



Integrating Geophysics and Geodesy

***Roberto Sabadini
Università degli Studi di Milano***

***XI Hotine-Marussi Symposium
Genoa, Italy, September 7-11, 2026***

Linearized Momentum and Poisson equations

$$\nabla \cdot \boldsymbol{\sigma}^\delta + \nabla (\mathbf{u} \cdot \nabla p_0) - \rho^\Delta \nabla \phi_0 - \rho_0 \nabla \phi^\Delta - \rho^L \nabla \phi_0 + \mathbf{M} = \mathbf{0}$$

$$\nabla^2 \phi^\Delta = 4\pi G (\rho^\Delta + \rho^L + \rho^T) - 2\omega^2$$

Mass conservation

$$\rho^\Delta = -\nabla \cdot (\rho_0 \mathbf{u}) = -\rho_0 \Delta - \mathbf{u} \cdot \nabla \rho_0$$

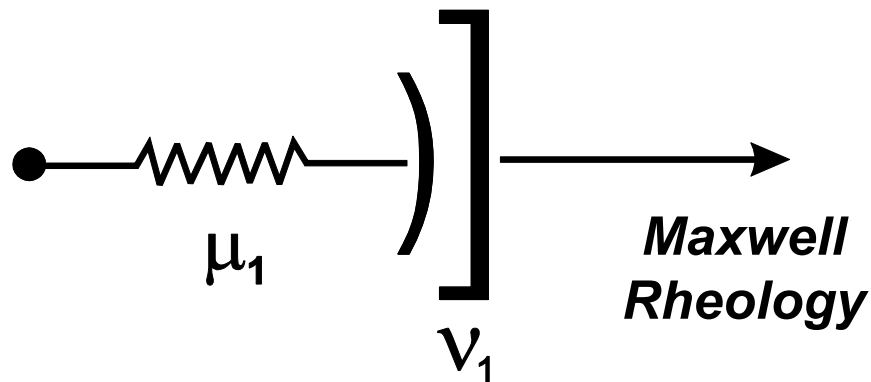
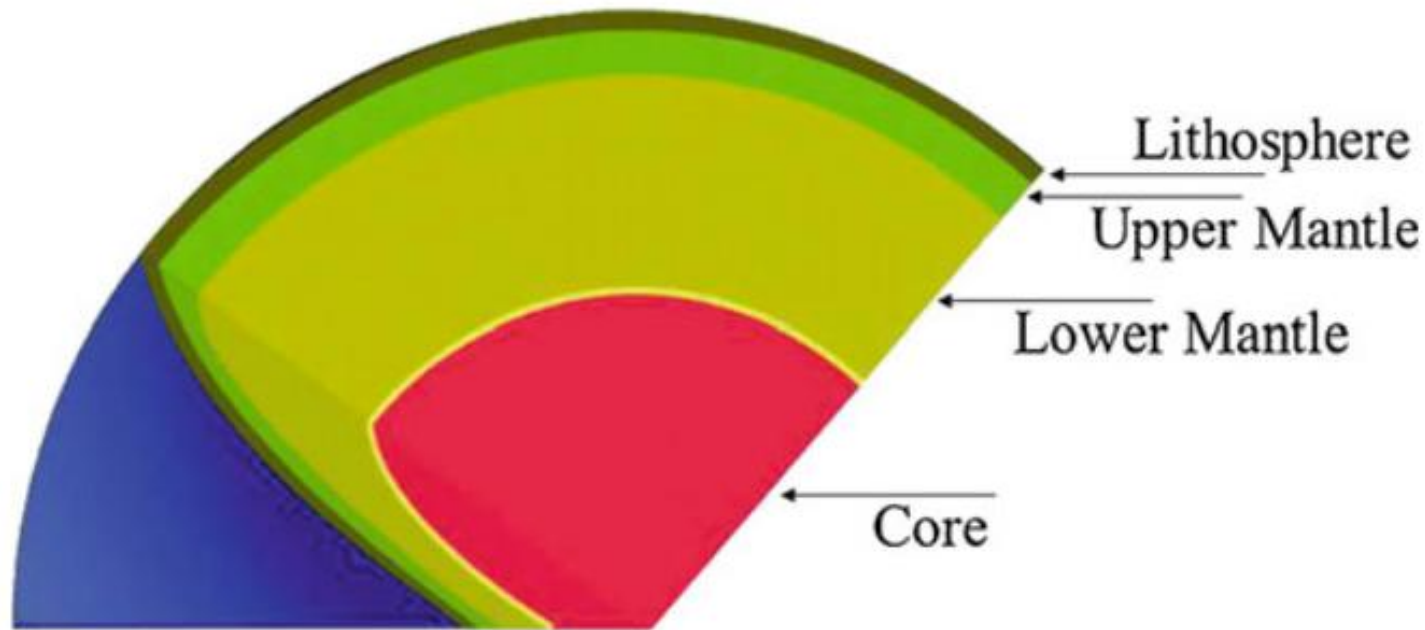
Energy conservation

$$\rho C_p \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (K \nabla T) + H$$

Angular momentum conservation

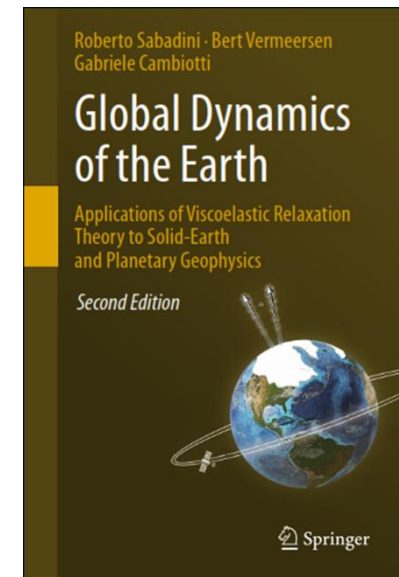
$$\frac{d\mathbf{H}}{dt} + \boldsymbol{\omega} \times \mathbf{H} = \mathbf{L}$$

angular $\mathbf{H} = \mathbf{I} \cdot \boldsymbol{\omega}$ angular momentum, containing the moment of inertia and angular velocity

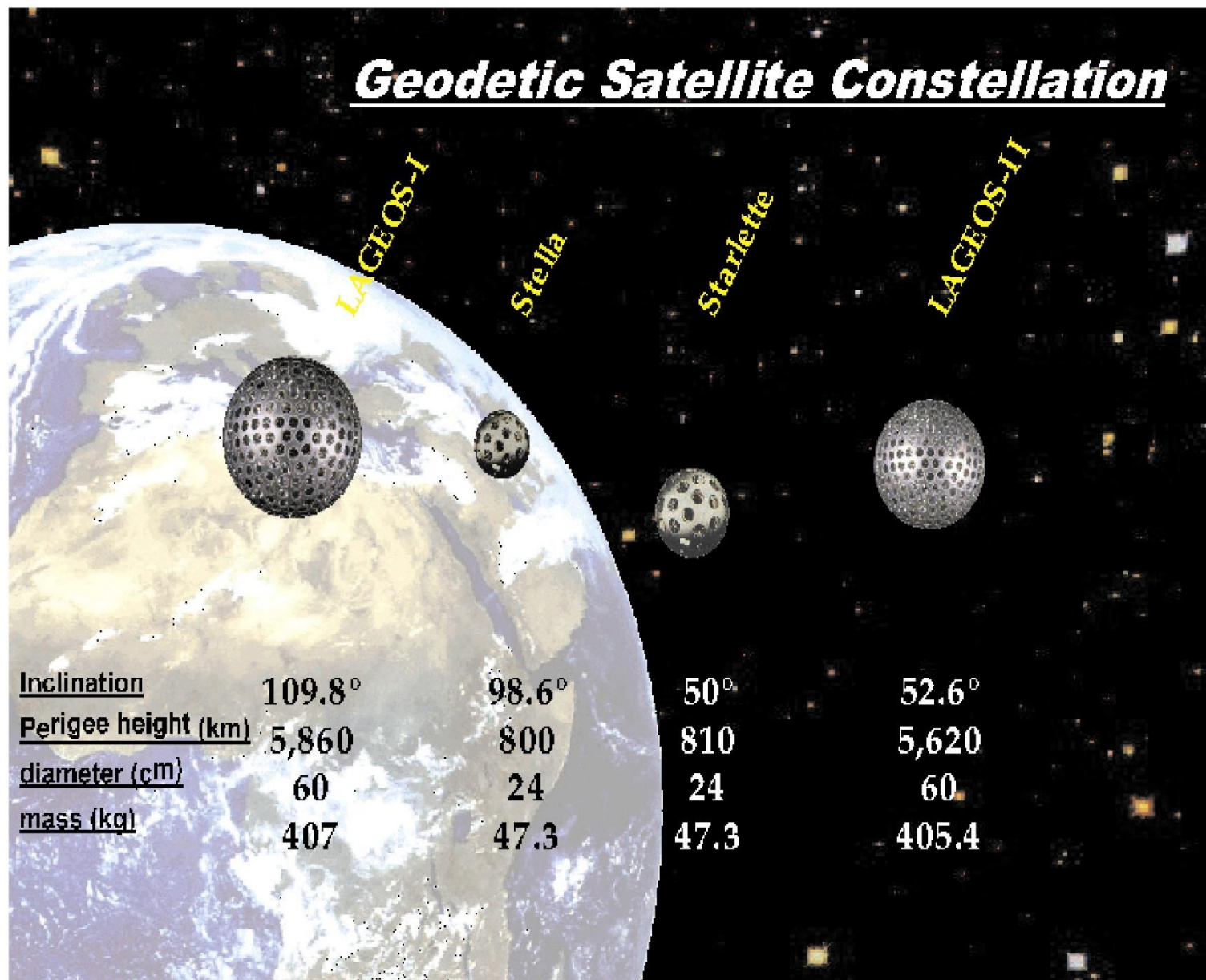


Roberto Sabadini, Bert Vermeersen and Gabriele Cambiotti, (2016). Global Dynamics of the Earth - Applications of Viscoelastic Relaxation Theory to Solid-Earth and Planetary Geophysics, Springer, ISBN: 978-94-017-7552-6 (second Edition),

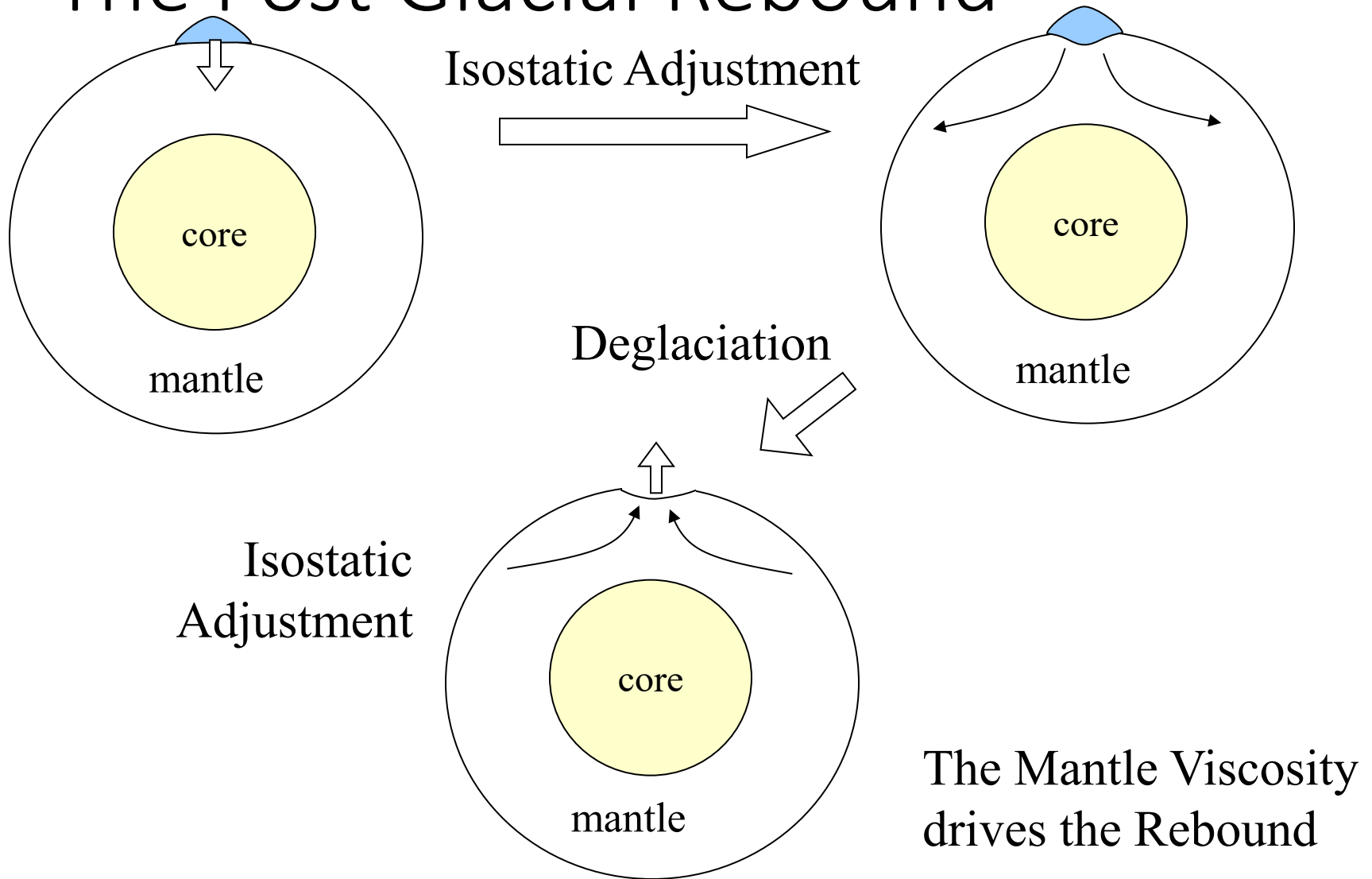
Email: roberto.sabadini@unimi.it, robertosabadini253@gmail.com



LAGEOS I lanciato il 4 maggio 1976, LAGEOS II lanciato il 23 ottobre 1992



The Post Glacial Rebound



Coherence between estimates of ice mass loss in Antarctica and Greenland from SLR

- Upper mantle viscosity 10^{21} Pa s, Lower mantle viscosity 2×10^{22} Pa s, Antarctica **-275 Gt/yr**, Greenland **-145 Gt/yr**, total ice mass loss of both Antarctica and Greenland **-420 Gt/yr**. (James and Ivins, 1997, Global geodetic signature of the Antarctic ice sheet, J. Geophys. Res., 102, 605-633.
- Upper mantle viscosity 1.3×10^{20} Pa s, Lower mantle viscosity 3.2×10^{22} Pa s, Antarctica **-250 Gt/yr**, Greenland **-144 Gt/yr**, total ice mass loss of both Antarctica and Greenland **-394 Gt/yr**. (Sabadini R, Di Donato G, Vermeersen LLA, Devoti R, Luceri V, Bianco G, 2002. Ice mass loss in Antarctica and stiff lower mantle viscosity inferred from the long wavelength time dependent gravity field. Geophys. Res. Lett., doi: 10.1029/2001GL014016

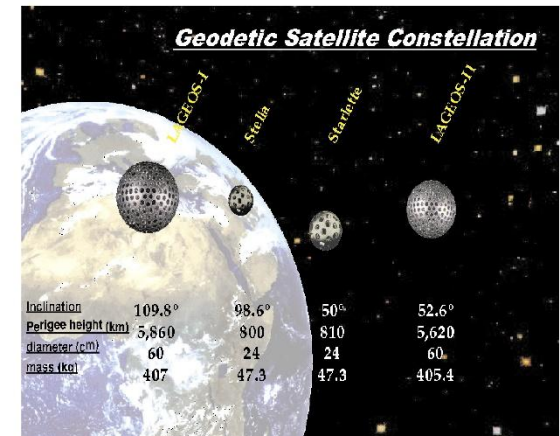
corresponding to an eustatic sea-level rise of 1.1 mm/yr

Table 1. Even Degree Zonal Secular Drift, Units: 10^{-11} /year

	$\dot{J}_2$	$\dot{J}_4$	$\dot{J}_6$
C97	-2.7 ± 0.4	-1.4 ± 1.0	0.3 ± 0.7
D01	-2.9 ± 0.2	0.6 ± 0.5	0.3 ± 0.3

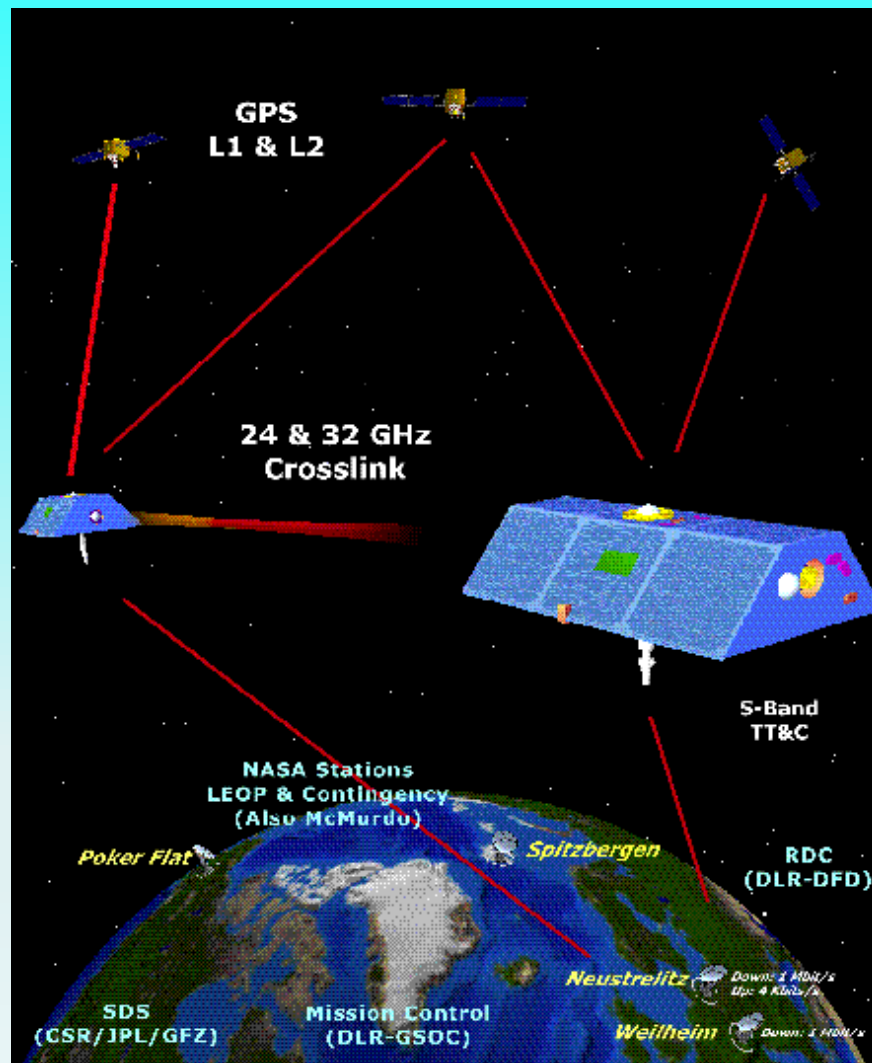
Table 2. Odd Degree Zonal Secular Drift, Units: 10^{-11} /year

	$\dot{J}_3$	$\dot{J}_5$	$\dot{J}_{odd}$
C97	-1.3 ± 0.5	2.1 ± 0.6	
D01			0.5 ± 0.2

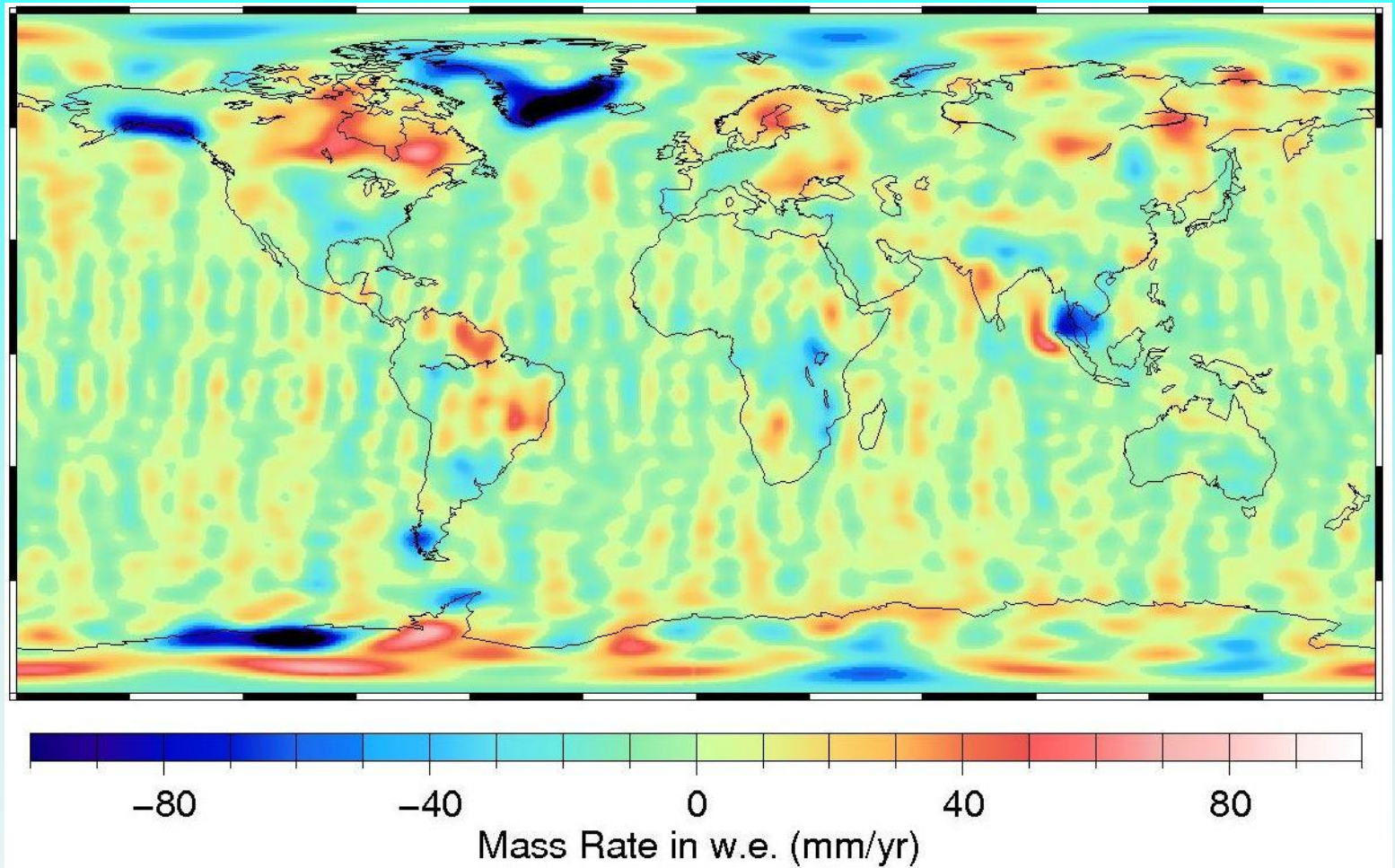


GRACE

March 17, 2002



The Map of Mass Variation Trend – Filtered from GRACE



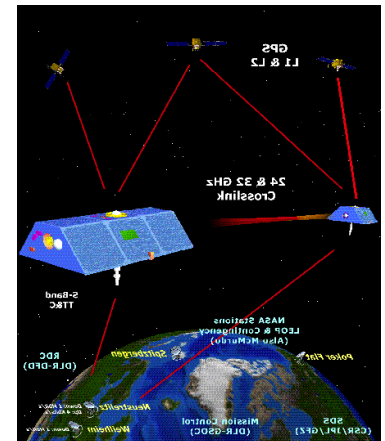
Ice mass loss in Antarctica and Greenland in Gt/yr from GRACE taking into account the preferred mantle viscosities for the upper and lower mantle and thus preferred PGR models, as well as a complete set of GRACE data releases from different teams.

Antarctica **-171 ± 39 Gt/yr**

Greenland **-101 ± 22 Gt/yr**

Total **-272 ± 61 Gt/yr**

V.R. Barletta, R. Sabadini, A. Bordonì (2008). Isolating the PGR signal in the GRACE data: impact on mass balance estimates in Antarctica and Greenland. GEOPHYSICAL JOURNAL INTERNATIONAL, vol. 172, p. 18-30, ISSN: 0956-540X, doi: 10.1111/j.1365-246X.2007.03630.x



Ice mass loss in Greenland from CryoSat and IceSat (ESA-NASA)

-196 $\pm$ 37 Gt/yr or -233 - **-159 Gt/yr**

Ravinder N., A. Sheperd, I. Ootosaka, T. Slater A. Muir and L. Gilbert, 2024,
Greenland Ice Sheet Elevation Change From CryoSat-2 and ICESat-2,
Geophysical Research Letters, doi.org/10.1029/2024GL110822

with our highest GRACE Greenland ablation estimate (-123 Gt/yr) being 23 % lower than the minimum ablation rate from ice satellites (**-159 Gt/yr**) .

Antarctic Ice Mass Loss 2002-2025 (NASA)

- **Released** Friday, March 8, 2024

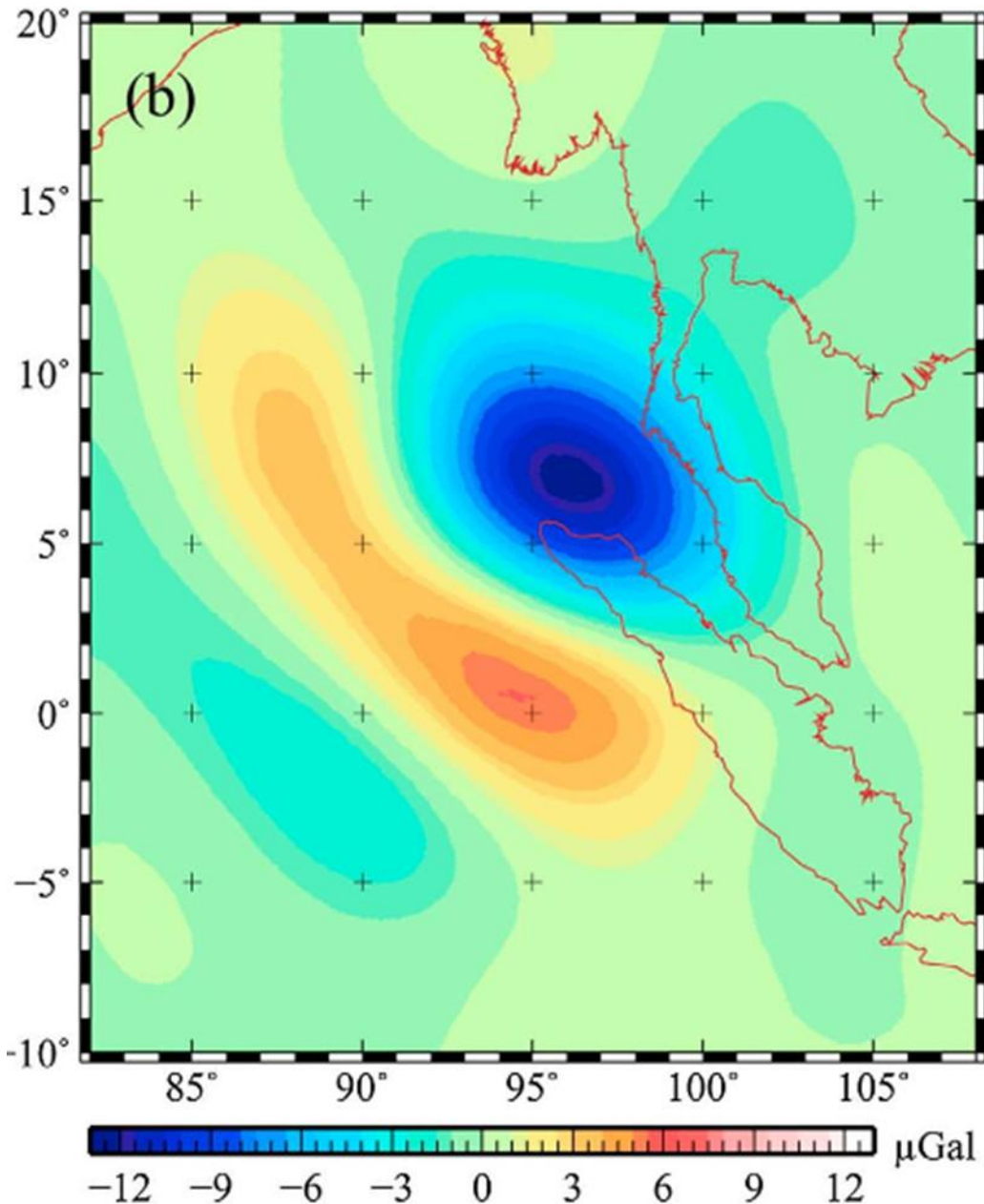
*Research based on observations from GRACE (2002-2017) and GRACE Follow-On (since 2018 -) indicates that between 2002 and 2025, Antarctica shed approximately **135 gigatons of ice per year**, causing global sea level to rise by 0.4 millimeters per year.*

If we sum the IceSat and CryoSat Greenland estimate and the NASA GRACE Antarctica estimate we obtain

-331 Gt/yr

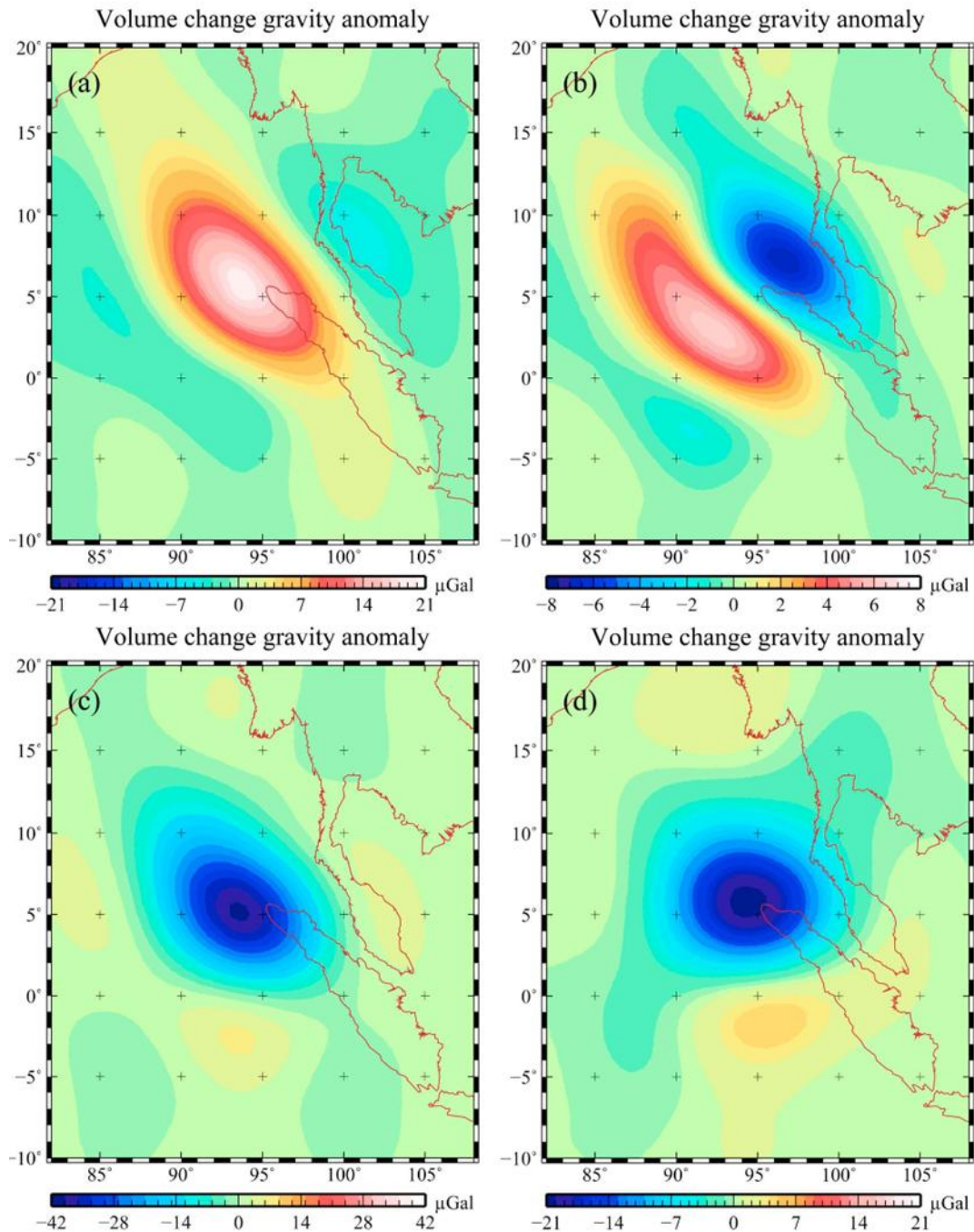
which is 16 % lower than our own (Sabadini et al., 2002) SRL estimates, except with Antarctica and Greenland contributions to total ice loss somehow interchanged in terms of highest contributors to total ice loss. It is higher than our preferred GRACE solution **-272 $\pm$ 61 Gt/yr by 18 %**, although is well within our own error bounds.

O-PREM gravity anomaly



***Modelled and filtered gravity anomaly
For the compressible model including
the Ocean (O-PREM) based on the
seismological solution
by Tsai et al. (2005)***

G. Cambiotti, A. Bordon, R.
Sabadini, L. Colli (2011). GRACE
gravity data help constraining seismic
models of the 2004 Sumatran
earthquake. *JOURNAL OF
GEOPHYSICAL RESEARCH. SOLID
EARTH*, vol. 116, ISSN: 0148-0227,
doi: 10.1029/2010JB007848



Gravity anomaly from volume changes

(a) in the mantle

(b) in the crust

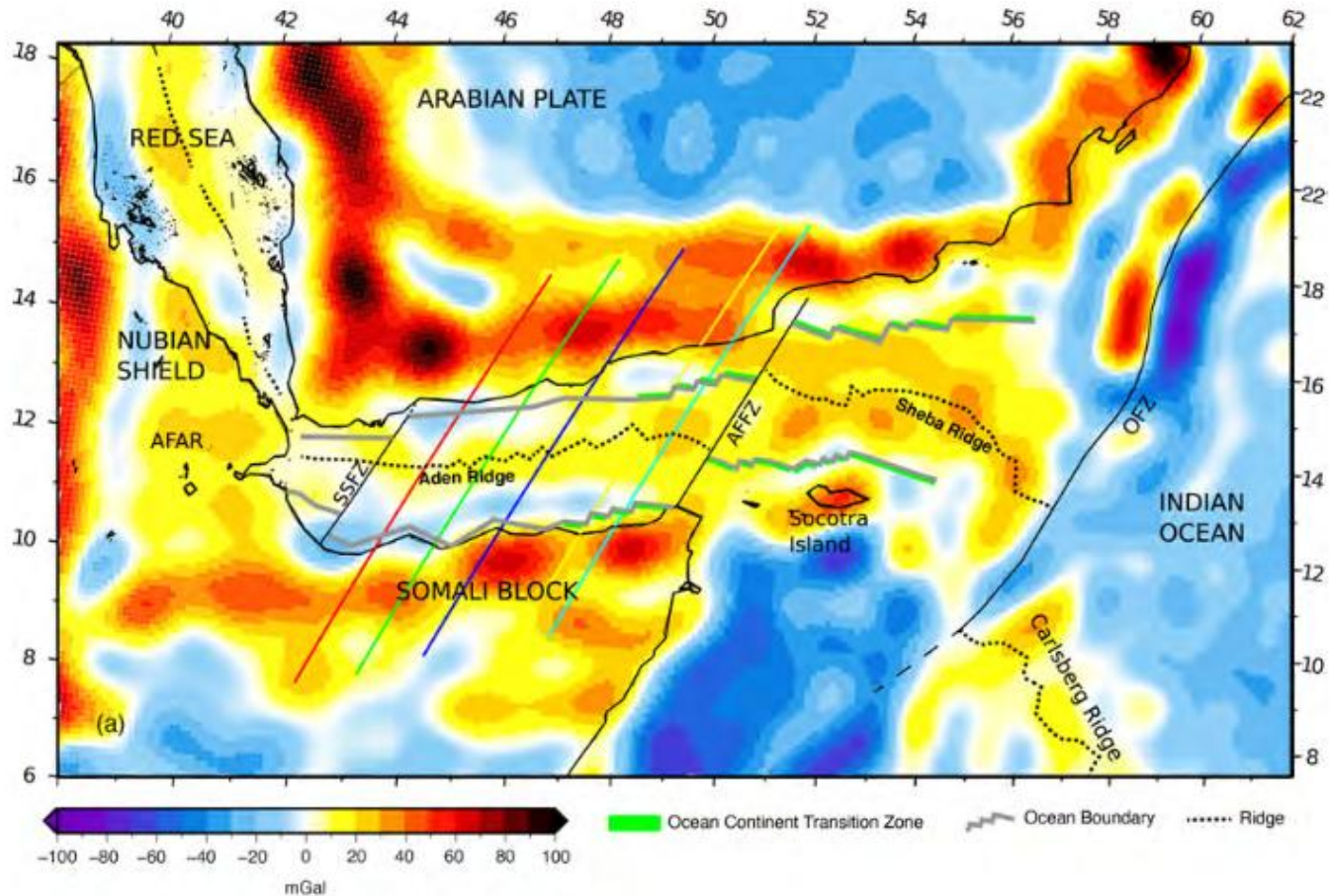
(c) in proximity of the fault

(d) sum of the previous ones

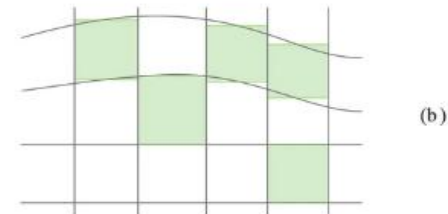
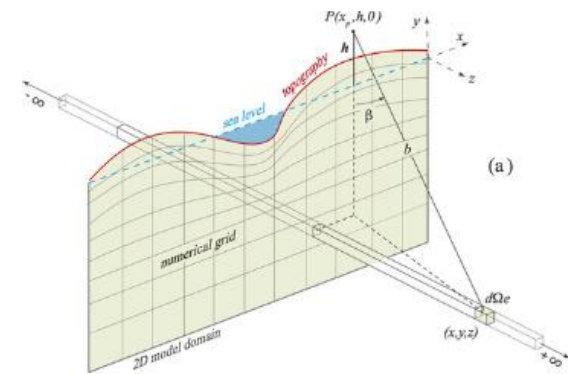
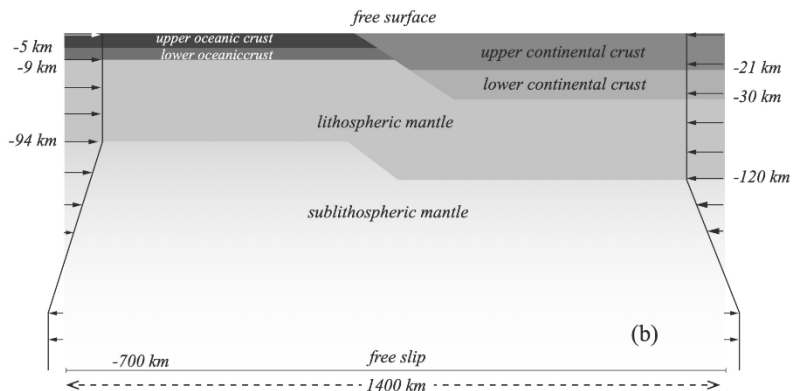
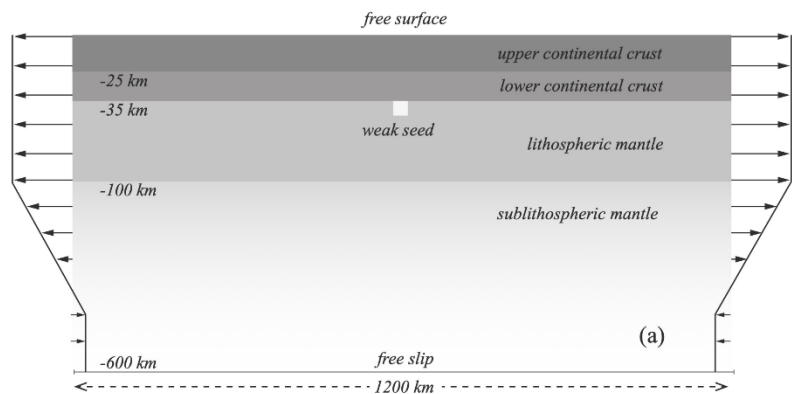
G. Cambiotti, A. Bordon, R. Sabadini, L. Colli (2011). GRACE gravity data help constraining seismic models of the 2004 Sumatran earthquake. *JOURNAL OF GEOPHYSICAL RESEARCH. SOLID EARTH*, vol. 116, ISSN: 0148-0227, doi: 10.1029/2010JB007848

GOCE (March 17, 2009)
Gravity and steady-state
Ocean Circulation Explorer





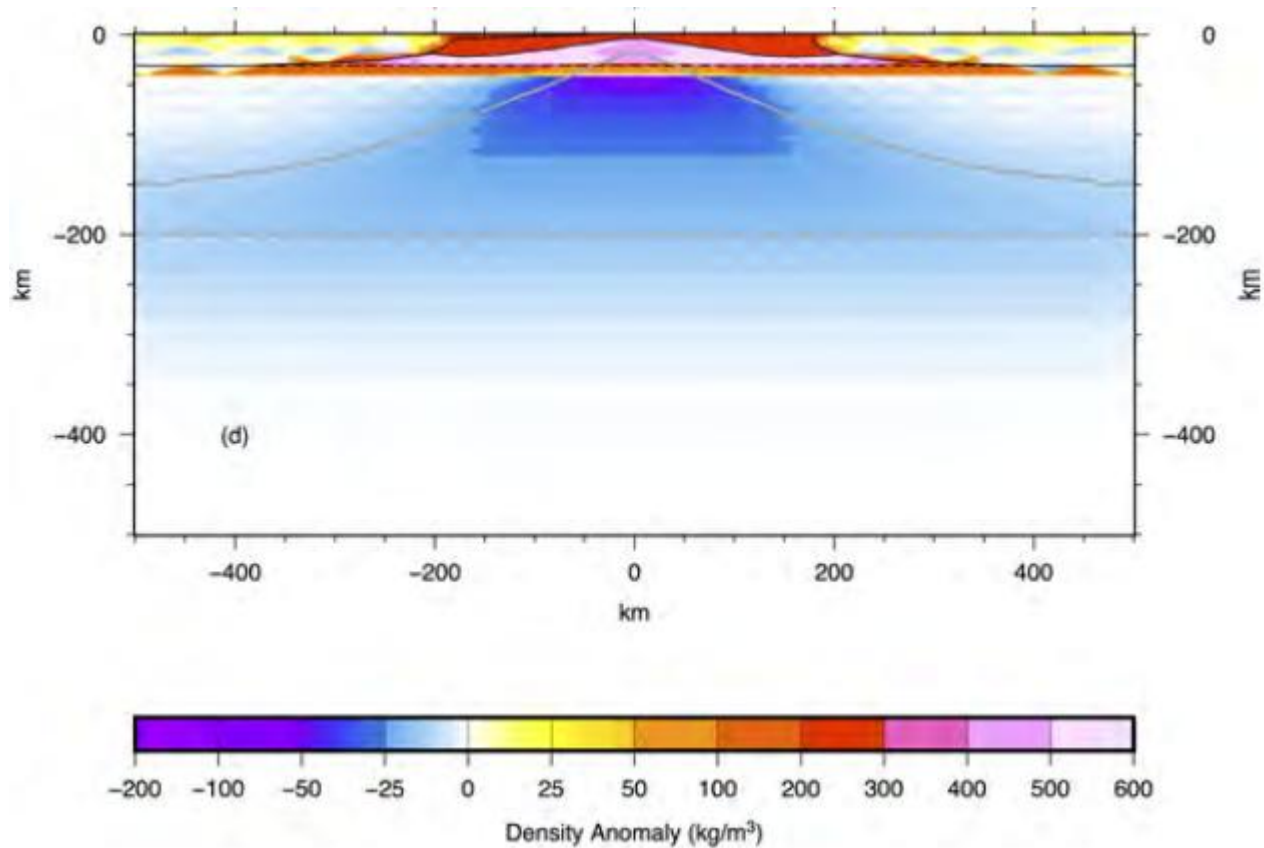
Marotta, A.M., Barzaghi, R., Bollino, A., Regorda, A., Sabadini, R., The gravitational signature of oceanization in the Gulf of Aden, Tectonophysics, 2023, Vol.869, DOI: 10.1016/j.tecto.2023.230110



Anna Maria Marotta, A.M., E. Spelta and C. Rizzetto, Gravity signal of crustal subduction from numerical modelling, *Geophysical journal international*, 2006-08, Vol.166 (2), p.923-938 (2006)

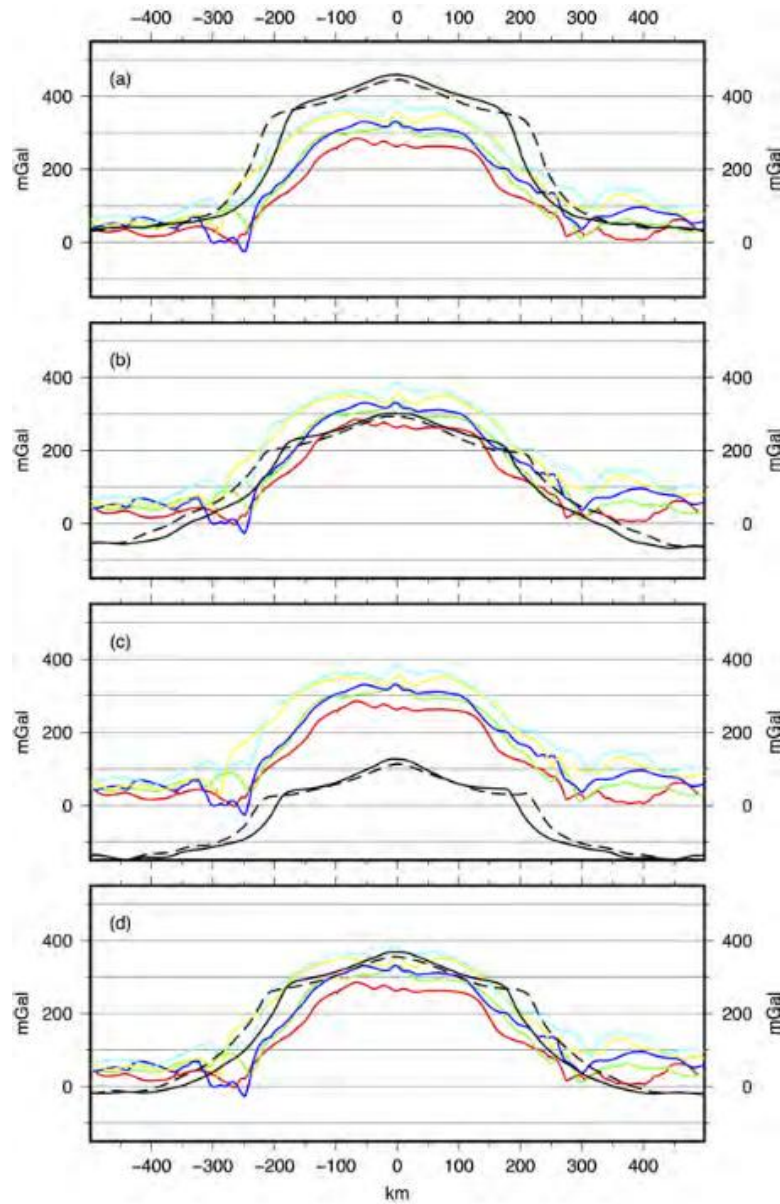
What is shown hereinafter for extensional tectonics is the result of an idea developed by Anna Maria Marotta, University of Milano, exploited within the ESA Study *Gravitational Seismology, EO Science for Society, July 2019: the effects of the horizontal motions of mantle-crust horizontal density contrasts on gravity rate of change*

FINAL Report , 4. Section 2, 4.4 Subsection SLOW TECTONICS AND QUASI STATIC GRAVITY CHANGES (Anna Maria Marotta, UNIMI)



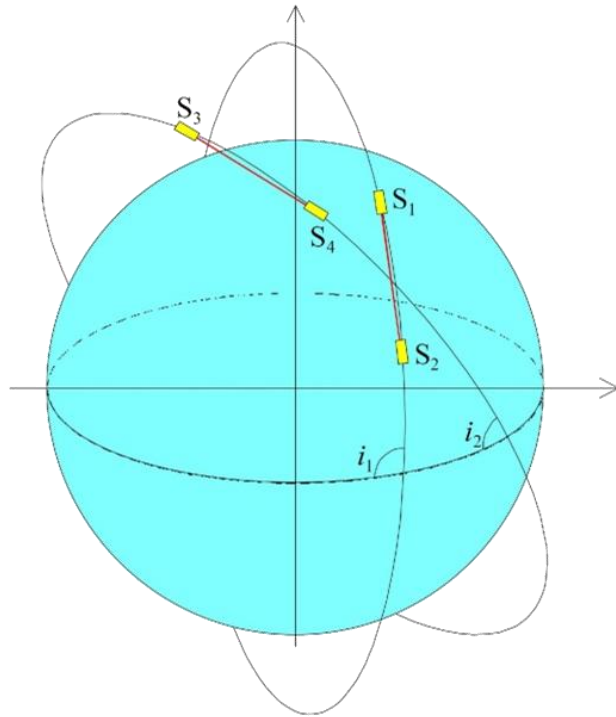
Marotta, A.M., Barzaghi, R., Bollino, A., Regorda, A., Sabadini, R., The gravitational signature of oceanization in the Gulf of Aden, *Tectonophysics*, 2023, Vol.869, DOI: 10.1016/j.tecto.2023.230110

-
-



Marotta, A.M., Barzaghi, R., Bollino, A., Regorda, A., Sabadini, R., The gravitational signature of oceanization in the Gulf of Aden, Tectonophysics, 2023, Vol.869, DOI: 10.1016/j.tecto.2023.230110

COURTESY FROM THALES ALENIA SPACE IN ITALY



Profiling the Earth, Gravitationally

MAGIC will sense the gravity rate of change with an accuracy of

1 microGal/yr

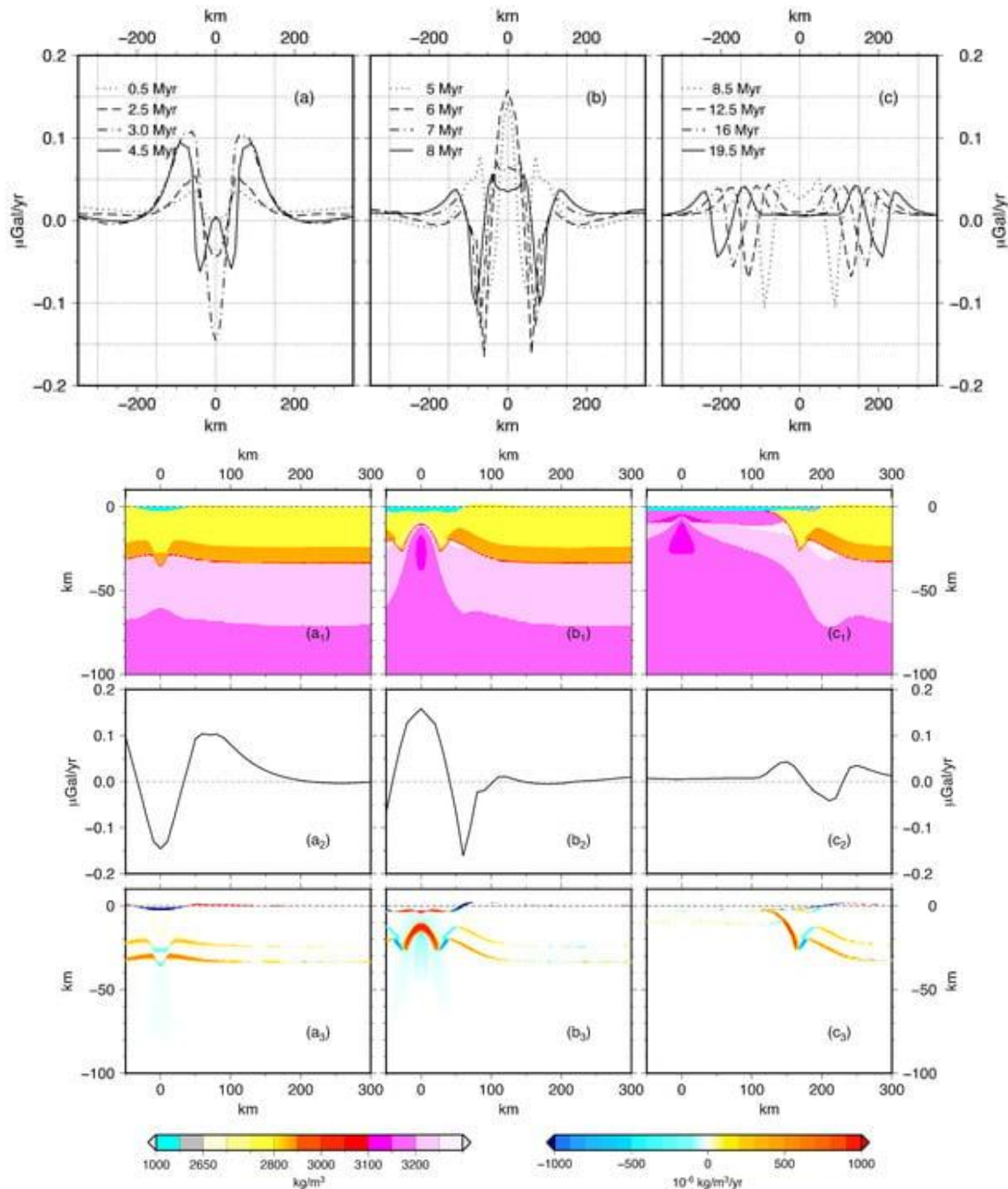
at high spatial resolution.

To get the flavor of the accuracy is like “sensing” the gravity of 1 cm crustal material, while sitting on it: it is obtained from the Bouguer Formula $\Delta g = 2\pi G \rho h$ compared to 981 Gal at the Earth’s surface

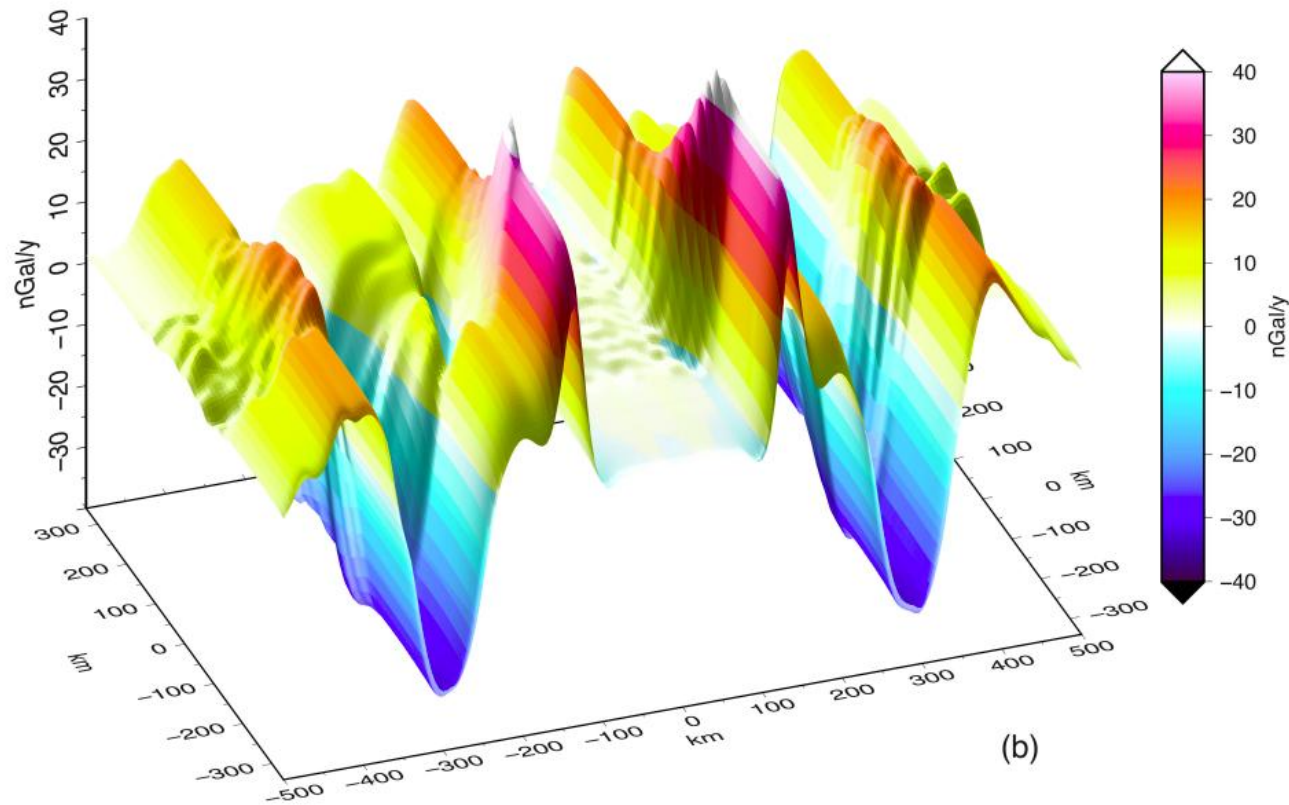
What is shown in the following is the result of the ASI funded Project:

NGGM-MAGIC : a breakthrough in the understanding of the dynamics of the Earth

**PI Anna Maria Marotta
University of Milano**



**Marotta, A.M., V. Fedeli,
A. Regorda, R. Sabadini,
Gravity rate of change
due to slow tectonics:
insights from numerical
modeling *Geosciences*
15(9), 359;
[https://doi.org/10.3390/
geosciences15090359](https://doi.org/10.3390/geosciences15090359)
(2025)**



Rossi L., A. Regorda, M. Reguzzoni, C.I. De Gaetani, R. Barzaghi, R. Sabadini and A.M. Marotta, 2026.
Sensitivity of the MAGIC satellite instrumentation to the gravity rate of change due to rifting in the Gulf of Aden,
Geophysical Journal International, doi.org/10.1093/gji/ggag237

Gulf of Aden rifting gravity signal as seen from MAGIC: E2E simulations and MAGIC sensitivity

Orbit **integration** is based on GO_055 and ESA Earth System Model, only defining the location of the satellites

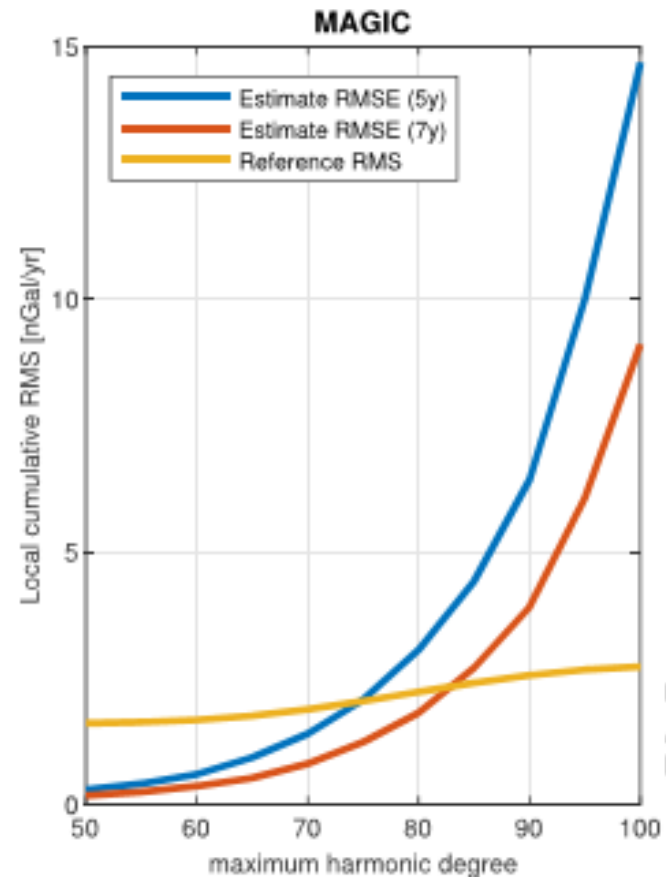
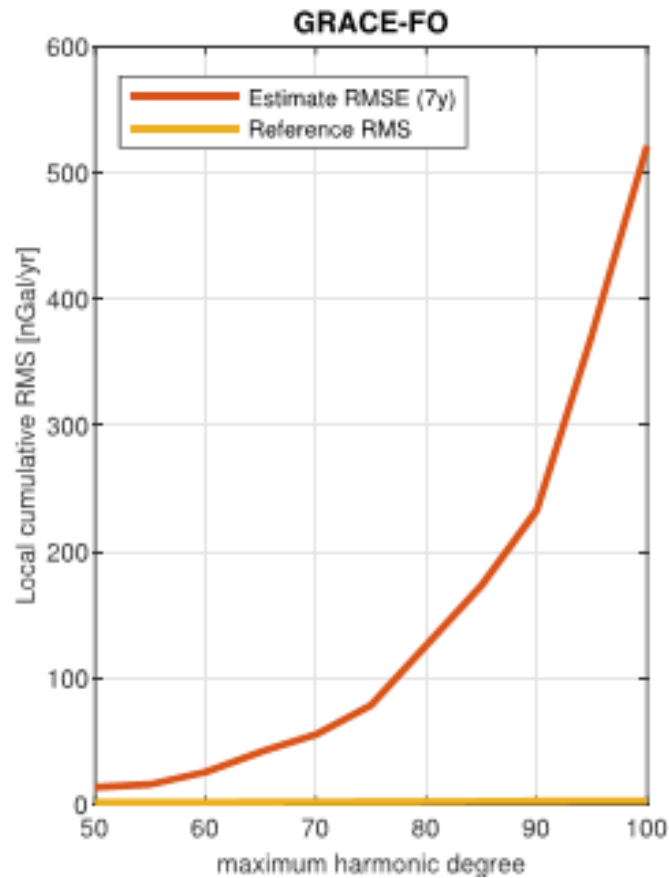
Along-orbit **observations** are simulated considering **only** the **modelled gravity signal** of the previous slide, which is thus the **true signal, so all the other time dependend phenomena from AOHIS are stripped away**

This gridded true signal is exploited in spherical harmoncs up to degree and order 900, upward continued at the satellite level in terms of **gravitational differences along the line-of-sight** of each pair of satellite, **considering also the observation time**, and then corrupted by the instrumental noise as given by the ASDs provided by ESA, for the accelerometer and interferometer.

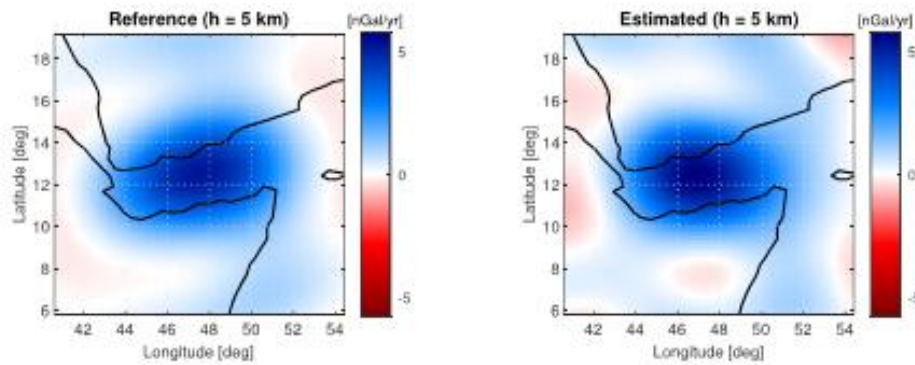
These simulated observations are then divided in terms of 7, 1-year subsets: each of this subset is used to obtain a 1-year solution in terms of spherical harmonic coefficients by means of 1-year **least square adjustment at each point of the grid** covering the whole Gulf of Aden, obtaining at the same time the **error of the estimated coefficients**

From these coefficients at at each period the gravity disturbace at 5 km height is synthetized at each point of the grid, and then by **linear regression the gravity rate of change is derived, propagating the cumulative estimation error RMSE for each harmonic degree**, to be compared with the **RMS of the modelled reference gravity rate of change**

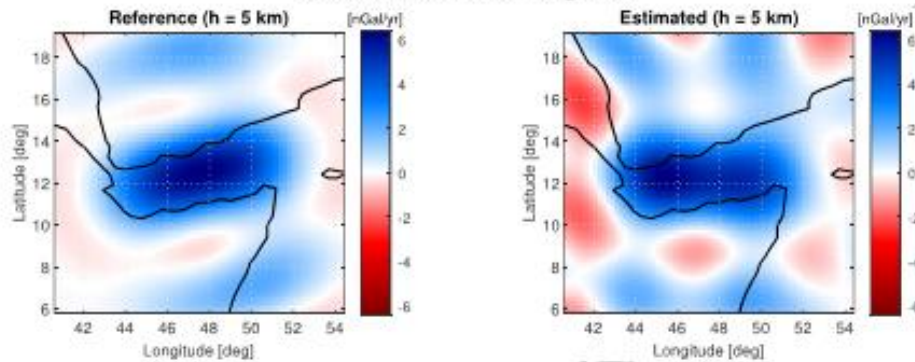
Rossi L., A. Regorda, M. Reguzzoni, C.I. De Gaetani, R. Barzaghi, R. Sabadini and A.M. Marotta, 2026. Sensitivity of the MAGIC satellite instrumentation to the gravity rate of change due to rifting in the Gulf of Aden, *Geophysical Journal International*, doi.org/10.1093/gji/ggag237



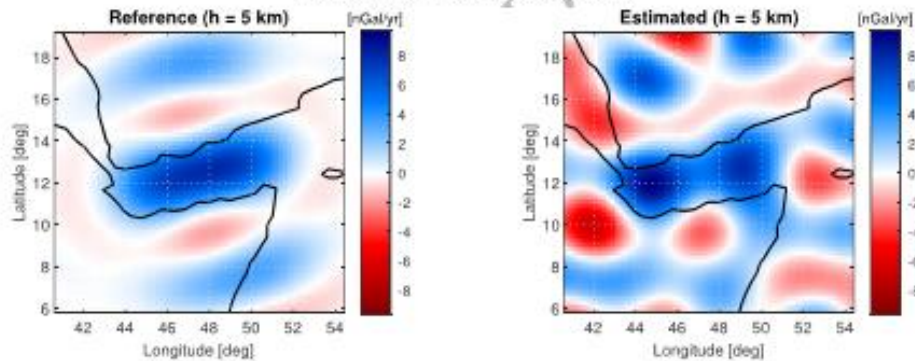
Rossi L., A. Regorda, M. Reguzzoni, C.I. De Gaetani, R. Barzaghi, R. Sabadini and A.M. Marotta, 2026.
 Sensitivity of the MAGIC satellite instrumentation to the gravity rate of change due to rifting in the Gulf of Aden,
Geophysical Journal International, doi.org/10.1093/gji/ggag237



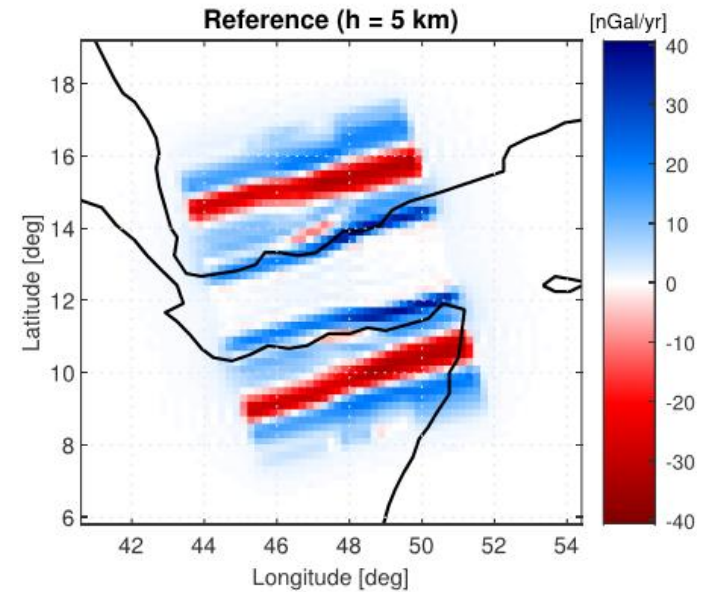
(a) Maximum harmonic degree 60



(b) Maximum harmonic degree 70



(c) Maximum harmonic degree 80



Rossi L., A. Regorda, M. Reguzzoni, C.I. De Gaetani, R. Barzaghi, R. Sabadini and A.M. Marotta, 2026.
Sensitivity of the MAGIC satellite instrumentation to the gravity rate of change due to rifting in the Gulf of Aden,
Geophysical Journal International, doi.org/10.1093/gji/ggag237



***MAGIC
will be launched
around 2030!***

***Thanks you all! And thank
you Genoa for hosting the
Hotine-Marussi
Symposium 2026!***

