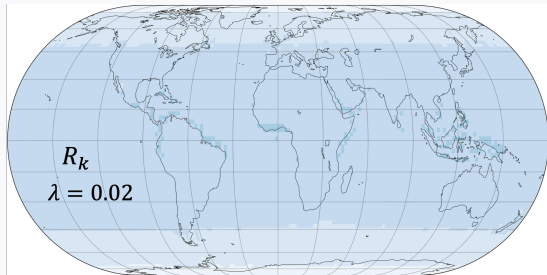


What can redundancy contribution matrices tell us about regularized *fast* mascon solutions?



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2. Institute of Geodesy, University of Stuttgart

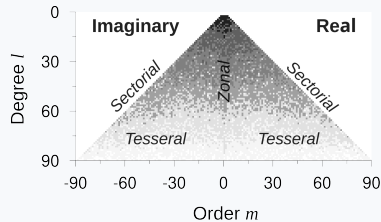
September 10, 2026

Background

From range between the satellites to gravity change

Spherical harmonic approach

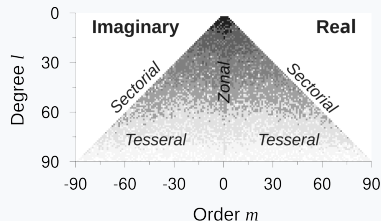
$$\delta g(\theta, \lambda, r, t) = \sum_{l=0}^L \sum_{m=-l}^l \Lambda_l(r) Y_{lm}(\theta, \lambda) \Delta K_{lm}(t)$$



From range between the satellites to gravity change

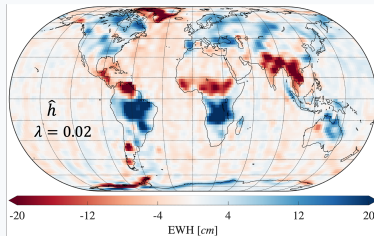
Spherical harmonic approach

$$\delta g(\theta, \lambda, r, t) = \sum_{l=0}^L \sum_{m=-l}^l \Lambda_l(r) Y_{lm}(\theta, \lambda) \Delta K_{lm}(t)$$



Mascon approach

$$\delta g_i(t) = G \sum_{j=1}^p \frac{r_i - a \cos \psi_{i,j}}{l_{i,j}^3} A_j h_j(t) \rho_{\text{water}}$$

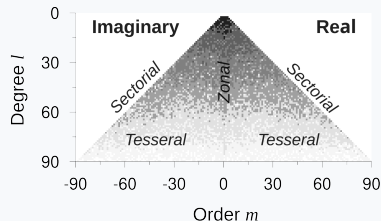


From range between the satellites to gravity change

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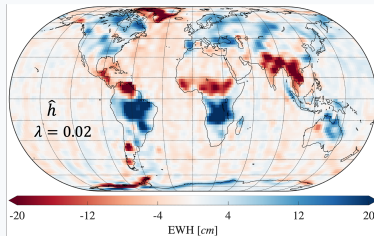
Continuous in space



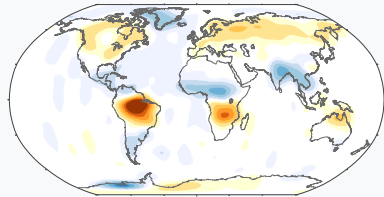
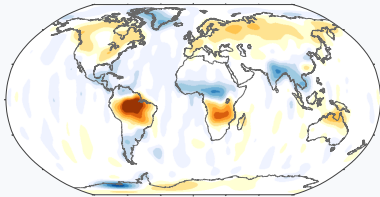
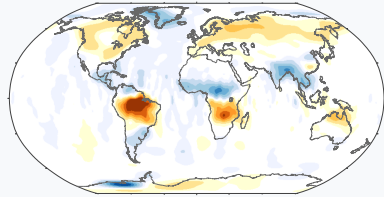
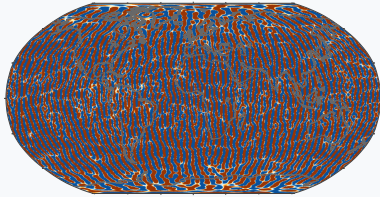
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Discrete in space



GRACE is noisy – The need for filtering



The fast mascons approach

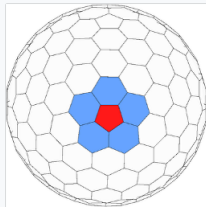
Spherical harmonic solutions to filtered area averaged mass anomalies

Formulation

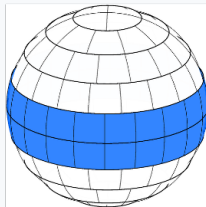
$$\underbrace{\Delta K_{lmj}(t)}_{\text{SH coefficients}} = \underbrace{\frac{a_E^2 \rho_{\text{water}}}{M} \frac{1 + k'_l}{2l + 1}}_{\text{isotropic transfer function}} \underbrace{h_j(t) \iint_{\Omega_j} Y_{lm}^\dagger(\theta, \lambda) d\Omega}_{\text{mascons}}$$

Save et al. (2016); Croteau et al. (2021)

CSR Mascons (40962)



GSFC Mascons (41168)



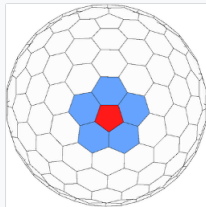
Formulation

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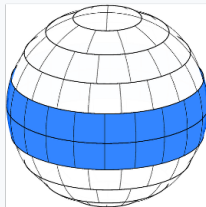
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Save et al. (2016); Croteau et al. (2021)

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Formulation

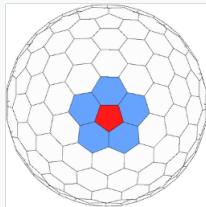
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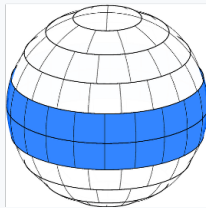
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Save et al. (2016); Croteau et al. (2021)

CSR Mascons (40962)



GSFC Mascons (41168)



Formulation

$$\underbrace{\Delta K_{lm}(t)}_{\text{SH Coefficients}} = \underbrace{\frac{a_E^2 \rho_{\text{water}}}{M} \frac{1 + k'_l}{2l + 1}}_{\text{isotropic transfer function}} \sum_{j=1}^N \underbrace{h_j(t)}_{\text{mass change}} \underbrace{Y_{lm}^\dagger(\theta_j, \lambda_j)}_{\text{SH functions}} \underbrace{W_j}_{\text{Area weights}}$$

Save et al. (2016); Croteau et al. (2021)

Formulation

$$\underbrace{\Delta K_{lm}(t)}_{\text{SH Coefficients}} = \underbrace{\frac{a_E^2 \rho_{\text{water}}}{M} \frac{1 + k'_l}{2l + 1}}_{\text{isotropic transfer function}} \sum_{j=1}^N \underbrace{h_j(t)}_{\text{mass change}} \underbrace{Y_{lm}^\dagger(\theta_j, \lambda_j)}_{\text{SH functions}} \underbrace{W_j}_{\text{Area weights}}$$

Save et al. (2016); Croteau et al. (2021)

Discrete spherical harmonic analysis

Formulation

$$\underbrace{\Delta K_{lm}(t)}_{\text{SH Coefficients}} = \underbrace{\frac{a_E^2 \rho_{\text{water}}}{M} \frac{1 + k'_l}{2l + 1}}_{\text{isotropic transfer function}} \sum_{j=1}^N \underbrace{h_j(t)}_{\text{mass change}} \underbrace{Y_{lm}^\dagger(\theta_j, \lambda_j)}_{\text{SH functions}} \underbrace{W_j}_{\text{Area weights}}$$

$$\underbrace{\mathbf{k}}_{\text{SH Coefficients}} = \underbrace{\mathbf{\Lambda}^{-1}}_{\text{isotropic transfer function}} \underbrace{\mathbf{Y}^\dagger}_{\text{SH functions}} \underbrace{\mathbf{W}}_{\text{Area weights}} \underbrace{\mathbf{h}}_{\text{mass change}}$$

Formulation

$$\underbrace{\Delta K_{lm}(t)}_{\text{SH Coefficients}} = \underbrace{\frac{a_E^2 \rho_{\text{water}}}{M} \frac{1 + k'_l}{2l + 1}}_{\text{isotropic transfer function}} \sum_{j=1}^N \underbrace{h_j(t)}_{\text{mass change}} \underbrace{Y_{lm}^\dagger(\theta_j, \lambda_j)}_{\text{SH functions}} \underbrace{W_j}_{\text{Area weights}}$$

$$\underbrace{\mathbf{k}}_{\text{SH Coefficients}} = \underbrace{\mathbf{\Lambda}^{-1}}_{\text{isotropic transfer function}} \underbrace{\mathbf{Y}^\dagger}_{\text{SH functions}} \underbrace{\mathbf{W}}_{\text{Area weights}} \underbrace{\mathbf{h}}_{\text{mass change}}$$

$$\mathbf{k} = \mathbf{A}^\dagger \mathbf{W} \mathbf{h} + \boldsymbol{\epsilon},$$

$$\mathcal{D}\{\mathbf{k}\} = \underbrace{\mathbf{Q}_k}_{\text{spectral covariance}}$$

No. of SH coefficients < No. of mascons $\Rightarrow$ System underdetermined

Solution

$$\mathcal{E}\{\mathbf{k}\} = \mathbf{A}^\dagger \mathbf{W} \mathbf{h},$$

$$\mathcal{D}\{\mathbf{k}\} = \mathbf{Q}_k \quad \text{Noise covariance}$$

Solution

$$\mathcal{E}\{\mathbf{k}\} = \mathbf{A}^\dagger \mathbf{W} \mathbf{h},$$

$$\mathcal{E}\{\mathbf{h}_0\} = \mathbf{h},$$

$$\mathcal{D}\{\mathbf{k}\} = \mathbf{Q}_k \quad \text{Noise covariance}$$

$$\mathcal{D}\{\mathbf{h}_0\} = \tilde{\mathbf{Q}}_h \quad \text{Signal covariance}$$

Solution

$$\mathcal{E}\{\mathbf{k}\} = \mathbf{A}^\dagger \mathbf{W} \mathbf{h},$$

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$$\mathcal{D}\{\mathbf{k}\} = \mathbf{Q}_k \quad \text{Noise covariance}$$

$$\mathcal{D}\{\mathbf{h}_0\} = \tilde{\mathbf{Q}}_h \quad \text{Signal covariance}$$

$$\hat{\mathbf{h}} = \left(\mathbf{W} \mathbf{A} \mathbf{Q}_k^{-1} \mathbf{A}^\dagger \mathbf{W} + \lambda \tilde{\mathbf{Q}}_h^{-1} \right)^{-1} \mathbf{W} \mathbf{A} \mathbf{Q}_k^{-1} \mathbf{k}$$

λ – Regularization parameter

$$\mathbf{h}_0 = \mathbf{0}$$

Solution

$$\mathcal{E}\{\mathbf{k}\} = \mathbf{A}^\dagger \mathbf{W} \mathbf{h},$$

$$\mathcal{E}\{\mathbf{h}_0\} = \mathbf{h},$$

$$\mathcal{D}\{\mathbf{k}\} = \mathbf{Q}_k \quad \text{Noise covariance}$$

$$\mathcal{D}\{\mathbf{h}_0\} = \tilde{\mathbf{Q}}_h \quad \text{Signal covariance}$$

$$\hat{\mathbf{h}} = \tilde{\mathbf{Q}}_h \mathbf{W} \mathbf{A} \left(\lambda \mathbf{Q}_k + \mathbf{A}^\dagger \mathbf{W} \tilde{\mathbf{Q}}_h \mathbf{W} \mathbf{A} \right)^{-1} \mathbf{k}$$

λ – Regularization parameter

$$\mathbf{h}_0 = \mathbf{0}$$

Croteau et al. (2021)

Solution

$$\mathcal{E}\{\mathbf{k}\} = \mathbf{A}^\dagger \mathbf{W} \mathbf{h},$$

$$\mathcal{E}\{\mathbf{h}_0\} = \mathbf{h},$$

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$$\hat{\mathbf{h}} = \text{Cov}_{sn} (\text{Cov}_{nn} + \text{Cov}_{ss})^{-1} \mathbf{k}$$

$$\mathcal{D}\{\mathbf{k}\} = \mathbf{Q}_k \quad \text{Noise covariance}$$

$$\mathcal{D}\{\mathbf{h}_0\} = \tilde{\mathbf{Q}}_h \quad \text{Signal covariance}$$

λ – Regularization parameter

$$\mathbf{h}_0 = \mathbf{0}$$

Least-squares collocation

e.g. Sneeuw (2000)

Redundancy contribution matrices

$$\hat{h} = \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} k$$

Redundancy contribution matrices

$$\begin{aligned}\hat{h} &= \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} k \\ &\quad + \left[I - \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} A^\dagger W \right] h_0\end{aligned}$$

Complete solution $h_0 = 0$

Redundancy contribution matrices

$$\begin{aligned}\hat{h} &= \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} k \\ &\quad + \left[I - \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} A^\dagger W \right] h_0\end{aligned}$$

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$$\begin{aligned}\mathcal{E}\{\hat{h}\} &= \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} \mathcal{E}\{k\} \\ &\quad + \left[I - \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} A^\dagger W \right] \mathcal{E}\{h_0\}\end{aligned}$$

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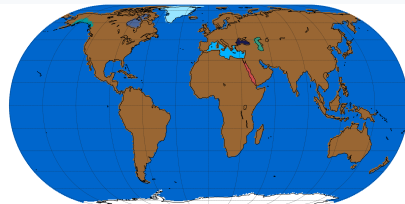
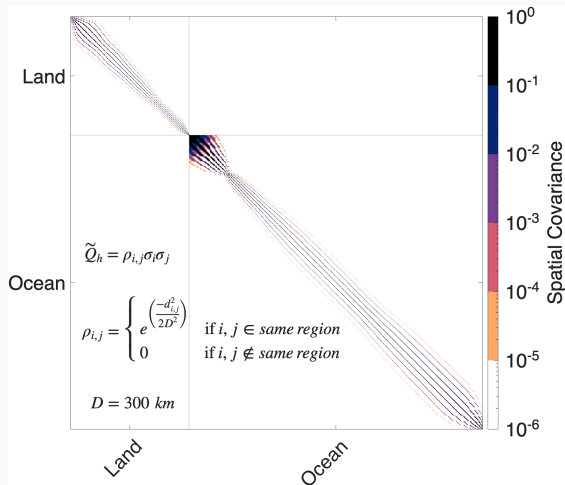
$$\begin{aligned}h &= \underbrace{\tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} A^\dagger W}_{R_k \text{ data contribution}} h \\ &\quad + \underbrace{\left[I - \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} A^\dagger W \right]}_{R_h \text{ contribution of prior}} h\end{aligned}$$

$$R_k + R_h = I \quad \text{All matrices are square}$$

tasting the pudding ...

Following Croteau et al. (2021)

Spatial covariance matrix – $\tilde{Q}_h$



Greenland Antarctic Gulf of Alaska Land Ocean Mediterranean Sea Black Sea Red Sea Caspian Sea Hudson Bay

18721 mascons. Different regions of the world are considered uncorrelated with each other

Spectral information

ITSG SH solutions $L = 96$

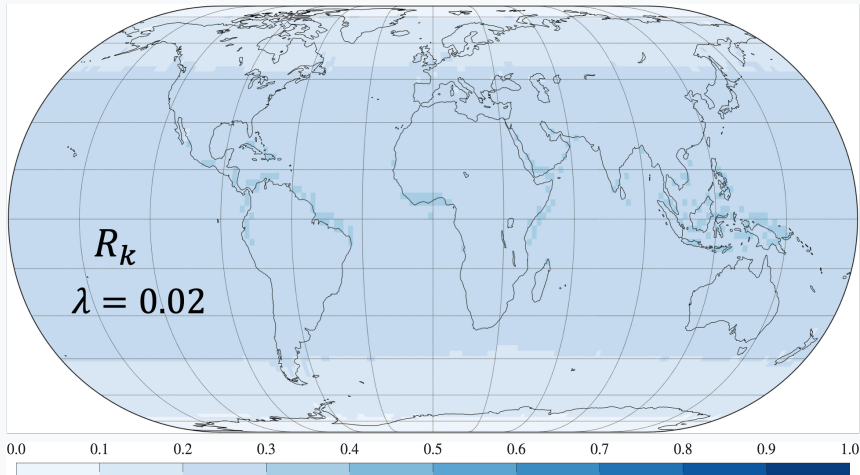
ITSG Normal matrices

k

Q_k^{-1}

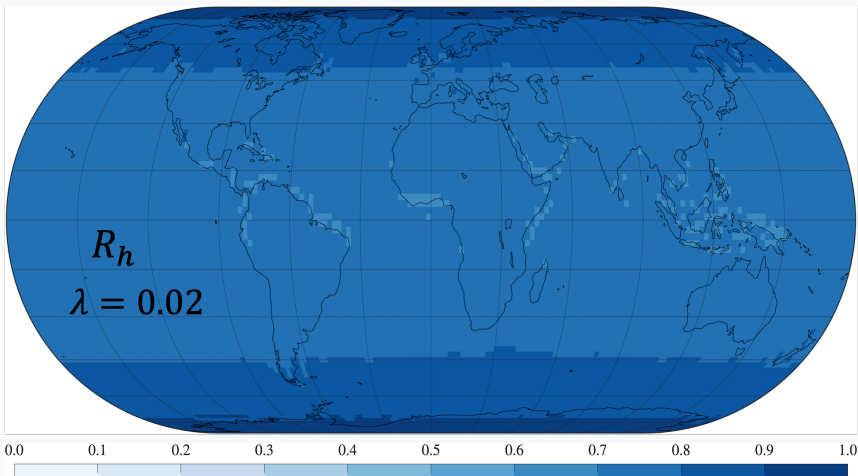
Spatial redundancy contribution of GRACE

$$R_k = \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} A^\dagger W$$



Spatial redundancy contribution of prior $\tilde{Q}_h$

$$R_h = I - \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} A^\dagger W$$



taking fast mascons further ...

A careful look at the fast mascons solution

$$h(\theta, \lambda) = \frac{M}{a_E^2 \rho_{\text{water}}} \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} \frac{2l+1}{1+k'_l} Y_{lm}(\theta, \lambda) \Delta K_{lm} \quad \text{Spherical harmonic synthesis}$$

A careful look at the fast mascons solution

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$$\mathbf{h} = \mathbf{Y} \mathbf{\Lambda} \mathbf{k}$$

Spherical harmonic synthesis

A careful look at the fast mascons solution

$$h(\theta, \lambda) = \frac{M}{a_E^2 \rho_{\text{water}}} \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} \frac{2l+1}{1+k'_l} Y_{lm}(\theta, \lambda) \Delta K_{lm} \quad \text{Spherical harmonic synthesis}$$

$$\mathbf{h} = \mathbf{Y} \mathbf{\Lambda} \mathbf{k}$$

$$= \mathbf{B} \mathbf{k}$$

$$\Rightarrow \mathbf{B} = \mathbf{Y} \mathbf{\Lambda}$$

Spherical harmonic synthesis

A careful look at the fast mascons solution

$$h(\theta, \lambda) = \frac{M}{a_E^2 \rho_{\text{water}}} \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} \frac{2l+1}{1+k'_l} Y_{lm}(\theta, \lambda) \Delta K_{lm}$$

Spherical harmonic synthesis

$$\mathbf{h} = \mathbf{Y} \mathbf{\Lambda} \mathbf{k}$$

$$= \mathbf{B} \mathbf{k}$$

$$\Rightarrow \mathbf{B} = \mathbf{Y} \mathbf{\Lambda}$$

$$\tilde{\mathbf{Q}}_h = \mathbf{B} \tilde{\mathbf{Q}}_k \mathbf{B}^\dagger$$

$$\tilde{\mathbf{Q}}_k \rightarrow \text{Spectral covariance}$$

Rummel & Schwarz, 1977; Devaraju, 2015

Spherical harmonic synthesis of the spectral covariance

A careful look at the fast mascons solution

$$\hat{h} = \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} k$$

A careful look at the fast mascons solution

$$\hat{h} = \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} k$$

$$= B \tilde{Q}_k B^\dagger W A (\lambda Q_k + A^\dagger W B \tilde{Q}_k B^\dagger W A)^{-1} k$$

$$\because \tilde{Q}_h = B \tilde{Q}_k B^\dagger$$

A careful look at the fast mascons solution

$$\begin{aligned}\hat{h} &= \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} k \\ &= B \tilde{Q}_k B^\dagger W A (\lambda Q_k + A^\dagger W B \tilde{Q}_k B^\dagger W A)^{-1} k && \because \tilde{Q}_h = B \tilde{Q}_k B^\dagger \\ &= B \tilde{Q}_k (\lambda Q_k + \tilde{Q}_k)^{-1} k && \iff A^\dagger W B = B^\dagger W A = I\end{aligned}$$

Klees et al. (2008)

$A^\dagger W B = B^\dagger W A = I$ depends on sampling on the sphere and W

Sneeuw (1994)

A careful look at the fast mascons solution

$$\hat{h} = \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} k$$

$$= B \tilde{Q}_k B^\dagger W A (\lambda Q_k + A^\dagger W B \tilde{Q}_k B^\dagger W A)^{-1} k$$

$$\because \tilde{Q}_h = B \tilde{Q}_k B^\dagger$$

$$= B \underbrace{\tilde{Q}_k (\lambda Q_k + \tilde{Q}_k)^{-1}}_{\text{Wiener filter}} k$$

$$\iff A^\dagger W B = B^\dagger W A = I$$

Klees et al. (2008)

A careful look at the fast mascons solution

$$\hat{h} = \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} k$$

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$$\iff A^\dagger W B = B^\dagger W A = I$$

$$= B \underbrace{(Q_k^{-1} + \lambda \tilde{Q}_k^{-1})^{-1} Q_k^{-1}}_{\text{Regularization filter}} k$$

Kusche et al. (2007)

A careful look at the fast mascons solution

$$\hat{h} = \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} k$$

$$= B \tilde{Q}_k B^\dagger W A (\lambda Q_k + A^\dagger W B \tilde{Q}_k B^\dagger W A)^{-1} k$$

$$\because \tilde{Q}_h = B \tilde{Q}_k B^\dagger$$

$$= B \underbrace{\tilde{Q}_k (\lambda Q_k + \tilde{Q}_k)^{-1}}_{\text{Wiener filter}} k$$

$$\iff A^\dagger W B = B^\dagger W A = I$$

$$= B \underbrace{(Q_k^{-1} + \lambda \tilde{Q}_k^{-1})^{-1} Q_k^{-1}}_{\text{Regularization filter}} k = B \bar{k}$$

Spherical harmonic synthesis of filtered $\bar{k}$

A careful look at the fast mascons solution

$$\hat{h} = \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} k$$

$$= B \tilde{Q}_k B^\dagger W A (\lambda Q_k + A^\dagger W B \tilde{Q}_k B^\dagger W A)^{-1} k$$

$$\because \tilde{Q}_h = B \tilde{Q}_k B^\dagger$$

$$= B \underbrace{\tilde{Q}_k (\lambda Q_k + \tilde{Q}_k)^{-1}}_{\text{Spectral redundancy contribution}} k$$

$$\iff A^\dagger W B = B^\dagger W A = I$$

$$= B \underbrace{(Q_k^{-1} + \lambda \tilde{Q}_k^{-1})^{-1} Q_k^{-1}}_{\text{Spectral redundancy contribution}} k = B \bar{k}$$

Spherical harmonic synthesis of filtered k

A careful look at the fast mascons solution

$$\hat{h} = \tilde{Q}_h W A (\lambda Q_k + A^\dagger W \tilde{Q}_h W A)^{-1} k$$

$$= B \tilde{Q}_k B^\dagger W A (\lambda Q_k + A^\dagger W B \tilde{Q}_k B^\dagger W A)^{-1} k$$

$$\because \tilde{Q}_h = B \tilde{Q}_k B^\dagger$$

$$= B \underbrace{\tilde{Q}_k (\lambda Q_k + \tilde{Q}_k)^{-1}}_{\text{Wiener filter}} k$$

$$\iff A^\dagger W B = B^\dagger W A = I$$

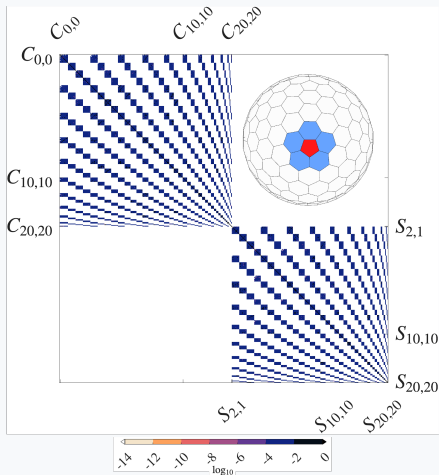
$$= B \underbrace{(Q_k^{-1} + \lambda \tilde{Q}_k^{-1})^{-1} Q_k^{-1}}_{\text{Regularization filter}} k = B \bar{k}$$

Spherical harmonic synthesis of filtered k

$$R_k = B \underbrace{\tilde{Q}_k (\lambda Q_k + \tilde{Q}_k)^{-1}}_{\text{Spectral redundancy contribution}} A^\dagger W$$

Sampling the mascons vs. $(A^\dagger W B) = I$?

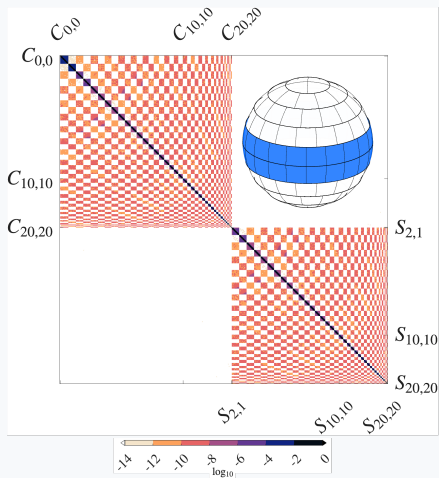
CSR – Icosahedron



Loses the orthogonality of discrete spherical harmonic functions with significant correlation between spherical harmonic functions in a structured pattern

Sampling the mascons vs. $(A^\dagger W B) = I$?

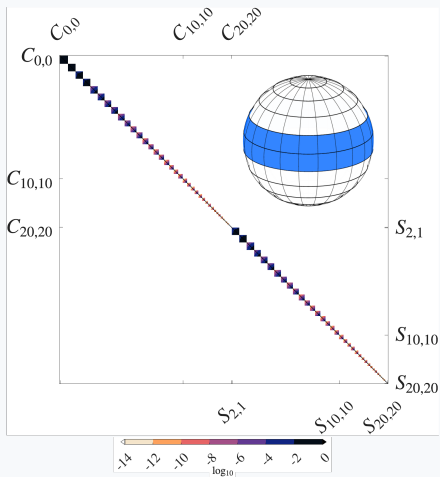
GSFC – Equal-area



Loses the orthogonality of discrete spherical harmonic functions with an order-leading block diagonal dominant structure in single precision

Sampling the mascons vs. $(A^\dagger W B) = I$?

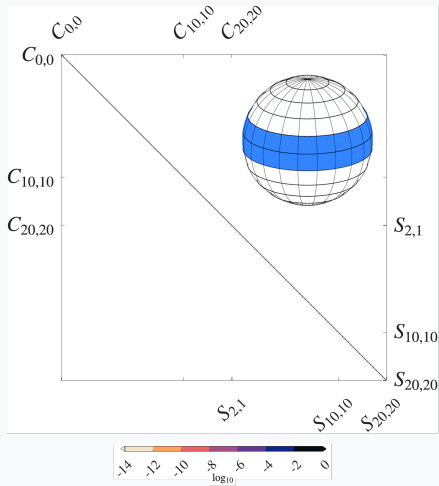
Equiangular



Loses the orthogonality of discrete spherical harmonic functions with an order-leading block diagonal dominant structure

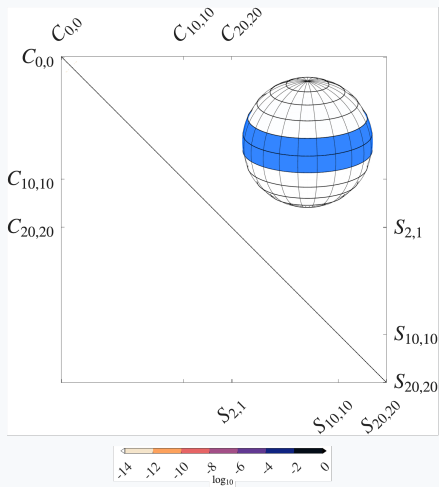
Sampling the mascons vs. $(A^\dagger W B) = I$?

Gauss-Legendre



Retains the orthogonality of discrete spherical harmonic functions

Sampling the mascons vs. $(A^\dagger W B) = I$?



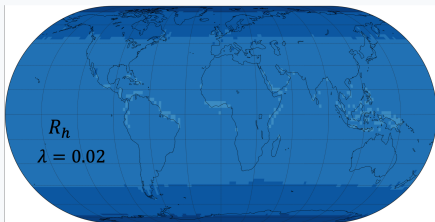
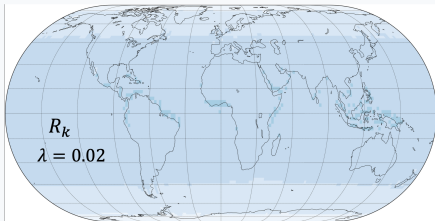
Retains the orthogonality of discrete spherical harmonic functions

Equivalence of fast mascons and stochastic filtering is guaranteed with a Gauss-Legendre sampling

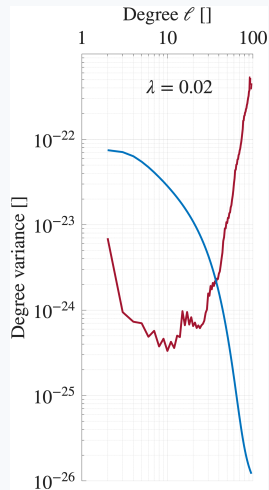
interpreting the spatial redundancy matrices ...

Comparing spatial and spectral redundancy contribution matrices

Spatial redundancy contribution – $\text{Diag}(R_k)$ and $\text{Diag}(R_h)$

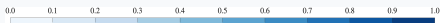
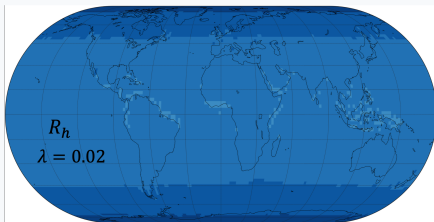
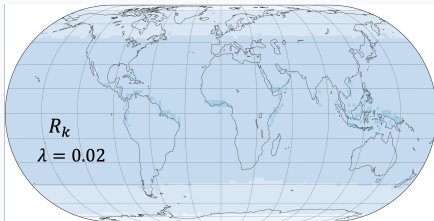


Degree variances of Q_k and $\tilde{Q}_h$

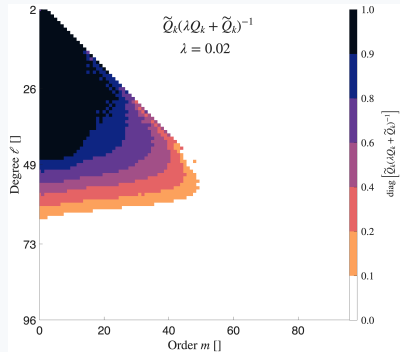


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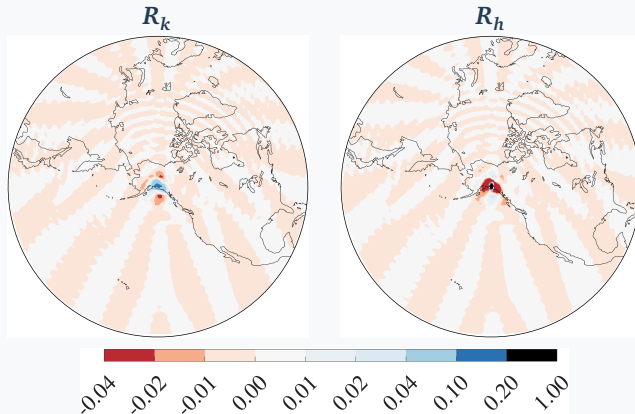


Spectral redundancy contribution

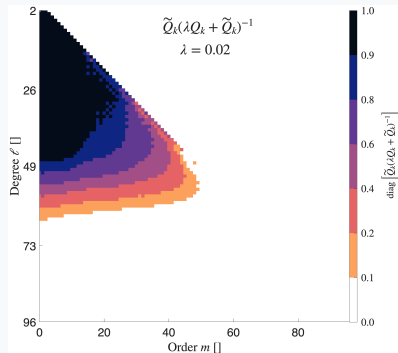


Comparing spatial and spectral redundancy contribution matrices

Spatial redundancy contribution – Rows of R_k and R_h

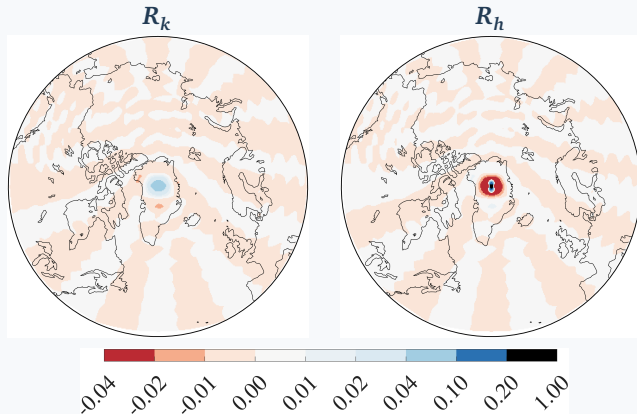


Spectral redundancy contribution

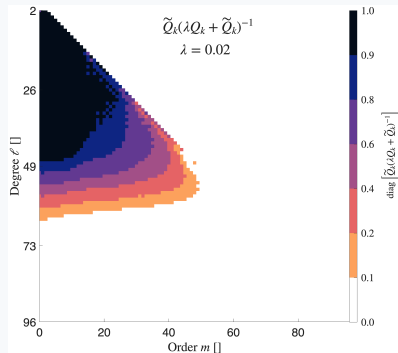


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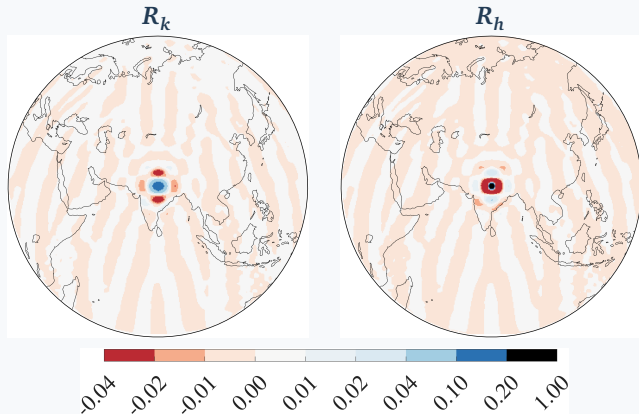


Spectral redundancy contribution

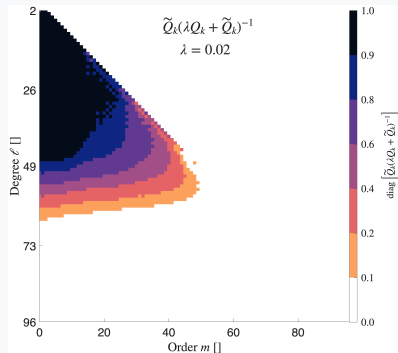


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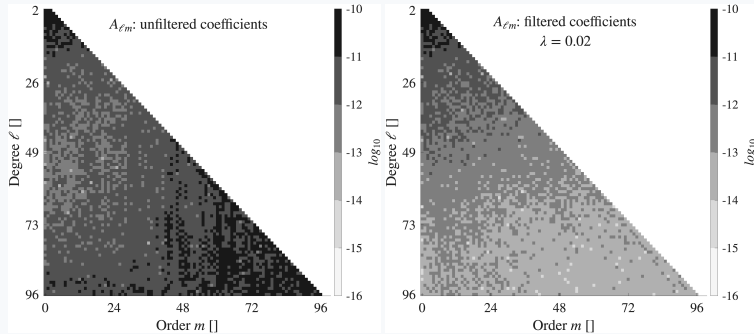


Spectral redundancy contribution

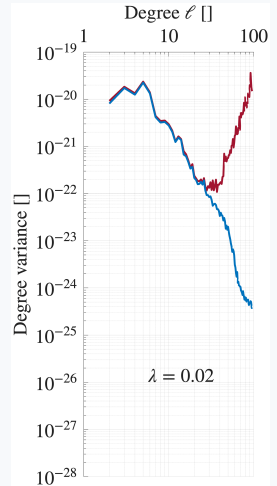


Spectra of mass anomalies – Amplitudes and degree variances

Fast mascons

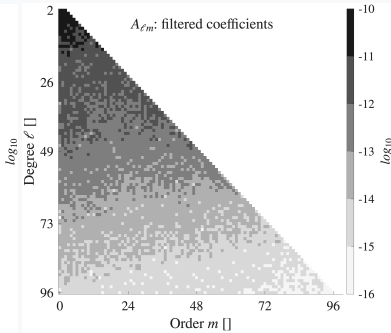
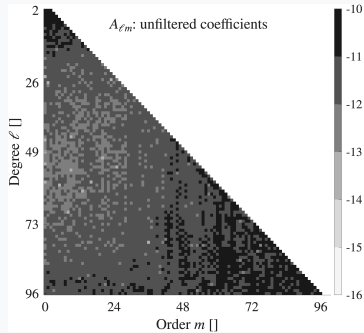


Degree variances of k and $\bar{k}$

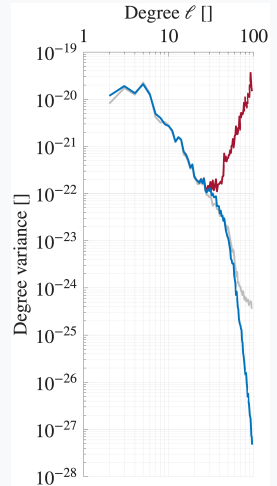


Spectra of mass anomalies – Amplitudes and degree variances

DDK5

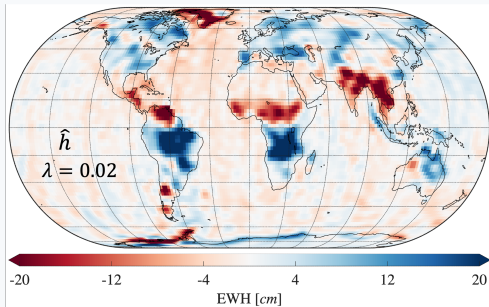


Degree variances of k and $\bar{k}$

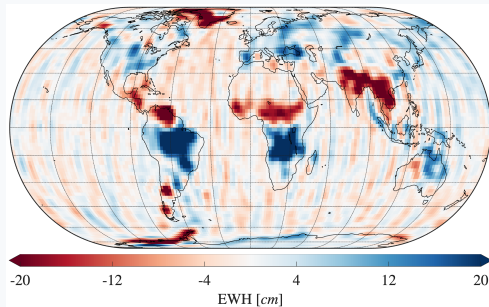


Spatial plots of mass anomalies

Fast mascons



DDK5



Summary

Key takeaways

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To understand the filtering properties and spatial resolution of fast mascons existing framework (Devaraju 2015) can be used

