

Spheroidal integral formulas for computing the disturbing gravitational potential and its first- and second-order directional derivatives from disturbing gravitational potential and its vertical and horizontal derivatives

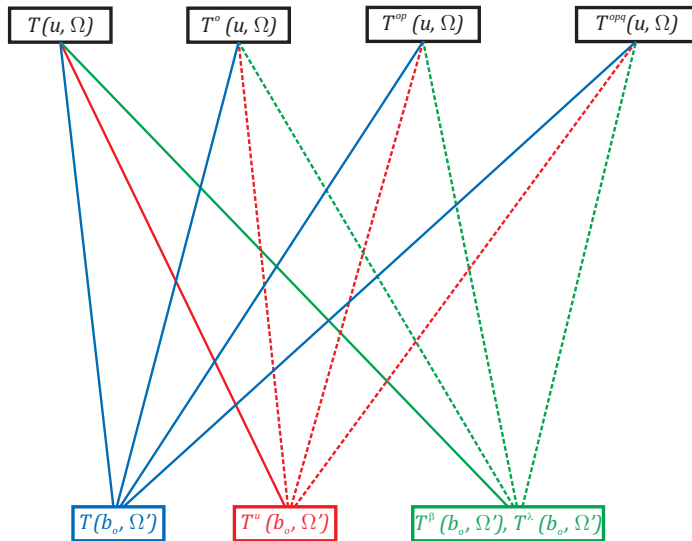
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Motivation





Spheroidal Integral Equations for Geodetic Inversion of Geopotential Gradients

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Abstract The static Earth's gravitational field has traditionally been described in geodesy and geophysics by the gravitational potential (geopotential for short), a scalar function of 3-D position. Although not directly observable, geopotential functionals such as its first- and second-order gradients are routinely measured by ground, airborne and/or satellite sensors. In geodesy, these observables are often used for recovery of the static geopotential at some simple reference surface approximating the actual Earth's surface. A generalized mathematical model is represented by a surface integral equation which originates in solving Dirichlet's boundary value problem of the potential theory defined for the harmonic geopotential, spheroidal boundary and globally distributed gradient data. The mathematical model can be used for combining various geopotential gradients without necessity of their re-sampling or prior continuation in space. The model extends the apparatus of integral equations which results from solving boundary-value problems of the potential theory to all geopotential gradients observed by current ground, airborne and satellite sensors. Differences between spherical and spheroidal formulations of integral kernel functions of Green's kind are investigated. Estimated differences reach relative values at the level of 3% which demonstrate the significance of spheroidal approximation for flattened bodies such as the Earth. The observation model can be used for combined inversion of currently available geopotential gradients while exploring their spectral and stochastic characteristics. The model would be even more relevant to gravitational field modelling of other bodies in space with more pronounced spheroidal geometry than that of the Earth.

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Vertical and horizontal spheroidal boundary-value problems

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Abstract

Vertical and horizontal spheroidal boundary value problems (BVVPs), i.e., determination of the external gravitational potential from the components of the gravitational gradient on the spheroid, are discussed in this article. The gravitational gradient is decomposed into the series of the vertical and horizontal vector spheroidal harmonics, before being orthogonalized in a weighted sense by two different approaches. The vertical and horizontal spheroidal BVVPs are then formulated and solved in the spectral and spatial domains. Both orthogonalization methods provide the same analytical solutions for the vertical spheroidal BVVP, and give distinct, but equivalent, analytical solutions for the horizontal spheroidal BVVP. A closed-loop simulation is performed to test the correctness of the analytical solutions, and we investigate analytical properties of the sub-integral kernels. The systematic treatment of the spheroidal BVVPs and the resulting mathematical equations extend the theoretical apparatus of geodesy and of the potential theory.

Keywords Differential operator · Gravitational gradient · Vector spheroidal harmonics · Spheroidal harmonic analysis

1 Introduction

Gravitational field is a fundamental property of our planet and belongs to the three main pillars of the modern geodesy (Flag et al. 2009). A accurate understanding of the properties of the gravitational field is indispensable for establishing reference datums and height systems revealing the inner structure of the field for geophysical and geodynamic studies, or monitoring sea-level changes, ice melting, and ocean currents. These applications allow geo-scientists to address challenging problems that arise in studying the complex Earth system.

Boundary-value problems (BVVPs) represent an important mathematical concept for the determination of the external gravitational field (Sano 1995). The Laplace differential equation together with the boundary conditions in the form of scalar, vector, and tensor quantities have been employed to formulate and solve numerous spheroidal BVVPs. See, for example, Gelfand et al. (1968), van Gelder and Blumenthal (2001), Fredson and Corbett (2013), Novák et al. (2017), and the references therein.

Close geometric proximity of the Earth's shape to the rotational ellipsoid, and the high-accuracy requirements for the gravitational field modelling, have motivated theoreticians to deal with the more difficult spheroidal BVVPs. This effort has resulted in the analytical solutions of the basic mathematical BVVPs, for example, the Dirichlet (Martinec and Gelfand 1997; Ye et al. 2003; Ye 2004; Chousen 2006), Neumann (Chousen 1997; Pao 2004; Ye et al. 2005; Moravetz and Yarkoni 2011; Najib-Alamoudy et al. 2012; Holota and Nováček 2014), and Stokes (Zagrebina 1996; Rotherham 1962; Madsen et al. 1982; Koon 1989; Marth 1973; Zhu 1981; Moritz 1989; Ye and Cao 1996; Martinec 1997; Rother 1999; Pao 2004; Arizumi and Martinec 2008; Heck and Seitz 2013; Sjöberg 2003), with the boundary conditions prescribed on the spheroid. The mixed (Holota 1987a, b; Janghor and Xiangping 1997; unclassified) (Arakhan and Sultan 2007; Sultan et al. 2001) and gradient-type (Hofberg and Gelfand 2007; Gelfand 2011) spheroidal BVVPs have also been formulated and their solutions successfully found. Moravetz and Novák (2007) suggested the one-step integration for the general determination in the spheroidal approximation.

Generally, the systematic treatment of BVVPs is allowed by the orthogonality of the corresponding basis functions. This remarkable property is taken for granted for the spherical case as it exactly holds for the scalar spherical harmonics and may be extended for their vector and tensor analogues (Fredson

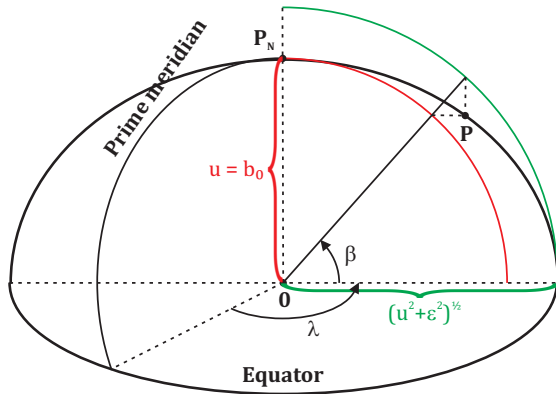
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Theoretical background

Preliminaries



a_0 , b_0 – the semi-major and the semi-minor axes of the reference spheroid (biaxial ellipsoid),

ε – the linear eccentricity, $\varepsilon = \sqrt{a_0^2 - b_0^2}$,

β – the reduced latitude,

λ – the longitude,

The disturbing gravitational potential T in mass-free space can be represented by a series of spheroidal harmonics as:

$$T(u, \Omega) = \frac{GM}{a_0} \sum_{n=0}^{\infty} \sum_{m=-n}^n \frac{Q_{n,|m|}\left(i \frac{u}{\varepsilon}\right)}{Q_{n,|m|}\left(i \frac{b_0}{\varepsilon}\right)} \times \bar{C}_{n,|m|}^e \bar{Y}_{n,m}(\Omega), \quad (1)$$

where G stands for Newton's gravitational constant, M is the mass of planetary body, $Q_{n,|m|}$ are the associated Legendre functions of the second kind of degree n and order m , $\bar{C}_{n,|m|}^e$ the fully normalized spheroidal coefficients, $\bar{Y}_{n,m}$ the fully normalized spheroidal harmonics, and $\Omega = (\beta, \lambda)$ the horizontal position.

Solution of the spheroidal BVPs

Poisson spheroidal integral equation

The Poisson spheroidal integral equation represents the solution of Dirichlet's boundary-value problem for the harmonic geopotential outside the spheroid (e.g., Ardalan 2000, Sect. 1.2):

$$T(u, \Omega) = \frac{1}{4\pi} \int_{\Omega'} T(b_0, \Omega') \mathcal{K}(u, \Omega, b_0, \Omega') d\Omega'. \quad (2)$$

The corresponding spheroidal integral kernel has the form:

$$\begin{aligned} \mathcal{K}(u, \Omega, b_0, \Omega') = & \sum_{n=0}^{\infty} \sum_{m=-n}^n \frac{Q_{n,|m|}\left(i\frac{u}{\varepsilon}\right)}{Q_{n,|m|}\left(i\frac{b_0}{\varepsilon}\right)} \\ & \times \bar{P}_{n,m}(\sin \beta) \bar{P}_{n,m}(\sin \beta') \cos(m(\lambda - \lambda')). \end{aligned} \quad (3)$$

Solution of the spheroidal BVPs

Vertical boundary-value problem

The integral transform of the disturbing potential gradient component T^u into the disturbing potential is defined as (Šprlák and Tangdamrongsu 2018):

$$T(u, \Omega) = \frac{1}{4\pi} \int_{\Omega'} T^u(b_0, \Omega') \mathcal{K}^u(u, \Omega, b_0, \Omega') d\Omega', \quad (4)$$

where the corresponding integral kernel is:

$$\begin{aligned} \mathcal{K}^u = & \frac{L(b_0, \beta')}{a_0} \sum_{n=0}^{\infty} \sum_{m=0}^n \frac{Q_{n,|m|}\left(i\frac{u}{\varepsilon}\right)}{\left. \frac{\partial}{\partial u} Q_{n,|m|}\left(i\frac{u}{\varepsilon}\right) \right|_{u=b_0}} \\ & \times \bar{P}_{n,m}(\sin \beta) \bar{P}_{n,m}(\sin \beta') \cos(m(\lambda - \lambda')). \end{aligned} \quad (5)$$

We omitted the full notation $\mathcal{K}^u(u, \Omega, b_0, \Omega')$ for practical reasons.

$$L(b_0, \beta') = \sqrt{b_0^2 + \varepsilon^2 \sin^2 \beta'}, \quad L(u, \beta) = \sqrt{u^2 + \varepsilon^2 \sin^2 \beta}, \quad v(u) = \sqrt{u^2 + \varepsilon^2} \quad (6)$$

Theoretical background

Solution of the horizontal BVP

The relation between the disturbing potential and the disturbing potential gradient components T^β and T^λ is described by the following integral transform:

$$T(u, \Omega) = \frac{1}{4\pi} \int_{\Omega'} \left[T^\beta(b_0, \Omega') \mathcal{K}^{\beta,o} - T^\lambda(b_0, \Omega') \mathcal{K}^{\lambda,o} \right] d\Omega', \quad (7)$$

where the corresponding integral kernels for $o = 1$ (according to Grafarend et al. 2006) is:

$$\mathcal{K}^{\beta,1} = L(b_0, \beta') \sum_{n=1}^{\infty} \sum_{m=-n}^n \frac{1}{n(n+1)} \frac{Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{Q_{n,|m|} \left(i \frac{b_0}{\varepsilon} \right)} \bar{Y}_{n,m}(\Omega') \frac{\partial}{\partial \beta'} \bar{Y}_{n,m}(\Omega),$$

$$\mathcal{K}^{\lambda,1} = \frac{a_0}{\cos \beta'} \sum_{n=1}^{\infty} \sum_{m=-n}^n \frac{1}{n(n+1)} \frac{Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{Q_{n,|m|} \left(i \frac{b_0}{\varepsilon} \right)} \bar{Y}_{n,m}(\Omega') \frac{\partial}{\partial \lambda'} \bar{Y}_{n,m}(\Omega),$$

Theoretical background

Solution of the horizontal BVP

and for $\sigma = 2$ (suggested by Lowes and Winch 2012)

$$\begin{aligned} \mathcal{K}^{\beta,2} = & a_0^2 L(b_0, \beta') \sum_{n=1}^{\infty} \sum_{m=-n}^n \frac{1}{a_0^2 n(n+1) - \varepsilon^2 m^2} \\ & \times \frac{Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{Q_{n,|m|} \left(i \frac{b_0}{\varepsilon} \right)} \bar{Y}_{n,m}(\Omega') \frac{\partial}{\partial \beta'} \bar{Y}_{n,m}(\Omega), \end{aligned}$$

$$\begin{aligned} \mathcal{K}^{\lambda,2} = & \frac{a_0 L^2(b_0, \beta')}{\cos \beta'} \sum_{n=1}^{\infty} \sum_{m=-n}^n \frac{1}{a_0^2 n(n+1) - \varepsilon^2 m^2} \\ & \times \frac{Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{Q_{n,|m|} \left(i \frac{b_0}{\varepsilon} \right)} \bar{Y}_{n,m}(\Omega') \frac{\partial}{\partial \lambda'} \bar{Y}_{n,m}(\Omega). \end{aligned}$$

First-order directional derivatives

Differential operators

The differential operators for the first-order directional derivatives read (Novák and Šprlák 2018):

$$D_{\beta} = \frac{1}{L(u, \beta)} \frac{\partial}{\partial \beta}, \quad D_{\lambda} = -\frac{1}{v(u) \cos \beta} \frac{\partial}{\partial \lambda}, \quad D_u = \frac{v(u)}{L(u, \beta)} \frac{\partial}{\partial u}. \quad (8)$$

Application of these operators to the kernel $\mathcal{K}^u$ provides first-order derivatives.

First-order directional derivatives

Derivatives of $\mathcal{K}^u$

$$\frac{\partial}{\partial u} \mathcal{K}^u = \frac{L(b_0, \beta')}{a_0} \sum_{n=0}^{\infty} \sum_{m=0}^n \frac{\frac{\partial}{\partial u} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{\frac{\partial}{\partial u} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right) |_{b_0}} \bar{P}_{n,m}(\sin \beta) \bar{P}_{n,m}(\sin \beta') \cos(m(\lambda - \lambda')), \quad (9)$$

$$\frac{\partial}{\partial \beta} \mathcal{K}^u = \frac{L(b_0, \beta')}{a_0} \sum_{n=0}^{\infty} \sum_{m=0}^n \frac{Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{\frac{\partial}{\partial u} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right) |_{b_0}} \frac{\partial \bar{P}_{n,m}(\sin \beta)}{\partial \beta} \bar{P}_{n,m}(\sin \beta') \cos(m(\lambda - \lambda')), \quad (10)$$

$$\frac{\partial}{\partial \lambda} \mathcal{K}^u = \frac{L(b_0, \beta')}{a_0} \sum_{n=0}^{\infty} \sum_{m=0}^n \frac{Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{\frac{\partial}{\partial u} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right) |_{b_0}} \bar{P}_{n,m}(\sin \beta) \bar{P}_{n,m}(\sin \beta') m \sin(m(\lambda' - \lambda)). \quad (11)$$

First-order directional derivatives

Generalized integral transformation

The generalized integral transformation from the vertical component of disturbing gravity to the first-order directional derivatives is defined as:

$$D_i T(u, \Omega) = \frac{1}{4\pi} \int_{\Omega'} T^u(b_0, \Omega') D_i \mathcal{K}^u(u, \Omega, b_0, \Omega') d\Omega', \quad (12)$$

where $i = \beta, \lambda, u$.

Second-order directional derivatives

Differential operators

The differential operators for the second-order directional derivatives have the following forms (Novák and Šprlák 2018):

$$D_{uu}\mathcal{K}^u = -\frac{1}{L(u, \beta)^2} \left[\frac{u\varepsilon^2 \cos^2 \beta}{L(u, \beta)^2} \frac{\partial}{\partial u} \mathcal{K}^u - \frac{\varepsilon^2 \sin \beta \cos \beta}{L(u, \beta)^2} \frac{\partial}{\partial \beta} \mathcal{K}^u - v(u)^2 \frac{\partial^2}{\partial u^2} \mathcal{K}^u \right], \quad (13)$$

$$D_{\beta\beta}\mathcal{K}^u = \frac{1}{L(u, \beta)^2} \left[\frac{uv(u)^2}{L(u, \beta)^2} \frac{\partial}{\partial u} \mathcal{K}^u - \frac{\varepsilon^2 \sin \beta \cos \beta}{L(u, \beta)^2} \frac{\partial}{\partial \beta} \mathcal{K}^u + \frac{\partial^2}{\partial \beta^2} \mathcal{K}^u \right], \quad (14)$$

$$D_{\lambda\lambda}\mathcal{K}^u = \frac{u}{L(u, \beta)^2} \frac{\partial}{\partial u} \mathcal{K}^u - \frac{\tan \beta}{L(u, \beta)^2} \frac{\partial}{\partial \beta} \mathcal{K}^u + \frac{1}{v(u)^2 + \cos^2 \beta} \frac{\partial^2}{\partial \lambda^2} \mathcal{K}^u, \quad (15)$$

$$D_{u\beta}\mathcal{K}^u = -\frac{v(u)}{L(u, \beta)^2} \left[\frac{\varepsilon^2 \sin \beta \cos \beta}{L(u, \beta)^2} \frac{\partial}{\partial u} \mathcal{K}^u + \frac{u}{L(u, \beta)^2} \frac{\partial}{\partial \beta} \mathcal{K}^u - \frac{\partial^2}{\partial u \partial \beta} \mathcal{K}^u \right], \quad (16)$$

$$D_{u\lambda}\mathcal{K}^u = \frac{1}{L(u, \beta) \cos \beta} \left[\frac{u}{v(u)^2} \frac{\partial}{\partial \lambda} \mathcal{K}^u - \frac{\partial^2}{\partial u \partial \lambda} \mathcal{K}^u \right], \quad (17)$$

$$D_{\beta\lambda}\mathcal{K}^u = -\frac{1}{v(u)L(u, \beta) \cos \beta} \left[\tan \beta \frac{\partial}{\partial \lambda} \mathcal{K}^u + \frac{\partial^2}{\partial \beta \partial \lambda} \mathcal{K}^u \right]. \quad (18)$$

Second-order directional derivatives

Derivatives of $\mathcal{K}^u$ (1/2)

$$\frac{\partial^2}{\partial u^2} \mathcal{K}^u = \frac{L(b_0, \beta')}{a_0} \sum_{n=0}^{\infty} \sum_{m=0}^n \frac{\frac{\partial^2}{\partial u^2} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{\frac{\partial}{\partial u} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right) |_{b_0}} \bar{P}_{n,m}(\sin \beta) \bar{P}_{n,m}(\sin \beta') \cos(m(\lambda - \lambda')), \quad (19)$$

$$\frac{\partial^2}{\partial \beta^2} \mathcal{K}^u = \frac{L(b_0, \beta')}{a_0} \sum_{n=0}^{\infty} \sum_{m=0}^n \frac{Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{\frac{\partial}{\partial u} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right) |_{b_0}} \frac{\partial^2 \bar{P}_{n,m}(\sin \beta)}{\partial \beta^2} \bar{P}_{n,m}(\sin \beta') \cos(m(\lambda - \lambda')), \quad (20)$$

$$\frac{\partial^2}{\partial \lambda^2} \mathcal{K}^u = \frac{L(b_0, \beta')}{a_0} \sum_{n=0}^{\infty} \sum_{m=0}^n \frac{Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{\frac{\partial}{\partial u} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right) |_{b_0}} \bar{P}_{n,m}(\sin \beta) \bar{P}_{n,m}(\sin \beta') \left[-m^2 \cos(m(\lambda - \lambda')) \right]. \quad (21)$$

Second-order directional derivatives

Derivatives of $\mathcal{K}^u$ (2/2)

$$\frac{\partial^2}{\partial u \partial \beta} \mathcal{K}^u = \frac{L(b_0, \beta')}{a_0} \sum_{n=0}^{\infty} \sum_{m=0}^n \frac{\frac{\partial}{\partial u} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{\frac{\partial}{\partial u} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right) |_{b_0}} \frac{\partial \bar{P}_{n,m}(\sin \beta)}{\partial \beta} \bar{P}_{n,m}(\sin \beta') \cos(m(\lambda - \lambda')), \quad (22)$$

$$\frac{\partial^2}{\partial u \partial \lambda} \mathcal{K}^u = \frac{L(b_0, \beta')}{a_0} \sum_{n=0}^{\infty} \sum_{m=0}^n \frac{\frac{\partial}{\partial u} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{\frac{\partial}{\partial u} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right) |_{b_0}} \bar{P}_{n,m}(\sin \beta) \bar{P}_{n,m}(\sin \beta') m \sin(m(\lambda' - \lambda)), \quad (23)$$

$$\frac{\partial^2}{\partial \beta \partial \lambda} \mathcal{K}^u = \frac{L(b_0, \beta')}{a_0} \sum_{n=0}^{\infty} \sum_{m=0}^n \frac{Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right)}{\frac{\partial}{\partial u} Q_{n,|m|} \left(i \frac{u}{\varepsilon} \right) |_{b_0}} \frac{\partial \bar{P}_{n,m}(\sin \beta)}{\partial \beta} \bar{P}_{n,m}(\sin \beta') m \sin(m(\lambda' - \lambda)). \quad (24)$$

Second-order directional derivatives

Generalized integral transformation

The generalized integral transformation for transforming the vertical component of disturbing gravity into the second-order directional derivatives is defined as:

$$D_{ij}T(u, \Omega) = \frac{1}{4\pi} \int_{\Omega'} T^u(b_0, \Omega') D_{ij}K^u(u, \Omega, b_0, \Omega') d\Omega', \quad (25)$$

where $ij = \beta\beta, \beta\lambda, \lambda\lambda, u\beta, u\lambda, uu$.

We derived the corresponding spheroidal integrals for the horizontal solution of the spheroidal BVP in the same way. The corresponding derivations of the Poisson integral equation are presented in (Novák and Šprlák 2018).

Numerical experiment

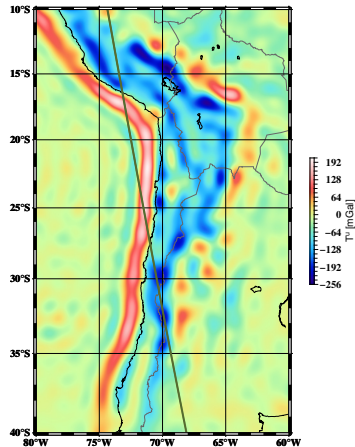
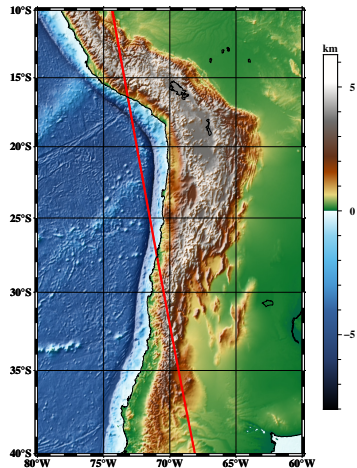
Setup

The newly derived spheroidal integral transformations were tested numerically in an extensive closed-loop test to check and eliminate potential mistakes in the derivation and software implementation.

- Test baseline: 450 sub-satellite points from the GOCE mission (EGG_TRF_2 product), September 28, 2013, starting at GPS time 1064363265 s.
- Test area: Andes Mountains and Pacific Ocean, characterised by extreme changes in topography and the gravitational field.
- Semi-minor axis of the selected track varies from 6590339.756 m to 6603586.934 m.
- EGM2008 (Pavlis et al., 2012) spherical harmonic coefficients (degree/order 290) were converted into spheroidal counterparts using the Hotine–Jekeli transformation (Jekeli, 1988), computed up to a maximum degree 40 higher to preserve total spectral power.
- Disturbing gravitational potential (T) and its vertical (T^u) and horizontal (T^β , T^λ) derivatives were computed globally on an equiangular 0.5° grid at the reference ellipsoid.
- In total, 40 spheroidal integral transformations were implemented and compared against spheroidal harmonic synthesis.

Numerical experiment

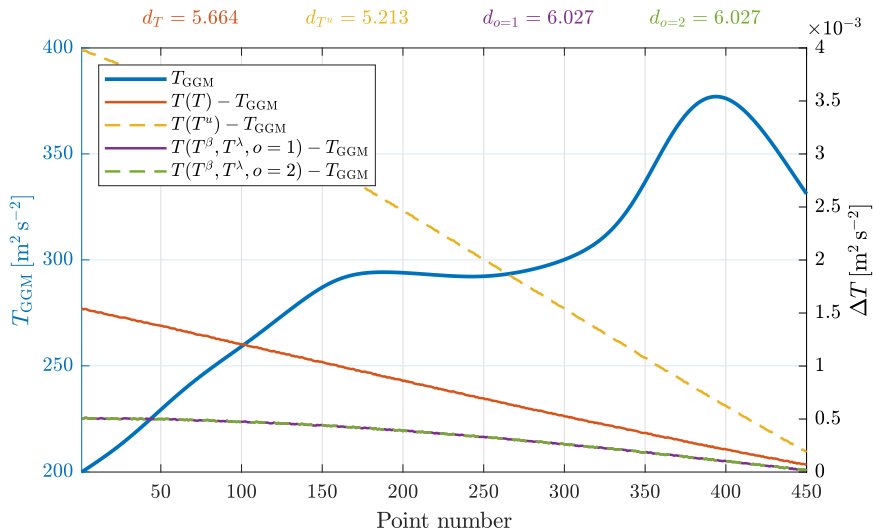
Test area



Topography and bathymetry over the test area (left), and the first-order radial derivative of the disturbing gravitational potential on the mean spheroid (right) from EGM2008 (Pavlis et al., 2012).

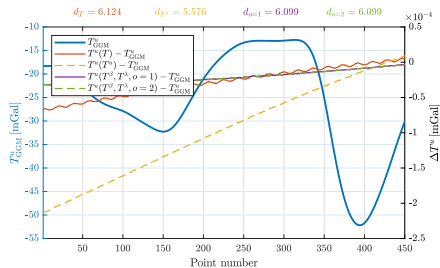
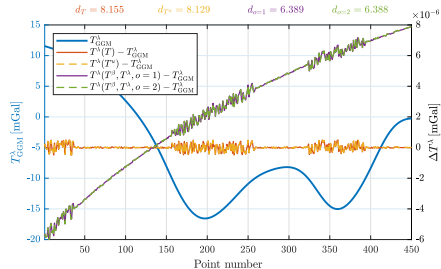
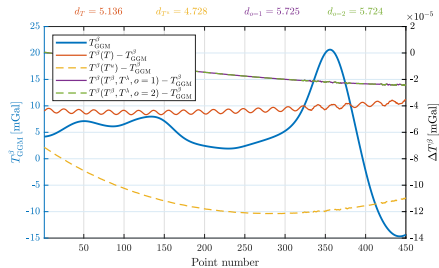
Numerical experiment

Results – disturbing potential



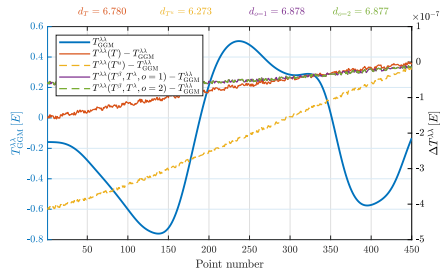
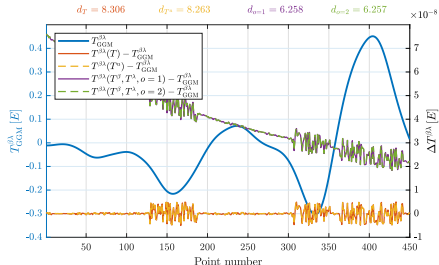
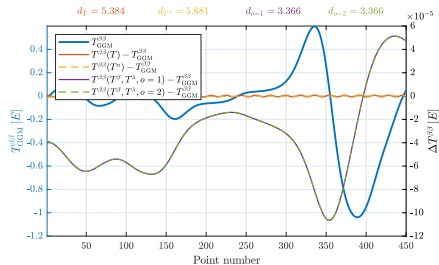
Numerical experiment

Results – first-order derivatives



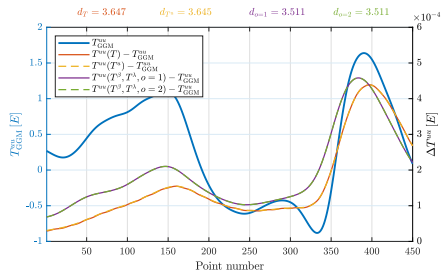
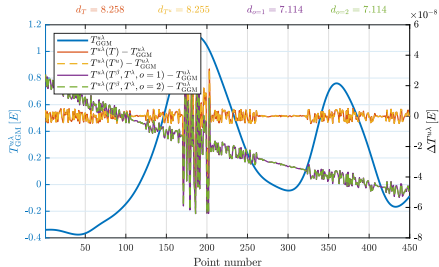
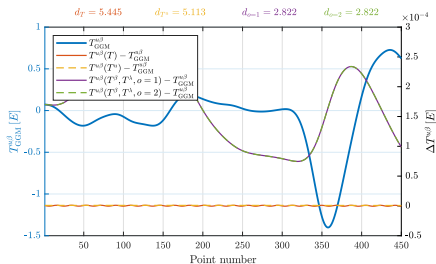
Numerical experiment

Results – second-order derivatives (horizontal-horizontal components)



Numerical experiment

Results – second-order derivatives (vertical-horizontal and vertical-vertical components)



- We derived 27 spheroidal integral formulas for computing the disturbing gravitational potential and its first- and second-order directional derivatives from the vertical and horizontal derivatives of disturbing potential given on the reference spheroid.
- The closed-loop numerical experiment along a realistic GOCE orbital arc confirmed the correctness of the derivation and software implementation.
- Spheroidal representation offers higher versatility with respect to irregular (flattened or elongated) planetary bodies.
- The manuscript with more detailed information is under preparation.

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Thank you for your attention

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