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IAG Inter-Commission Committee on Theory (ICCT)



On the definition of vertical velocity in Space Geodesy

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Introduction

- Why the definition of vertical velocity matters ?
 - vertical velocities from GNSS observations and geophysical models (e.g. GIA) do not usually refer to same type of vertical land motion
- GNSS-derived vertical velocity is not a unique quantity, but the projection of the 3D station velocity vector onto a chosen direction
 - standard choice → ellipsoidal normal direction (ENU)
 - alternate choice → geocentric radial direction
- The difference between the two types is primarily driven by the horizontal velocity field (!) **and it may not be negligible**

Types of vertical velocities in practice

Ellipsoidal vertical velocity → vertical motion along the ellipsoidal normal

Rationale: directly associated with changes in geodetic height

- GNSS height time series and geodetic coordinate monitoring
- land subsidence and crustal deformation studies
- coastal VLM and GNSS corrections to TG records, etc.

Geocentric radial velocity → vertical motion relative to the Earth's center

Rationale: compatible with geophysical loading theory and SH formulations

- GIA and surface loading models, mass re-distribution studies (GRACE/GRACE-FO)
- SH representation of Earth deformation and shape change
- geocentric sea-level change from SA, etc.

Mathematical definitions

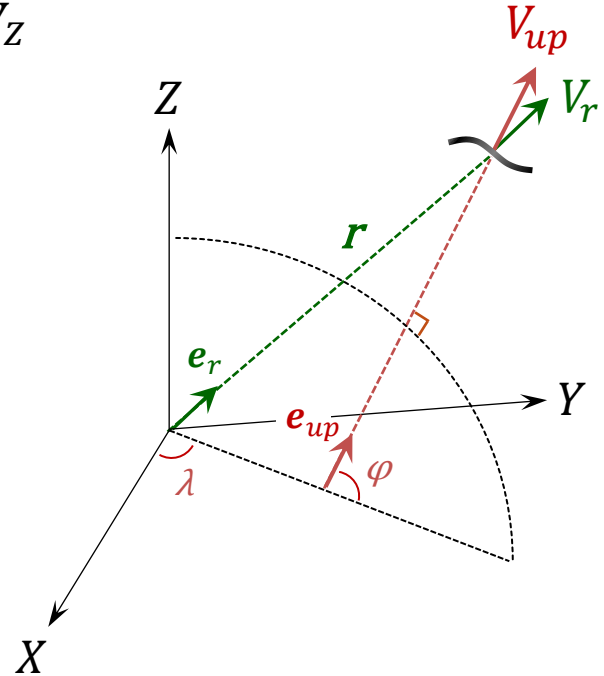
Ellipsoidal vertical velocity

$$V_{up} = \mathbf{e}_{up} \cdot \mathbf{V} = \cos\varphi \cos\lambda V_X + \cos\varphi \sin\lambda V_Y + \sin\varphi V_Z$$

Geocentric radial velocity

$$V_r = \mathbf{e}_r \cdot \mathbf{V} = \frac{\mathbf{r} \cdot \mathbf{V}}{|\mathbf{r}|} = \frac{X V_X + Y V_Y + Z V_Z}{|\mathbf{r}|}$$

Neither type is a projection of the other; both types are distinct projections of the same 3D station velocity vector



Beware of “mixed” vertical motion definitions

Example 1: Comparison of GNSS vertical velocities with predictions from GIA and geophysical loading models

Example 2: Inversion of GNSS vertical displacements for estimating surface mass load SHCs

Example 3: Euler pole-based rigid plate motion $\neq$ zero ellipsoidal vertical velocity

Key point: Ensure that the adopted definition of “vertical motion” is consistent across all datasets and models being compared or combined

Question to be answered in this study

How significant is the difference between ellipsoidal and geocentric vertical velocities ?

The role of the horizontal velocity field

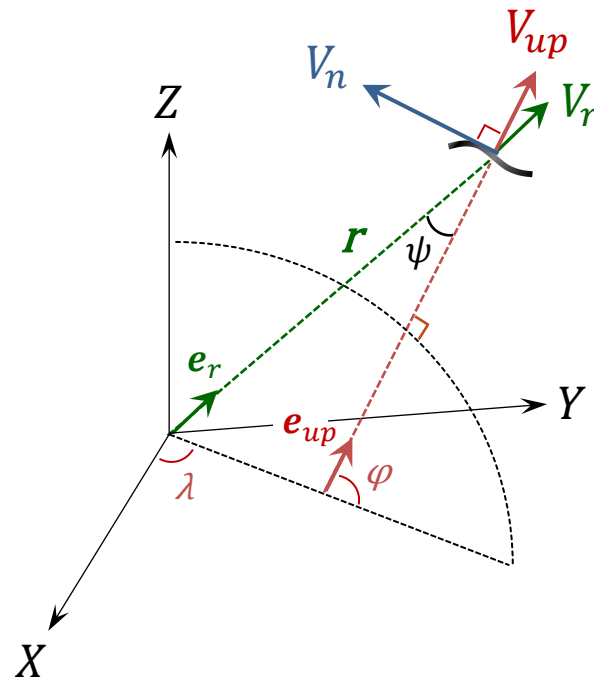
The **east-west component** of the horizontal velocity vector is normal to the local meridian plane. Therefore,

$$V_r = \mathbf{e}_r \cdot \overbrace{(\mathbf{V}_n + \mathbf{V}_e + \mathbf{V}_{up})}^{\mathbf{V}} = \mathbf{e}_r \cdot \mathbf{V}_n + \mathbf{e}_r \cdot \mathbf{V}_{up}$$

It is easily shown: $V_r = -\sin\psi V_n + \cos\psi V_{up}$

$$\max \psi \approx 11.5' \longrightarrow \approx -\psi V_n + V_{up}$$

To first order, the difference btw **ellipsoidal vertical velocity** and **geocentric radial velocity** is controlled entirely by the **north-south component** of the local horizontal velocity.



Analytic formulation

After straightforward derivations..

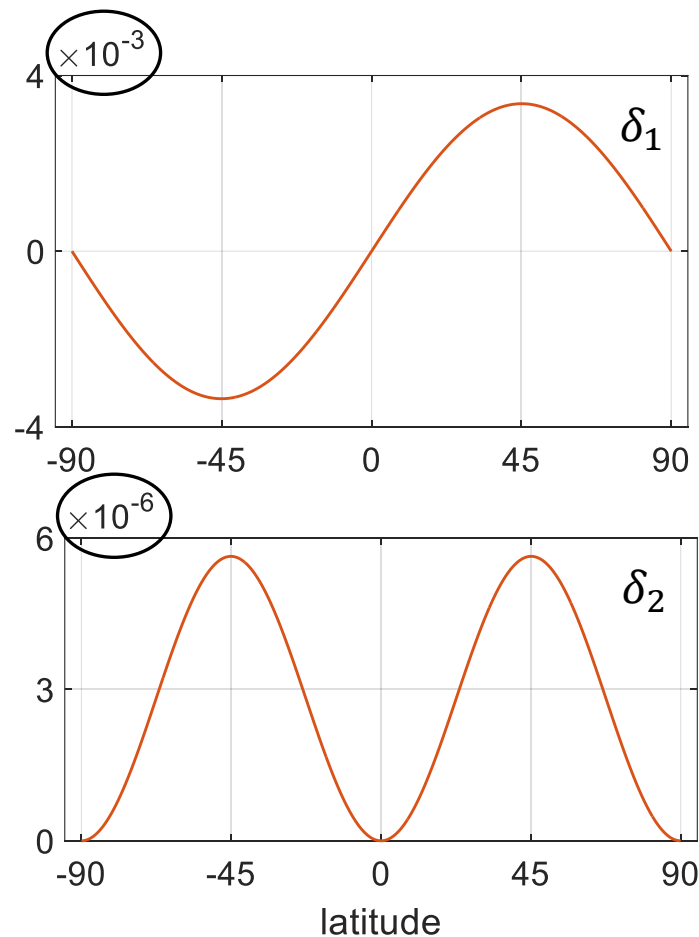
$$\Delta V = V_{up} - V_r = \delta_1 * V_n + \delta_2 * V_{up}$$

$$\delta_1 = \frac{N e^2 \sin\varphi \cos\varphi}{r} = \sin\psi$$

$$\delta_2 = \frac{r - N (1 - e^2 \sin^2\varphi) - h}{r} = 1 - \cos\psi$$

* The sensitivity of δ_1 and δ_2 on h is negligible

* The geocentric distance r is known function of φ and h



Analytic formulation

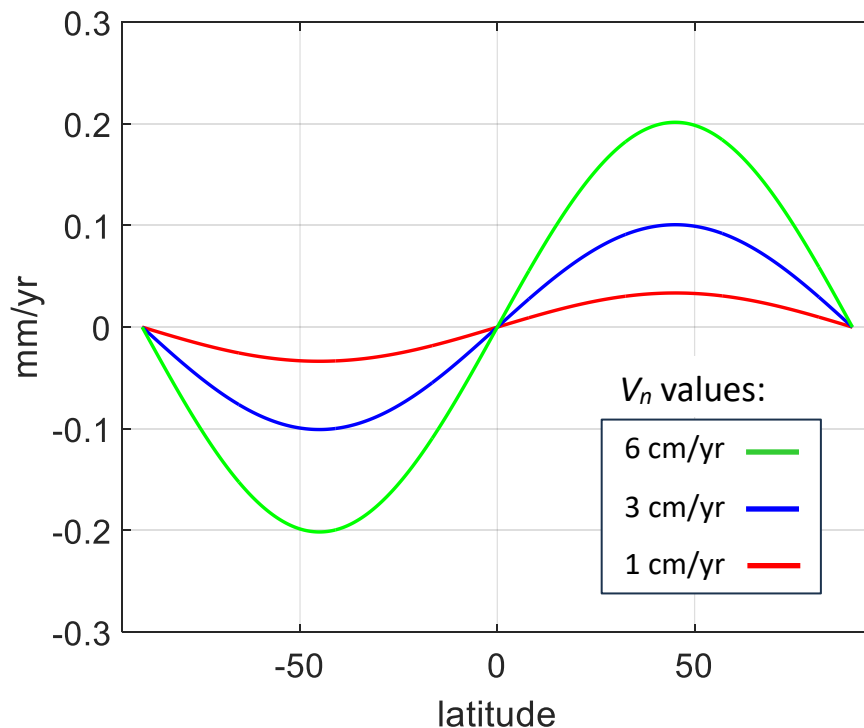
After straightforward derivations..

$$\Delta V = V_{up} - V_r = \delta_1 * V_n + \delta_2 * V_{up}$$

The difference can reach 0.1-0.2 mm/yr at mid-latitude stations with large north-south component of horizontal motion

Are there such regions on Earth ?

Sensitivity of ΔV on V_n



Alternative formulation

Instead of the following formula:

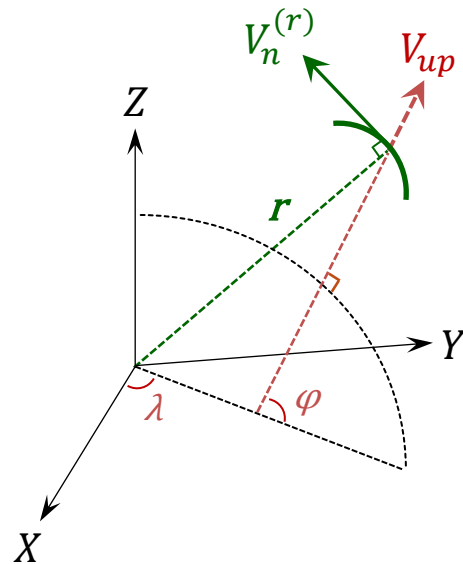
$$\Delta V = V_{up} - V_r = \delta_1 * V_n + \delta_2 * V_{up}$$

a similar expression can be easily derived by using the “spherical motion components”:

$$\Delta V = V_{up} - V_r = \delta_1 * V_n^{(r)} - \delta_2 * V_r$$

↪ **Contribution of Euler pole-based rigid plate motion to ellipsoidal vertical velocity ($V_r = 0$)**

$$V_{up} = \delta_1 * V_n^{(r)}$$



Example with NGL GNSS database

~20,000 GNSS stations

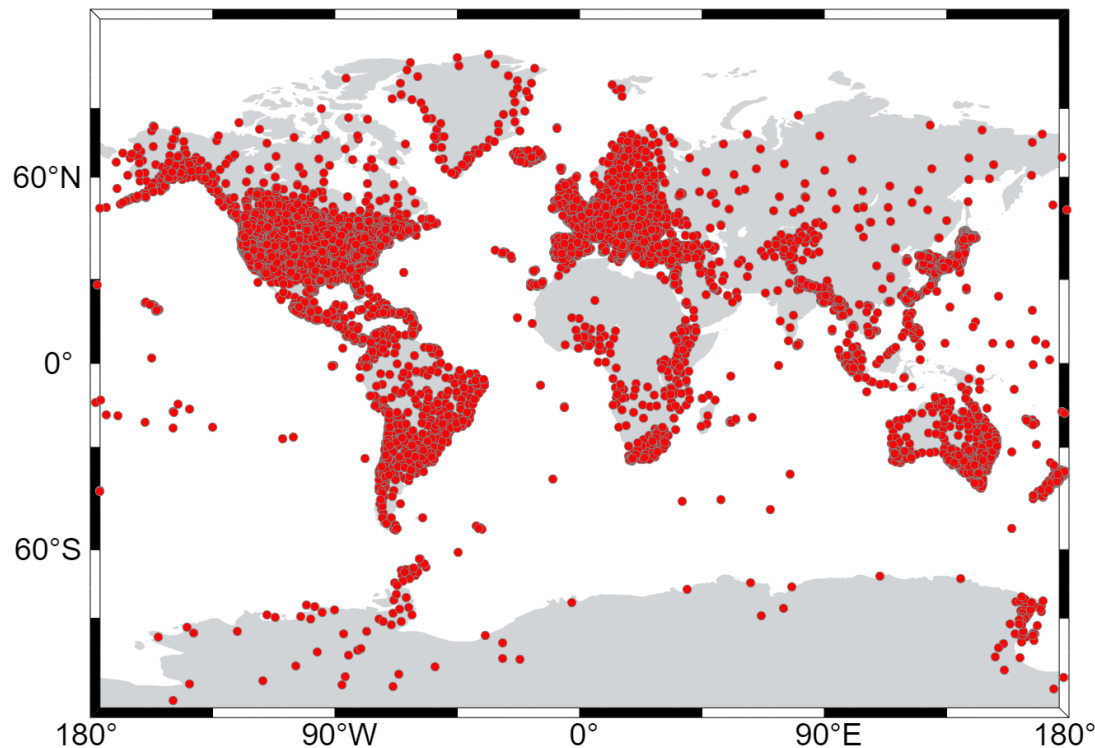
MIDAS velocities (IGS14)

Exclude stations with:

- data span < 3 yrs
- $\sigma_v > 5$ mm/yr
- post-seismic deformation

	V_{up}	V_n
max	29.9	73.7
min	-29.3	-79.4

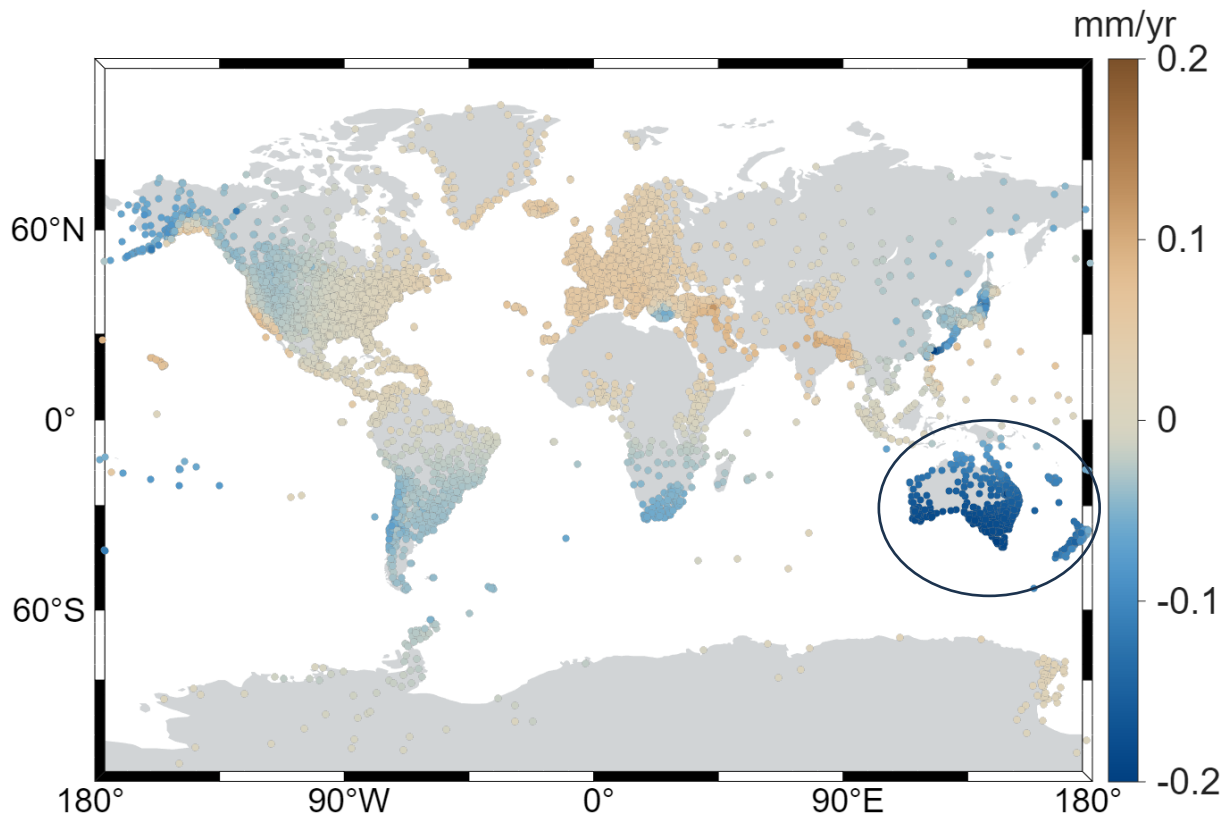
in mm/yr



Blewitt et al. (2016) MIDAS robust trend estimator for accurate GPS station velocities without step detection, JGR 10.1002/2015JB012552

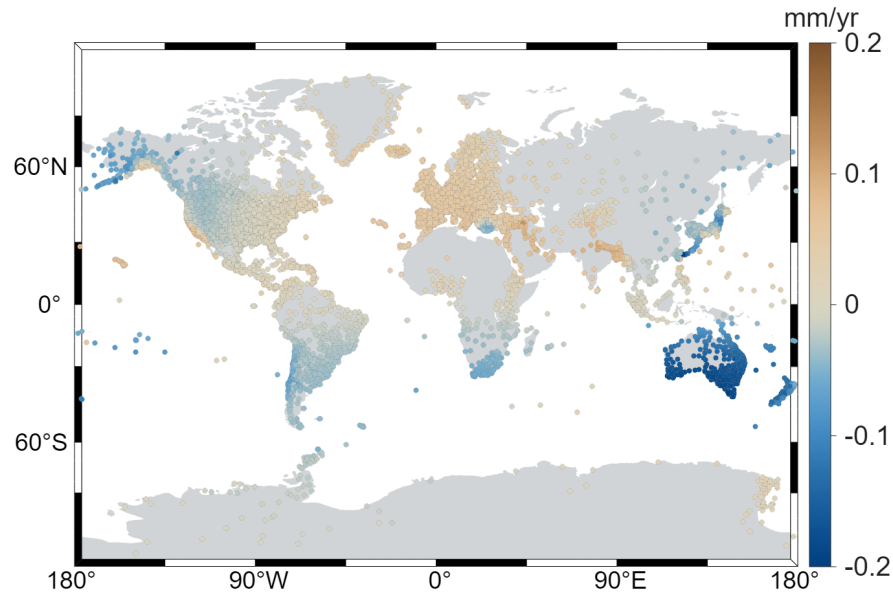
Global differences of vertical velo types

$$\Delta V = V_{up} - V_r$$

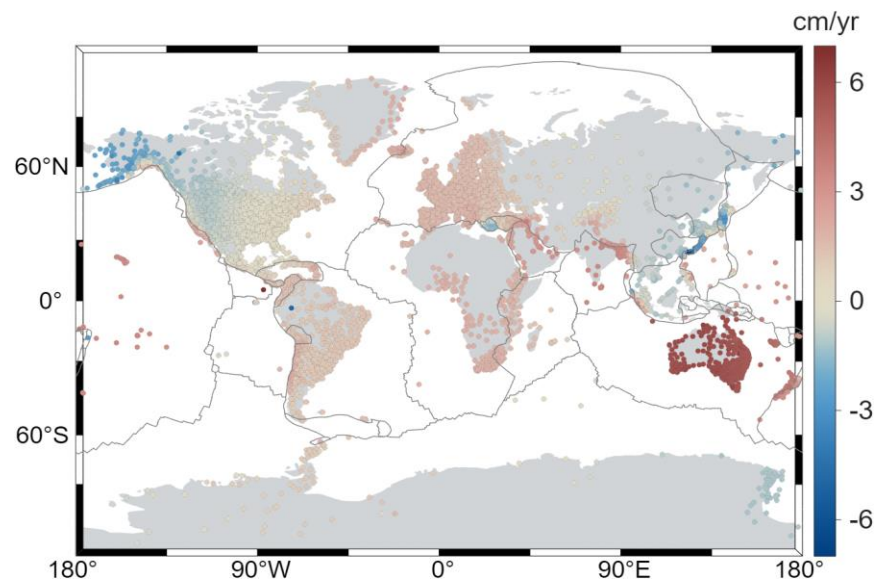


Max = 0.22 mm/yr , Min = -0.21 mm/yr , RMS = 0.05 mm/yr

Global differences of vertical velocity types



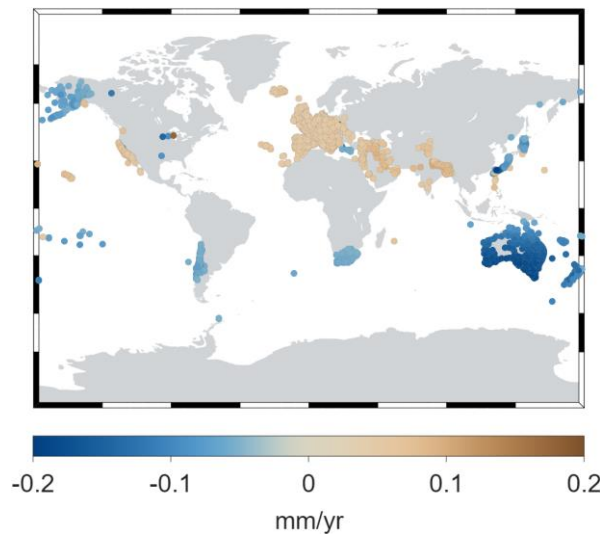
$$\Delta V = V_{up} - V_r$$



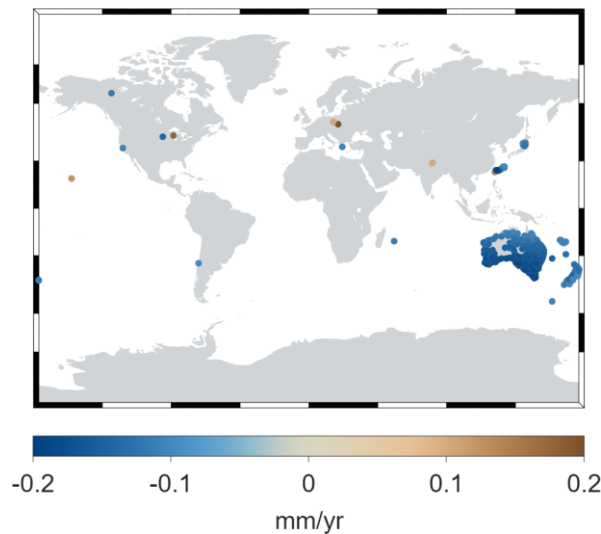
$$V_n$$

Global differences of vertical velo types

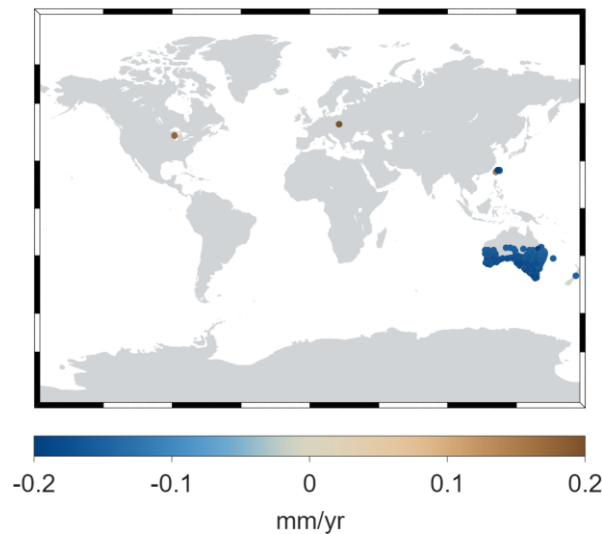
$|\Delta V| > 0.05 \text{ mm/yr}$



$|\Delta V| > 0.10 \text{ mm/yr}$



$|\Delta V| > 0.15 \text{ mm/yr}$



Why does $\Delta V \approx 0.1-0.2 \text{ mm/yr}$ matter ?

Comparable to weak geophysical signals we seek to explain

- Same order as GIA and surface-loading secular signals in several regions

Mostly systematic rather than random

- Forms a coherent regional bias (e.g. Australia)
- Cannot be reduced by longer time series or denser GNSS networks

Can bias high-precision Earth monitoring

- GNSS observations vs. GIA/surface-loading model comparisons
- Sea-level and long-term deformation studies

A relevant case

JGR Solid Earth

RESEARCH ARTICLE

10.1029/2019JB018034

Key Points:

- GPS-based vertical land motion at many individual sites across Australia is substantially different to GIA model predictions
- GPS velocities better reflect the solid

Present-Day Vertical Land Motion of Australia From GPS Observations and Geophysical Models

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¹Geography and Spatial Sciences, School of Technology, Environments and Design, University of Tasmania, Hobart, Tasmania, Australia, ²Geoscience Australia, Canberra, ACT, Australia

~ 30% of the reported difference can be explained by the geometrical difference btw ellipsoidal and geocentric vertical velocities in AUST !

In Conclusions, p. 13

The average vertical subsidence of the Australian continent is substantially different to vertical motion predicted by postglacial rebound models. This is based on the evidence that the median vertical trend of land motion across the Australian continent is -0.8 mm/year with a median uncertainty of 0.4 mm/year and the first and third quartile values of -1.2 and -0.4 mm, respectively. The background rates of motion from postglacial models across Australia range between -0.20 and 0.03 mm/year. We note that it is challenging to have a statistically significant difference given the irregular spatial distribution of sites across the continent, the non-Gaussian distribution of rates and their associated uncertainties, and the unknown uncertainty of the ICE 6G/VM5a vertical rate predictions.

GNSS velo

GIA velo

$$V_{up}^{GNSS} - V_r^{GIA} \approx -0.7 \text{ mm/yr}$$

Conclusions

- “Vertical velocity” in space geodesy is direction-dependent
- Ellipsoidal vertical velocities and geocentric radial velocities are distinct quantities
differ up to 0.2 mm/yr in mid-lat regions with strong N-S horizontal motion
- Such differences are comparable to subtle secular geophysical signals and may bias comparisons of GNSS observations with model predictions
- Future GNSS products and Earth-system studies should explicitly specify and consistently adopt the underlying definition of vertical motion