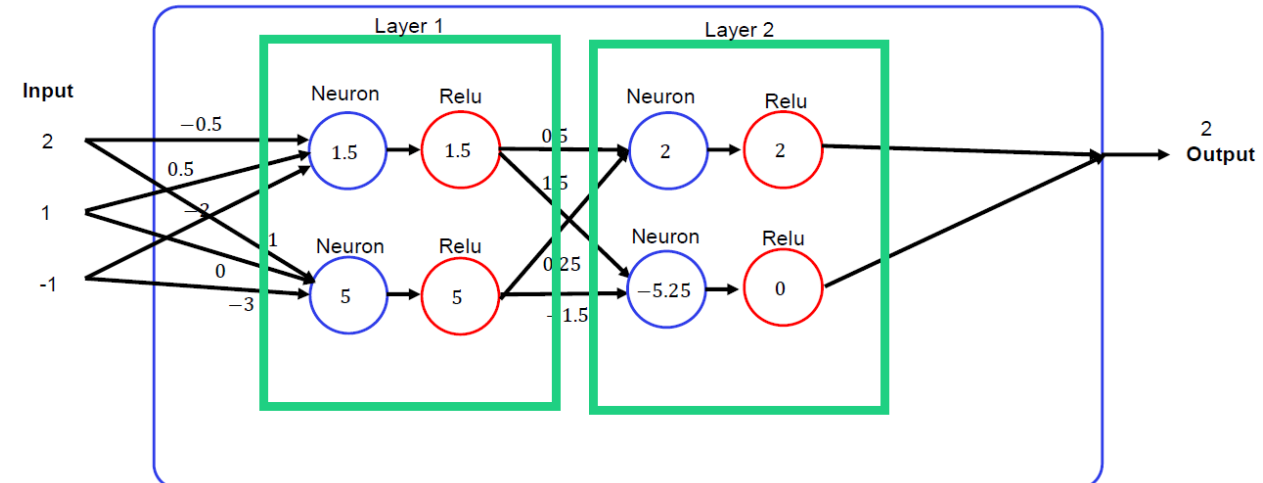
Neural network



Neural network - example

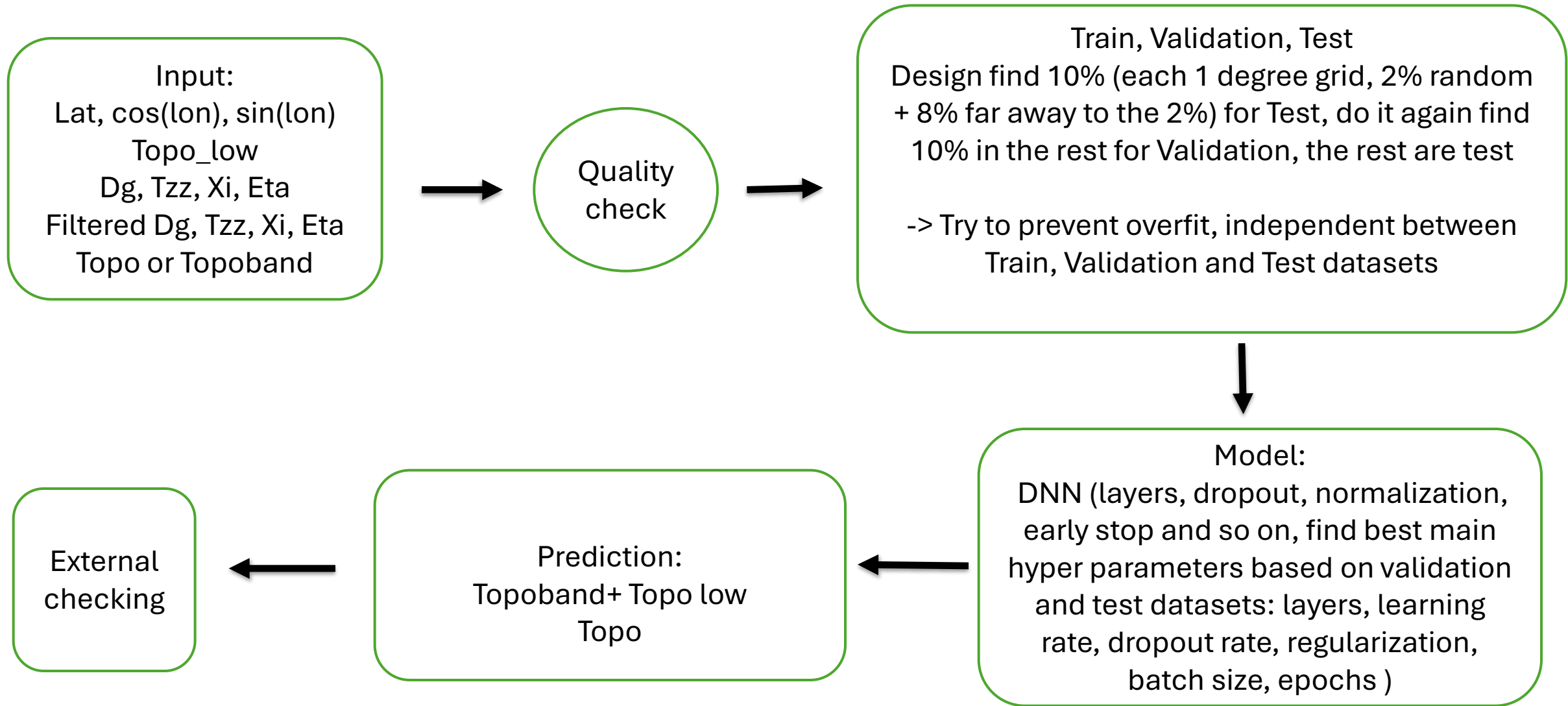


Neural network - example



Solving the problems: Non-linear problem + Multi gravity signals

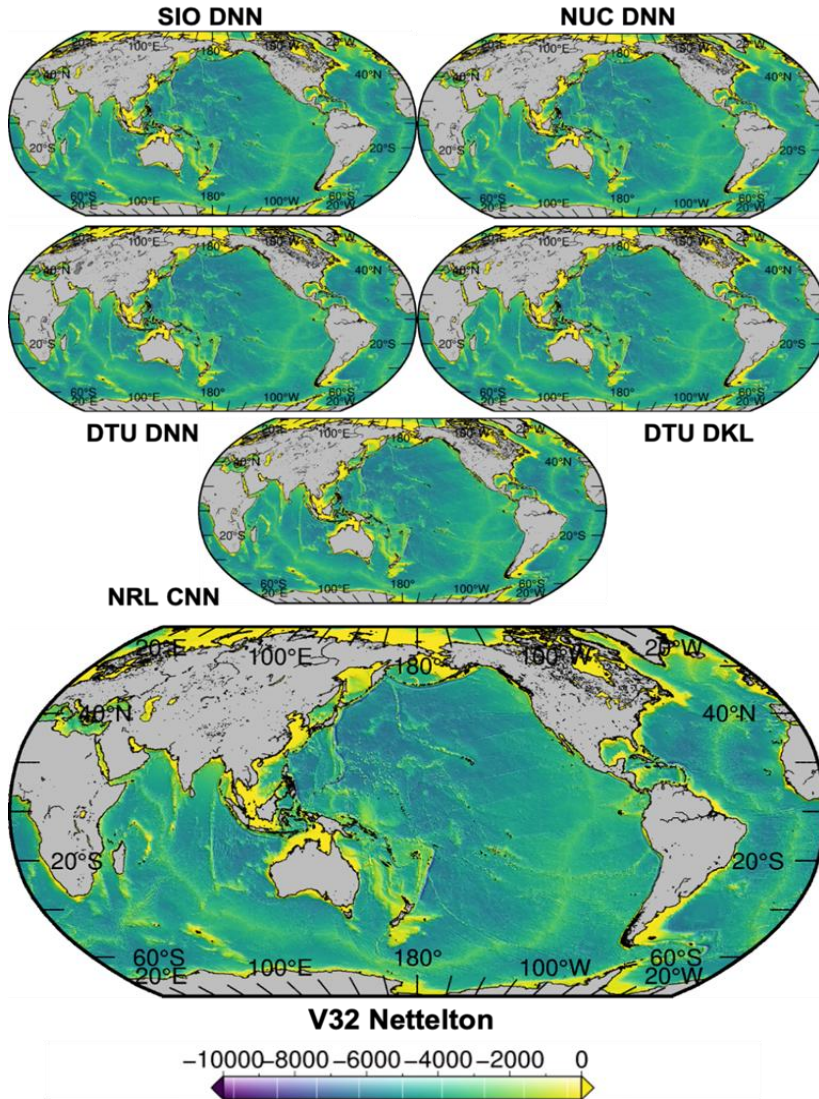
Flowchart



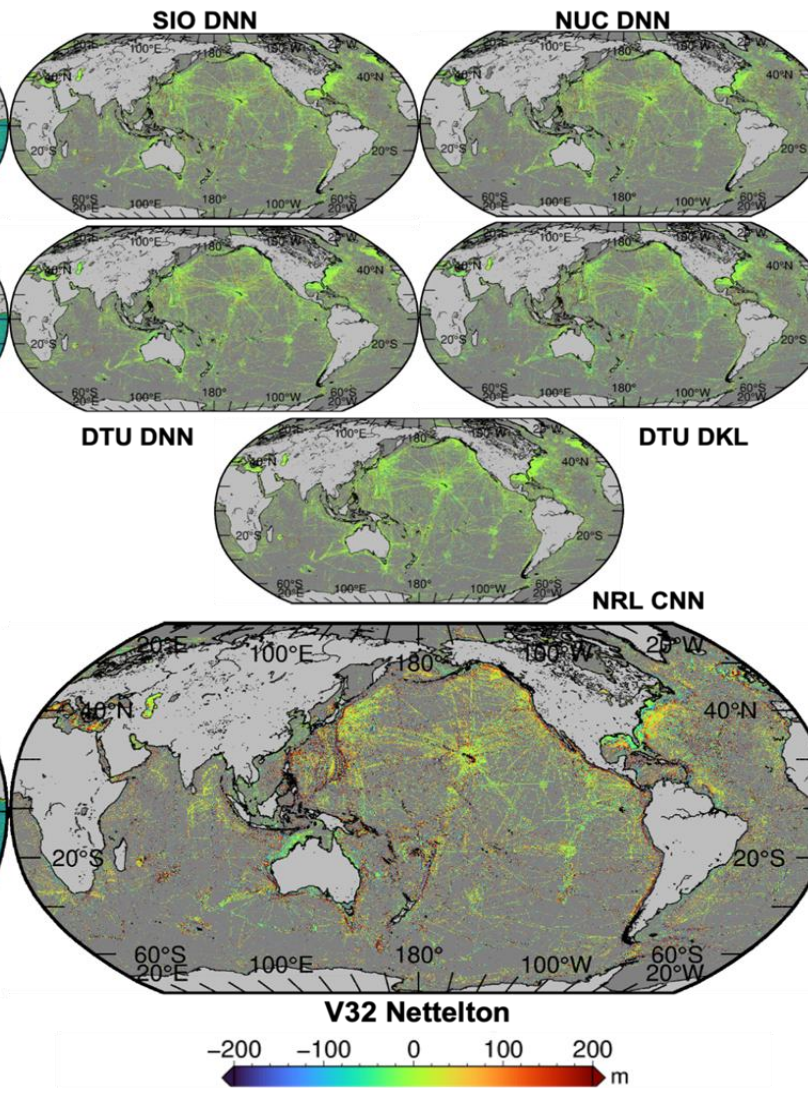
HPC + GPUs

Results: Global Bathymetry Models

Bathymetric Predictions



Training Error

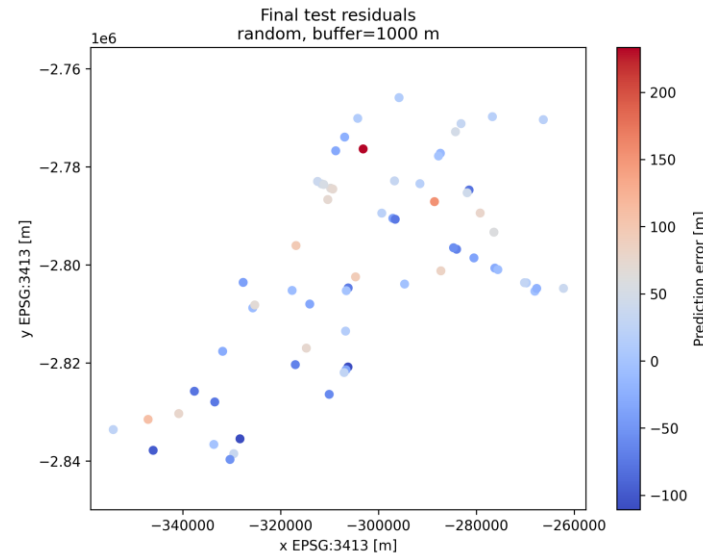
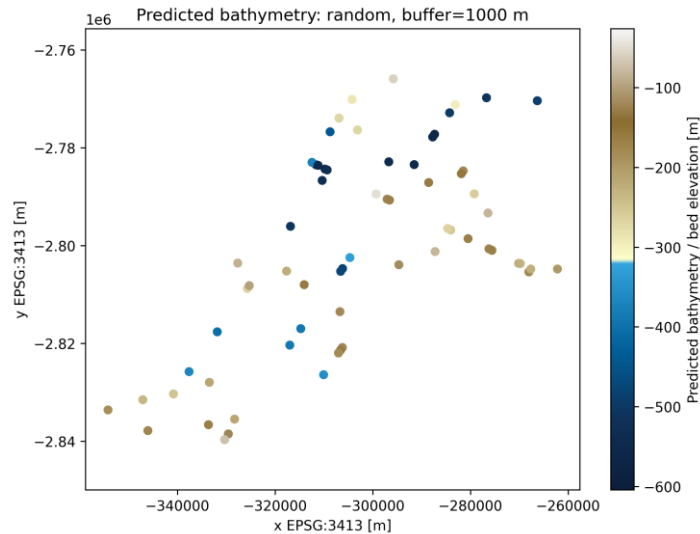
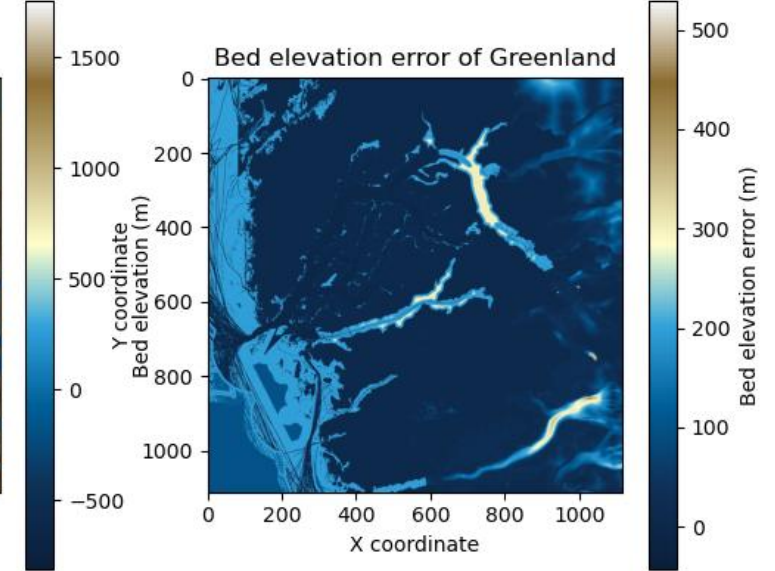
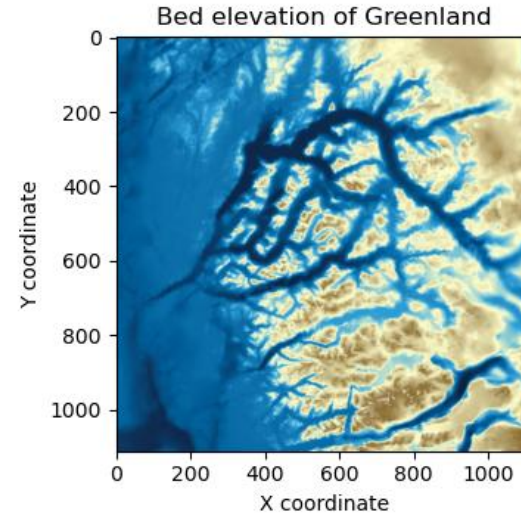
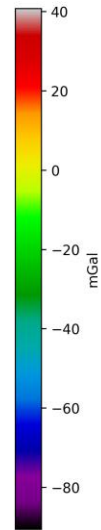
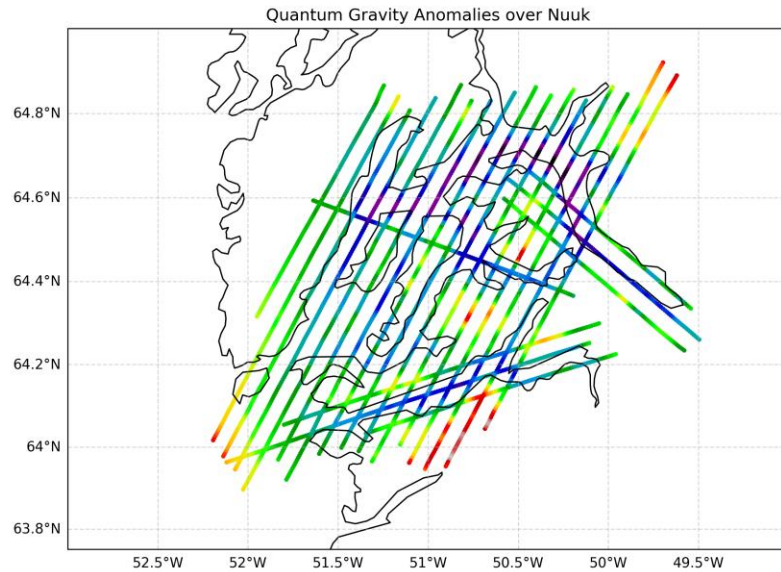


- SWOT-derived gravity improves global bathymetric predictions, especially in areas of complex seafloor.
- Five independent ML models produce remarkably consistent global predictions with accuracy improvements of 23-46% over classical methods.
- The greatest improvements are at the summits of seamounts and the deep ocean trenches.

Sandwell, D., Phrampus, B.J., Salajegheh, F., Nilsson, B., Lu, B., Yu, Y., Harper, H., Andersen, O.B., Smith, W., Elmore, P. and Kirby, J., 2026. Bathymetry prediction with SWOT gravity anomaly using machine learning methods: Paper 1—model development. *Journal of Geophysical Research: Solid Earth*, 131(8), p.e2025JB032798.

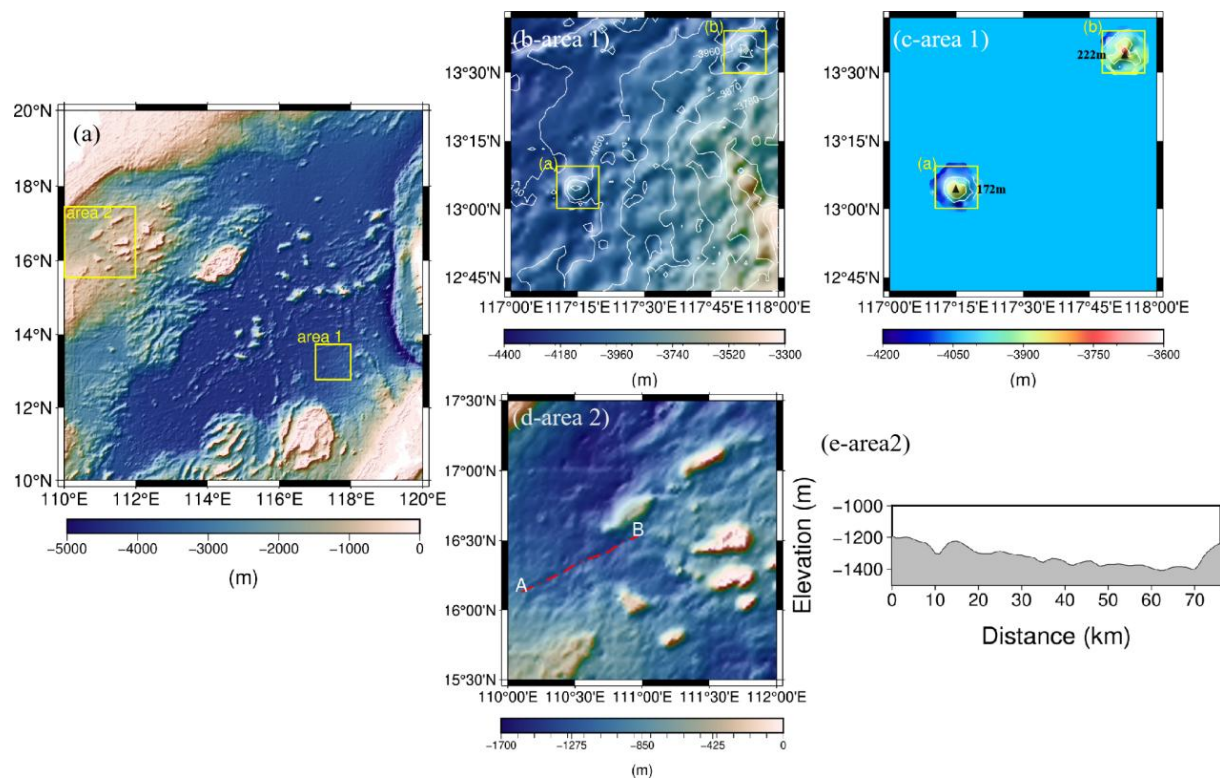
Nilsson, B., Phrampus, B.J., Salajegheh, F., Lu, B., Yu, Y., Sandwell, D., Andersen, O.B., Smith, W.H., Elmore, P., Kirby, J.F. and Altamirano, L., 2026. Bathymetry prediction with SWOT gravity anomaly using machine learning methods: Paper 2—model evaluation and uncertainty analysis. *Journal of Geophysical Research: Solid Earth*, 131(8), p.e2025JB033502.

Bathymetry Prediction – AI -Greenland

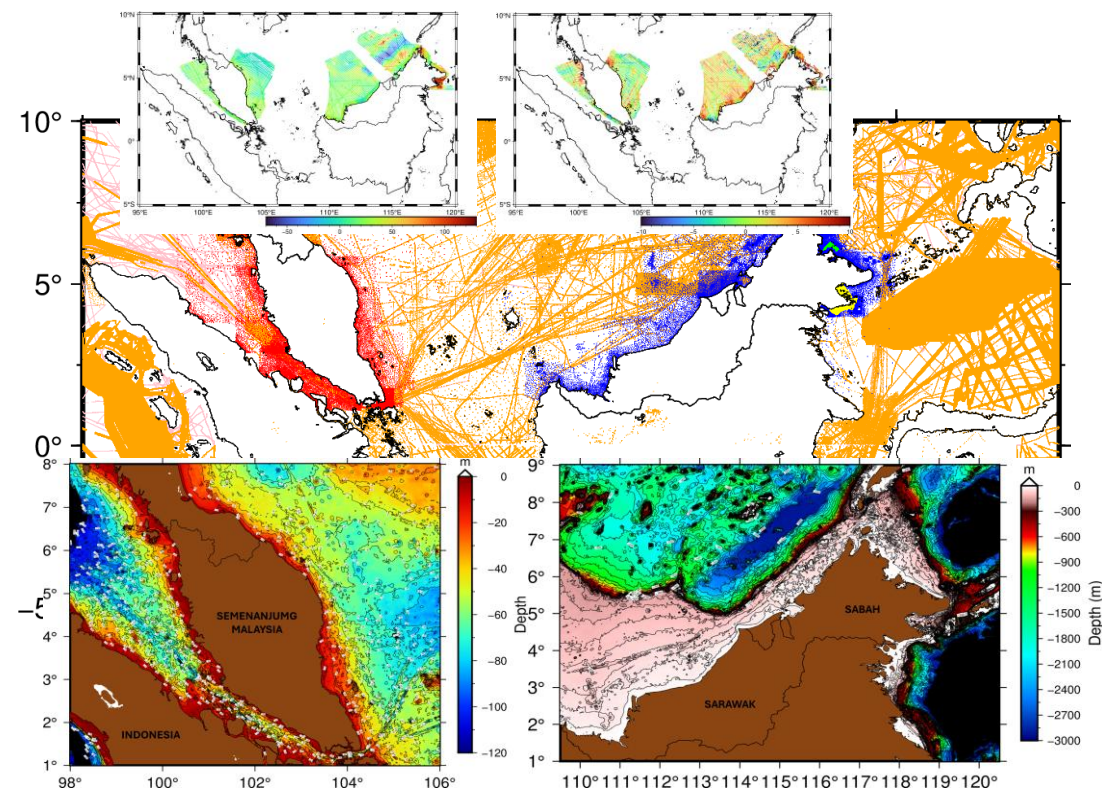


Model	XGBoost	DNN
RMS	63.34 m	77.96 m
Features	Coordinates Gravity Anomaly Height of Flight Distance from Land Centerline	

Relevant Works



South China Sea
SWOT+AI



Malaysia coastal areas
SWOT+Airborne Gravity+AI+Collocation

Xu et. al., Unveiling Hidden Seafloor Features in the South China Sea Using SWOT-Derived Multi-Gravity Signals and Machine Learning, submitted, 2026.

Lu et. al., need to revise, local bathymetry prediction in Malaysia, 2026.

TAKE HOME

- **Conventional geodetic methods remain essential**, providing the physical basis, constraints, and interpretability needed to support AI-driven approaches.
- **AI is a powerful tool, but not a replacement for physics.** Geodesists can make AI more reliable, interpretable, and effective by integrating mathematical and physical knowledge.
- **Quantum sensing and quantum technologies** represent an important emerging direction in geodesy, with strong potential for future gravity and navigation applications.
- In the **AI era, high-quality geodetic data will become increasingly critical.** Better data, together with physical knowledge, will ultimately determine the reliability of AI-based geoscience.
- **Bathymetry prediction still requires further advances**, particularly in data coverage, uncertainty quantification, multi-source integration, and physics-informed AI.
- **Geodesy must continue to identify and develop new scientific and technological frontiers**, connecting fundamental physics, Earth observation, AI, and emerging sensing technologies.

Thank you for your attention!

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