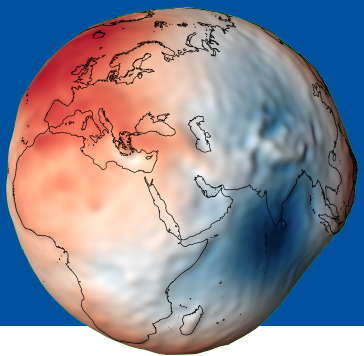
Combination of time-variable gravity field solutions from multi-satellite datasets via Constrained Collocation Model (1993–2024)

Lin Zhang[1], Yunzhong Shen[1], **Nico Sneeuw**[2], Peyman Saemian[2], Kunpu Ji[3], Qiujie Chen[1], Fengwei Wang[1]

[1]Tongji University
[2]University of Stuttgart
[3]The Hong Kong Polytechnic University

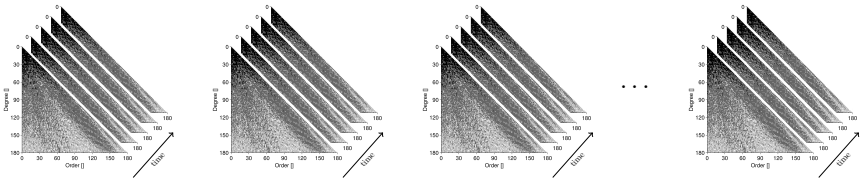


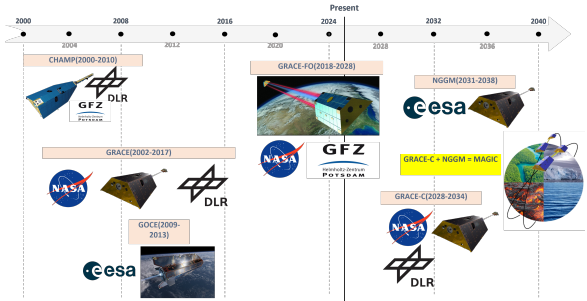
7–11 September 2026

University of Stuttgart
Germany

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香港理工大學

Time-variable gravity field





Combination “motivations” in literature

References	Input data	Basic principle	Limitations
COST-G	GRACE(-FO), multi-center combination	VCE, parameter domain	single mission

Single-mission combination Gap filling Multi-satellite combination Hydro-meteorological reconstruction

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COST-G	GRACE(-FO), multi-center combination	VCE, parameter domain	single mission
Yi and Sneeuw, 2021	GRACE(-FO)	SSA in SH domain	–
Weigelt et al., 2024	Multi LEO satellite & SLR	VCE based combination	limited resolution & explicit filtering
Grombein et al., 2022	Multiple LEO satellites	VCE based combination	limited resolution & explicit filtering

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Humphrey et al., 2019	Precipitation, temperature	Regression	climate oriented, no anthropogenic effect
Gentner et al., 2026	Climate RA & anthropogenic indicator	CNN/LSTM	input data accuracy limited
Saemian et al., 2026	Global hydro-meteorological + RA	ML ensemble	model quality dependent

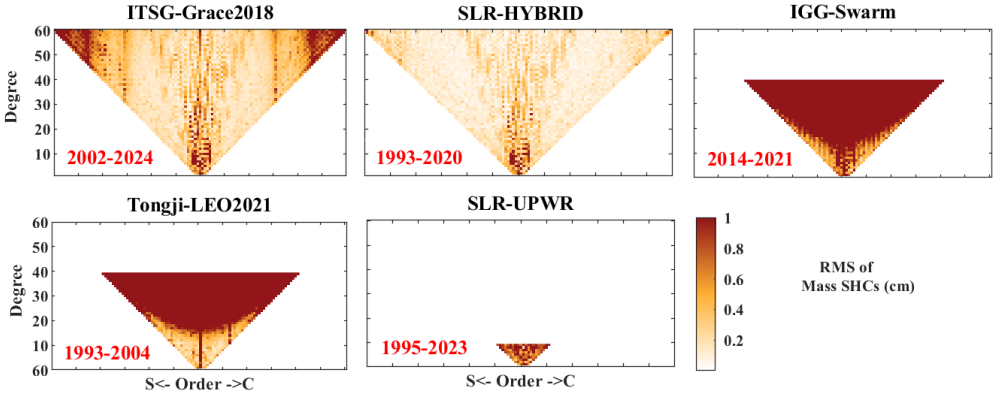
Single-mission combination Gap filling Multi-satellite combination Hydro-meteorological reconstruction

Our objective

A new time series of **monthly gapless gravity field solutions** up to **d/o 60 from January 1993 to December 2024** that

1. directly integrates multi-satellite observations without relying on external hydro-meteorological model inputs;
2. simultaneously performs combination and denoising without requiring additional explicit filtering;
3. incorporates covariance matrices of both observation errors and signals.

Input Data


ITSG-Grace2018
SLR-HYBRID
IGG-Swarm
Degree
60
50
40
30
20
10
2002-2024
1993-2020
2014-2021
Tongji-LEO2021
SLR-UPWR
1993-2004
1995-2023
S<- Order ->C
S<- Order ->C
1
0.8
0.6
0.4
0.2
RMS of
Mass SHCs (cm)

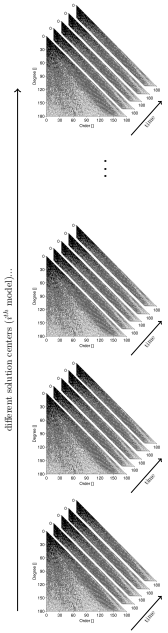
Methodology 1/3

Functional model – collocation

$$
\begin{aligned}
y_{k,i}^{lm} = {} & a_{0,i}^{lm} + a_{1,i}^{lm}\Delta t_k + a_{2,i}^{lm}\Delta t_k^2 \\
& + a_{3,i}^{lm}\sin 2\pi\Delta t_k + a_{4,i}^{lm}\cos 2\pi\Delta t_k \\
& + a_{5,i}^{lm}\sin 4\pi\Delta t_k + a_{6,i}^{lm}\cos 4\pi\Delta t_k \\
& + s_{k,i}^{lm} + e_{k,i}^{lm}
\end{aligned}
\tag{1}
$$

i^{th} model, k^{th} month

$s_{k,i}^{lm}$ – non-seasonal signal (NSS)



Methodology 1/3

Functional model – collocation

stacking all SHCs of all months for each model into $\mathbf{y}_k$

$$\mathbf{y}_k = \mathbf{A}_k \mathbf{x} + \mathbf{B}_k \mathbf{s}_k + \mathbf{e}_k \tag{2}$$

$$\begin{bmatrix} \mathbf{y}_{k,1}^{\mathrm{T}} & \mathbf{y}_{k,2}^{\mathrm{T}} & \cdots & \mathbf{y}_{k,q_k}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{A}_k = \begin{bmatrix} \mathbf{A}_{k,1} \\ \mathbf{A}_{k,2} \\ \vdots \\ \mathbf{A}_{k,q_k} \end{bmatrix}$$

$$\mathbf{B}_k = \begin{bmatrix} \mathbf{B}_{k,1} \\ \mathbf{B}_{k,2} \\ \vdots \\ \mathbf{B}_{k,q_k} \end{bmatrix}$$

Methodology 1/3

Functional model – collocation

stacking all SHCs of all months for each model into $\mathbf{y}_k$

$$\mathbf{y}_k = \mathbf{A}_k \mathbf{x} + \mathbf{B}_k \mathbf{s}_k + \mathbf{e}_k \tag{2}$$

$$\mathbf{B}_{k,i} = \begin{bmatrix} \mathbf{I}_{M_{k,i}} & \mathbf{O}_{M-M_{k,i}} \end{bmatrix}$$

$$\mathbf{A}_{k,i} = \begin{bmatrix} 1 \\ \Delta t_k \\ \Delta t_k^2 \\ \sin 2\pi\Delta t_k \\ \cos 2\pi\Delta t_k \\ \sin 4\pi\Delta t_k \\ \cos 4\pi\Delta t_k \end{bmatrix}^{\mathrm{T}} \otimes \mathbf{B}_{k,i}$$

$$\begin{bmatrix} \mathbf{y}_{k,1}^{\mathrm{T}} & \mathbf{y}_{k,2}^{\mathrm{T}} & \cdots & \mathbf{y}_{k,q_k}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$

$$\mathbf{A}_k = \begin{bmatrix} \mathbf{A}_{k,1} \\ \mathbf{A}_{k,2} \\ \vdots \\ \mathbf{A}_{k,q_k} \end{bmatrix}$$

$$\mathbf{B}_k = \begin{bmatrix} \mathbf{B}_{k,1} \\ \mathbf{B}_{k,2} \\ \vdots \\ \mathbf{B}_{k,q_k} \end{bmatrix}$$

$M_{k,i}$ & M denote monthly SHCs from q^{th} model

Methodology 1/3

Time evolution of signal

Multi-order Markov process

$$\mathbf{s}_k = \sum_{j=k-r}^{k-1} \Phi_{k,j}\mathbf{s}_j + \mathbf{w}_k$$

- r – model order
- τ_s – correlation factor
- σ_s^2 – variance factor

$$\Phi_{k,j} = \exp\left(\frac{-(t_k - t_j)}{\tau_s}\right) \otimes \mathbf{I}_M$$

$$\mathbf{Q}_k = \sigma_s^2 \left[1 - \sum_{j=k-r}^{k-1} \exp\left(\frac{-2(t_k - t_j)}{\tau_s}\right) \otimes \mathbf{I}_M\right]$$

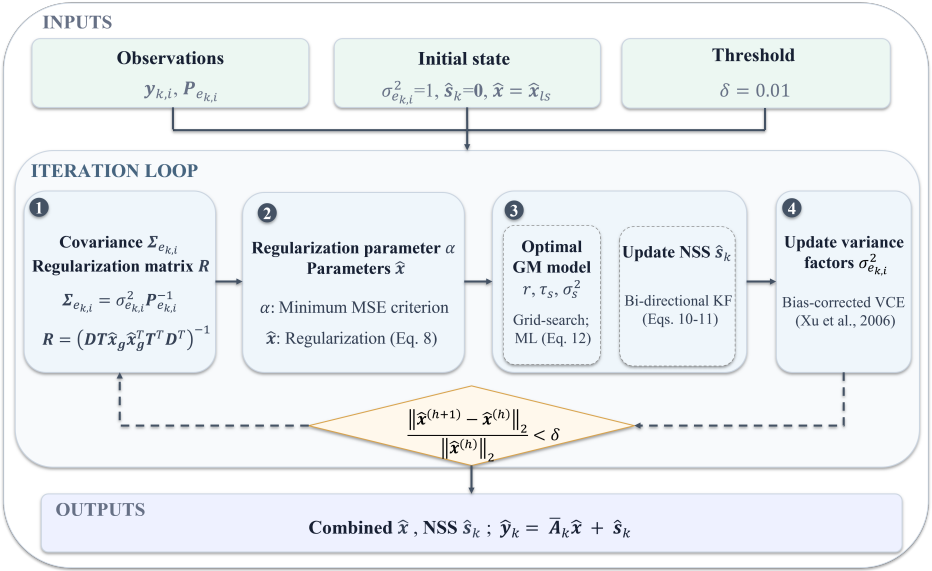
Methodology 2/3

Constrained least squares collocation solution

$$
\begin{aligned}
\hat{\mathbf{x}} &= \left[\sum_{k=1}^{n} \mathbf{A}_k^T \Sigma_{e_k}^{-1} \mathbf{A}_k + \alpha \mathbf{R}\right]^{-1} \left[\sum_{k=1}^{n} \mathbf{A}_k^T \Sigma_{e_k}^{-1} \left(\mathbf{y}_k - \mathbf{B}_k \hat{\mathbf{s}}_k\right)\right] \\
\bar{\mathbf{s}}_k &= \sum_{j=k-r}^{k-1} \Phi_{k,j} \hat{\mathbf{s}}_j \\
\hat{\mathbf{s}}_k &= \bar{\mathbf{s}}_k + \mathbf{K}_k \left(\mathbf{y}_k - \mathbf{A}_k \hat{\mathbf{x}} - \mathbf{B}_k \bar{\mathbf{s}}_k\right)
\end{aligned}
$$

- mutually nested estimation of $\hat{\mathbf{x}}$ and $\hat{\mathbf{s}}_k$
- iteration
- solving meta parameters: variance factors $\sigma_{\hat{e}_i}^2$, regularization parameter α, regularization matrix $\mathbf{R}$, model order r, time constant τ_s and variance component σ_s^2 of GM-model

Methodology 3/3



Results

Earth Syst. Sci. Data, 18, 5167–5186, 2026
https://doi.org/10.5194/essd-18-5167-2026
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Open Access Earth System Science Data

A combination of time-variable gravity field solutions from multi-satellite datasets (1993–2024) via constrained collocation model

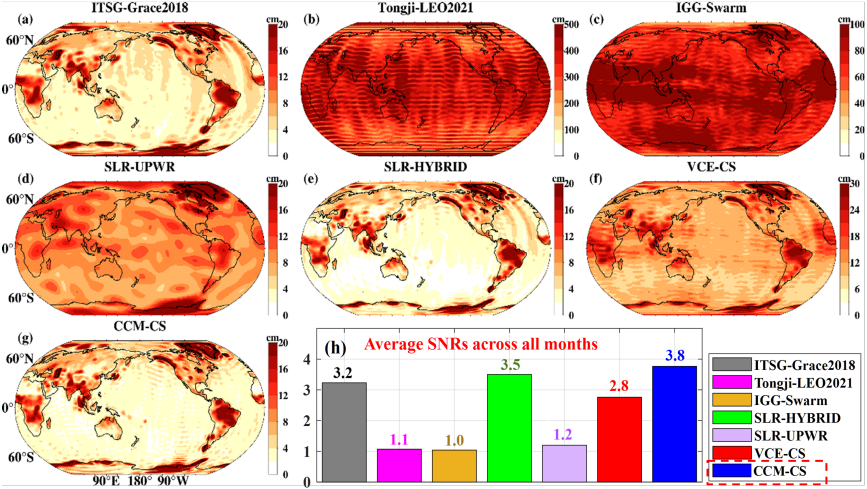
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[1]College of Surveying and Geo-informatics, Tongji University, Shanghai, China
[2]Institute of Geodesy, University of Stuttgart, Stuttgart, Germany
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Correspondence: Yunzhong Shen (yzshen@tongji.edu.cn)

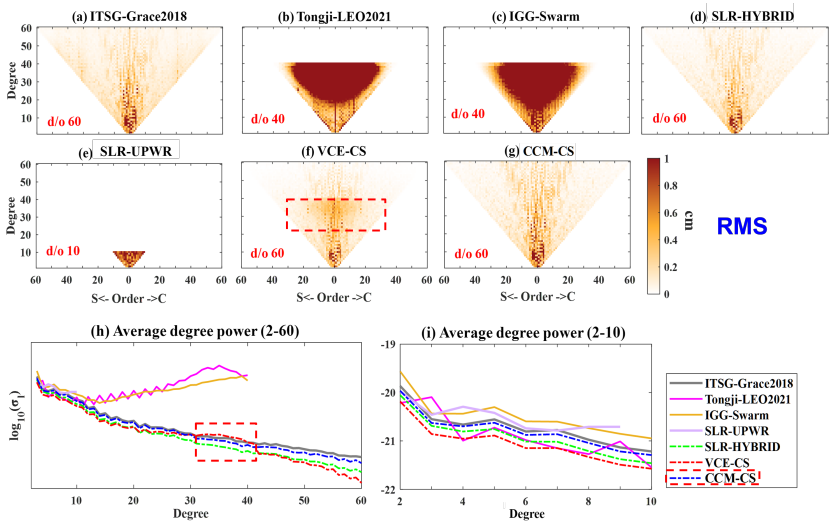
Received: 30 January 2026 – Discussion started: 16 March 2026
Revised: 17 June 2026 – Accepted: 24 June 2026 – Published: 21 July 2026

Results – spatial domain



CCM-CS – Constrained Collocation Model for Combined Solution

Results – spectral domain



CCM-CS – Constrained Collocation Model for Combined Solution

Validation

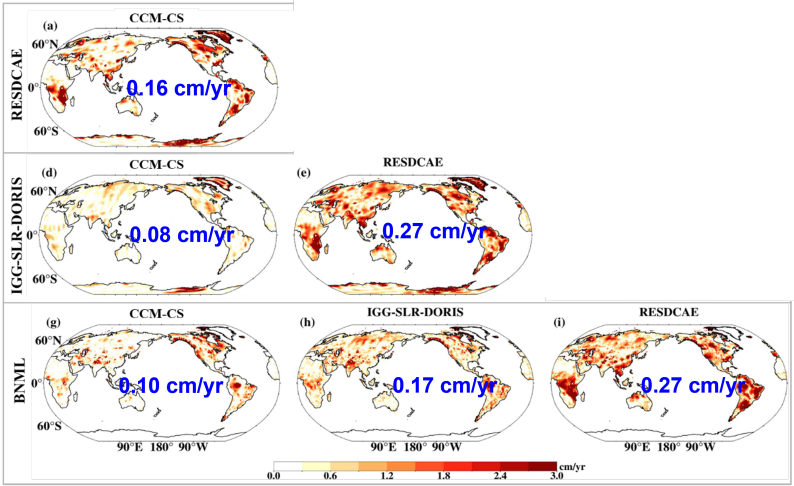
internal (see above)

- 5 input data sets
- VCE-based combination

external

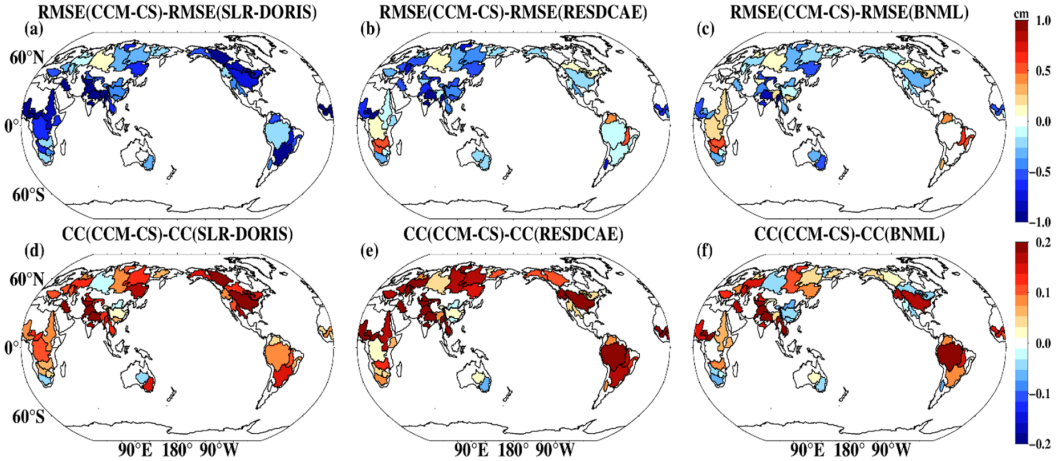
- 3 models: IGG-SLR-DORIS (Löcher et al., 2025), REDSCAE (Uz et al., 2024), BNML (Mandal et al., 2025)
- mass anomalies GrIS, AIS vs. IMBIE
- catchment level TWS anomalies
- water flux balance
- sea level budget

Validation, e.g., absolute trend differences



CCM-CS – Constrained Collocation Model for Combined Solution

Validation, e.g., water balance: $P - ET - R = \mathrm{d}M/\mathrm{d}t$



CCM-CS – Constrained Collocation Model for Combined Solution

Conclusions

- Monthly SH coefficients are modelled by trend parameters and time-variable signals. Using gravity field models with different time spans, degrees/orders, and accuracies, the **constrained collocation model can solve for a long gapless combined time-variable gravity field model with consistent spatial resolution**.
- Using SLR, GRACE/GRACE-FO, Swarm, and LEO satellite time-variable gravity field models, **a monthly gravity field model of $L_{\max} = 60$ for 1993–2024** is constructed, with a longer series length and higher accuracy than any individual gravity field model.
- CCM-based TWS change **better closes the water flux balance**, global TWS anomaly shows **stronger consistency with the sea level budget**, and ice-sheet mass-change trends **agree more closely with IMBIE**.

Open access paper

Zenodo dataset

Thank You!



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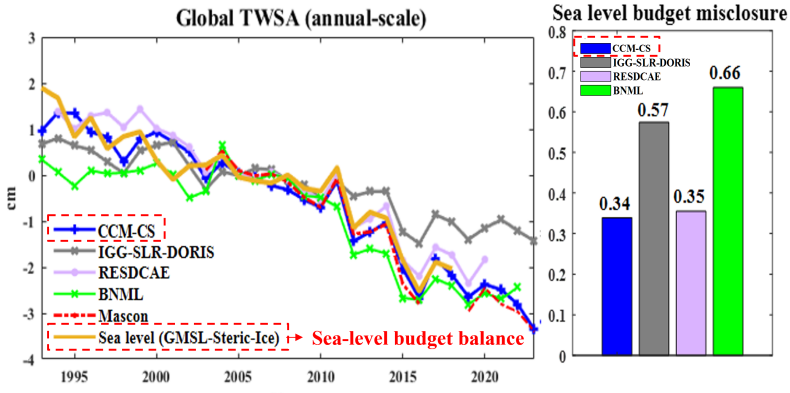
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Bonus slides

More results



More results

