



POLITECNICO
MILANO 1863

DEPARTMENT OF CIVIL AND
ENVIRONMENTAL ENGINEERING

GNSS-based spatial prediction of zenith and slant tropospheric delays

Xiangyang Song* - Giovanna Venuti* – Eugenio Realini**

* DICA – Politecnico di Milano – ** GReD srl Como

Hotine Marussi Symposium – Genova 7-11 September 2026



Contents

1. Background

GNSS tropospheric delay

The use of GNSS to correct InSAR APS, GACOS service and the underpinning ITD model

2. GNSS ZTDs interpolation – from ITD to AITD model

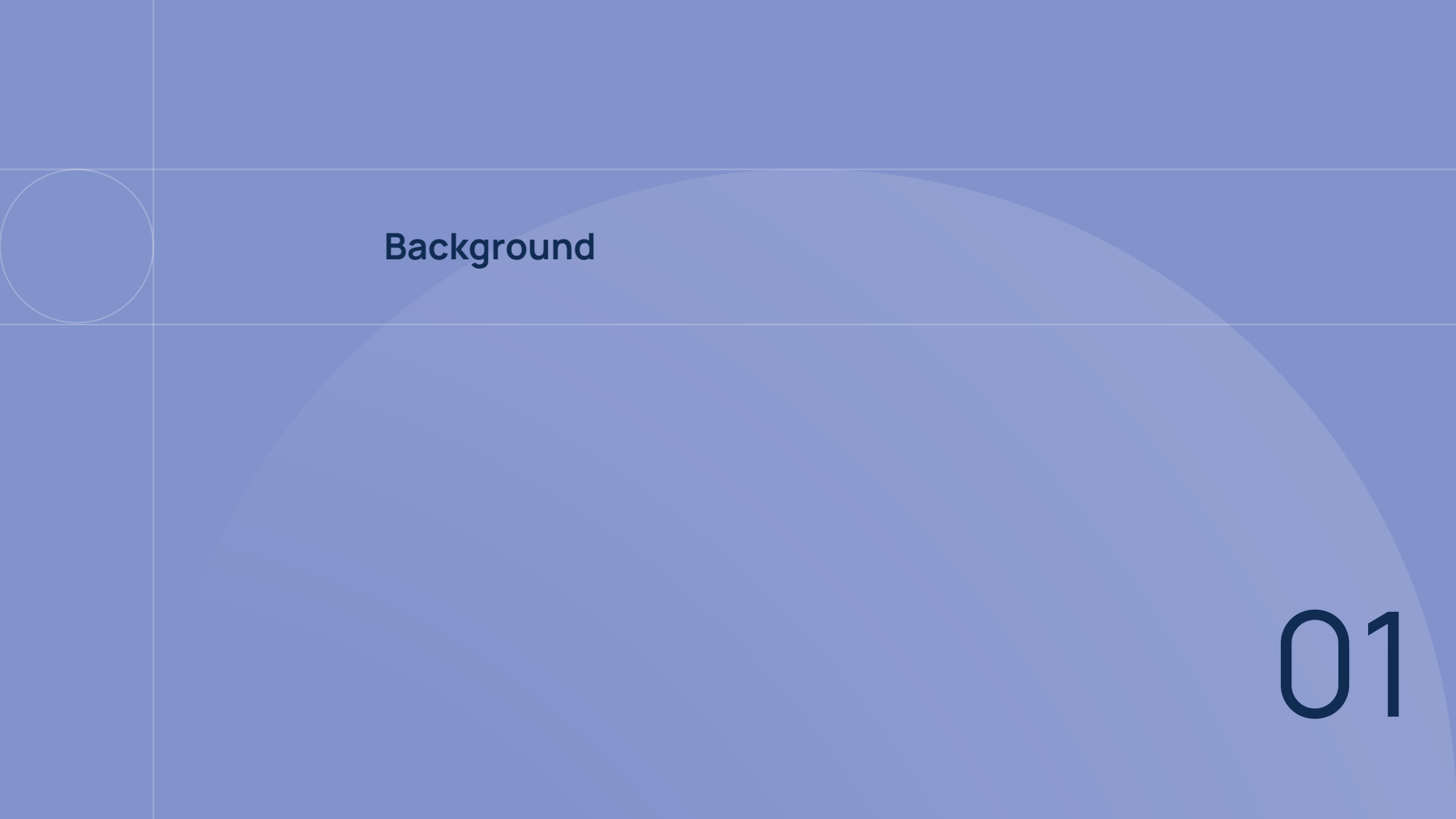
model improvement

3. STD prediction based on GNSS only

the data driven model

test settings and results

4. Future work



Background

01

GNSS Zenith / Slant Tropospheric Delay

GNSS signal propagation in the troposphere occurs at an unknown varying velocity, depending on the atmospheric gas content and distribution in this layer.

The difference between the actual velocity modulus and the known constant speed of the light in vacuum, which is used to evaluate the satellite-receiver distance from the signal travel time, results in an extra-path which is properly accounted for in GNSS positioning.

SLANT Tropospheric Delay - STD: extra-path along the satellite-receiver direction (the bending of the signal propagation can be neglected). It is measured in unit length.

ZENITH Tropospheric Delay - ZTD:

$$STD = ZTD \cdot \text{mapping function} + \text{gradient}$$

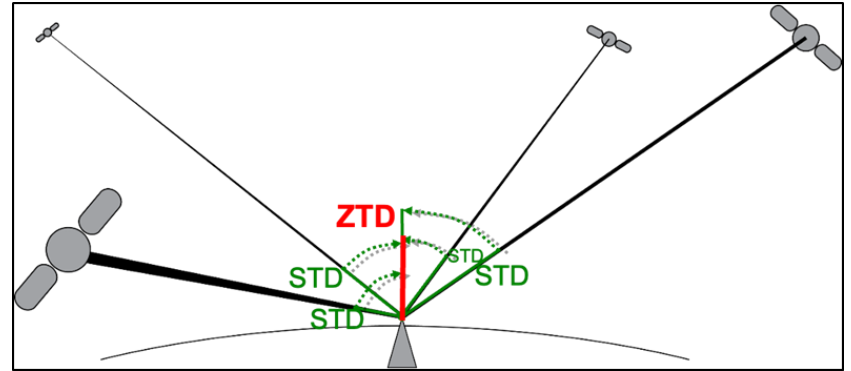
STD is expressed in terms of an unknown parameter, common to all the simultaneous observations to the satellites in view, called **ZENITH Tropospheric Delay - ZTD**.

ZTD represents in fact the component of the SLANT delay in the zenith direction .

mappingfunction: $VMP3(E, a, b, c)$ it depends on satellite elevation (E) and on empirical parameters (a, b, c) computed from NWP model

gradient(E, A) accounts for the anisotropic distribution of the gases in the atmosphere, it depends on satellite elevation E and is azimuthal direction (A) and on two unknown parameters. (Chen and Herring, 1997)

Figure 1: Schematic representation of ZTD as the average projection of the STD along the paths of all the satellites simultaneously in view.



GNSS and SAR water-vapor-induced delays

In this presentation, we focus only on the so-called Tropospheric Wet delay, namely on the delay component due to tropospheric water vapour.

$$STD = SHD + SWD = ZHD \cdot mpf_h + ZWD \cdot mpf_w$$

Synthetic Aperture Radar - SAR phase imagery are affected by the same kind of delay. The contribution of atmospheric delay is called **Atmospheric Phase Screen - APS**.

It is the differential delay with respect to a reference epoch.

Moreover, this differential delay is varying within the interferogram along the SAR signal Line Of Sight – LOS.

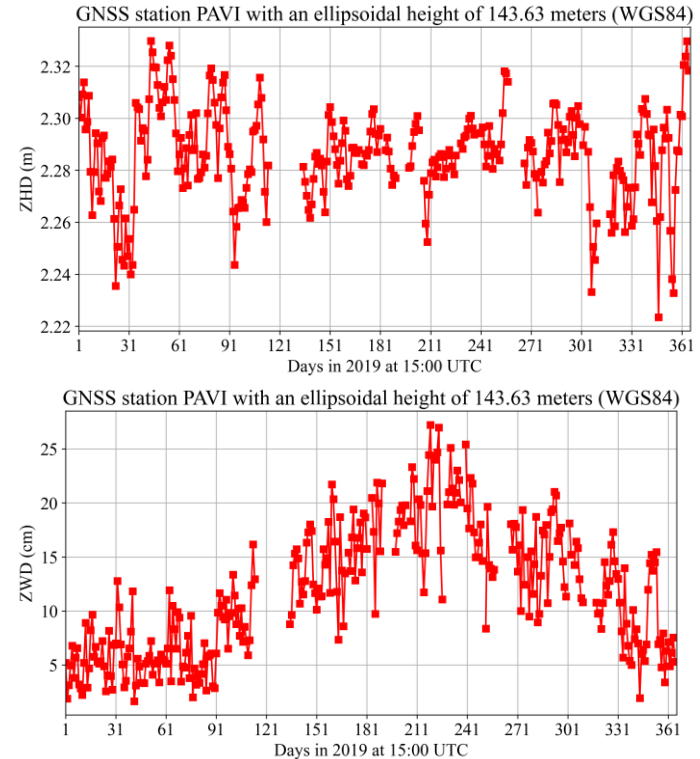


Figure 2: Time series of ZHD and ZWD at GNSS station PAVI.

GNSS ZTD/STD to correct SAR APS

The use of GNSS estimated tropospheric delays to correct SAR APS is our starting point and it is useful to clarify why we proceed in a certain way.

There exists a service, named **GACOS**, which operationally produces APS for correcting SAR interferograms.

The service produces ZTD maps starting from Numerical Weather Prediction model outputs (mainly ERA5 reanalysis data).

The NWP ZTD is computed by a discrete integration of the NWP derived refractivity index along the zenith direction. The refractivity N is in turn derived from 3D predicted grids of temperature, pressure and relative humidity.

$$SWD = 10^{-6} \int_s N_w ds$$

$$N_w = K \times \frac{e}{T} + K' \times \frac{e}{T^2}$$

e is the water vapor pressure in Hectopascal, and T is the temperature in Kelvin. K and K' are given constant parameters.

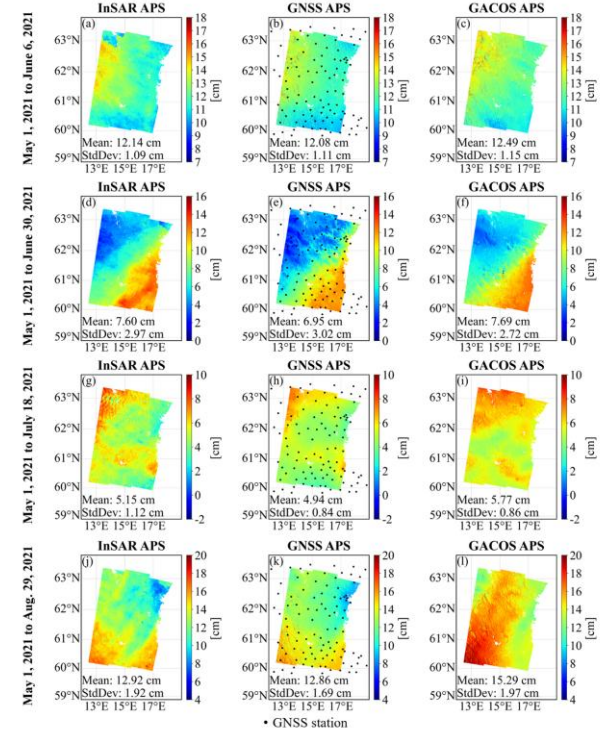


Figure 3: APSs from SAR, GNSS and GACOS data.

GNSS ZTD interpolation

02

GNSS ZTD/STD to correct SAR APS

Questions:

Is it possible to produce, with the same level of accuracy, ZTD delay maps starting from sparse GNSS ZTDs, such as those derived from the PPP processing of geodetic permanent networks data?

Can we apply the same interpolating model (strategy) implemented in GACOS?

Can we go further by exploiting not only the zenith but also the 'noisy' slant delays?

Iterative Tropospheric Decomposition Model (GACOS)

ZTD is modeled as the sum of a *stratified component* (S), depending on height (h), a *turbulent component* (T), depending on the horizontal coordinates ($\mathbf{x}$), and *unmodeled residuals* (ε):

$$ZTD(h, \mathbf{x}) = S(h) + T(\mathbf{x}) + \varepsilon(h, \mathbf{x})$$

The stratified delay (S), is further expressed as an exponential function of the height (h) depending on two unknown parameters:

$$S(h) = Ae^{-Bh}$$

A light dependency of the stratified component from the horizontal position is accounted for by a moving window interpolation strategy.

The interpolation strategy consists in an initial iterative decomposition of the two components performed on all the prediction point whose horizontal and vertical coordinates are known.

Iterative Decomposition step:

The decorrelation window is centered in the prediction point.

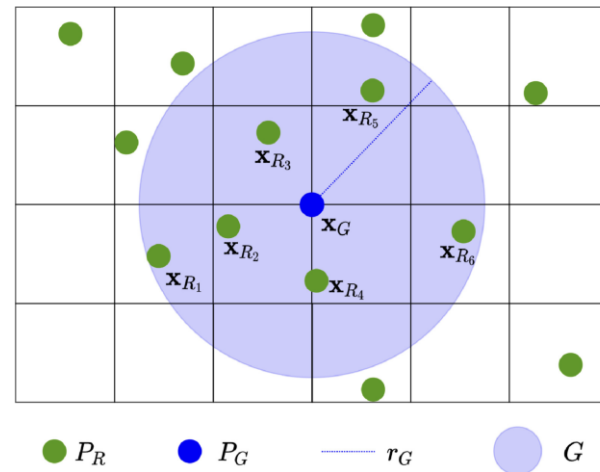
First, the parameters of the stratified model are Least Squares estimated from all the ZTD in the decorrelation window.

Secondly, an Inverse Distance Interpolation is applied to predict the turbulent component of each observation point from the residuals of the remaining points.

The first step is then repeated, till convergence, from the residual with respect to the turbulent component.

Interpolation step:

The decomposed stratified and turbulent components are then predicted in the central point of the moving window.



Limits and the augmented strategy

Decomposition step

The decorrelation window is centered in the observation points. From the decomposition step we save the stratified model parameters and the final residuals

Interpolation step

The decorrelation window is now centered in the prediction point. The decomposition outputs are all IDW interpolated in the center of the decorrelation window.

A modified weighting function is also introduced

$$w_{ik} = \frac{1}{1 + \frac{d_{ik}^2}{\alpha}} \bigg/ \sum_{k \neq i} \frac{1}{1 + \frac{d_{ik}^2}{\alpha}}$$

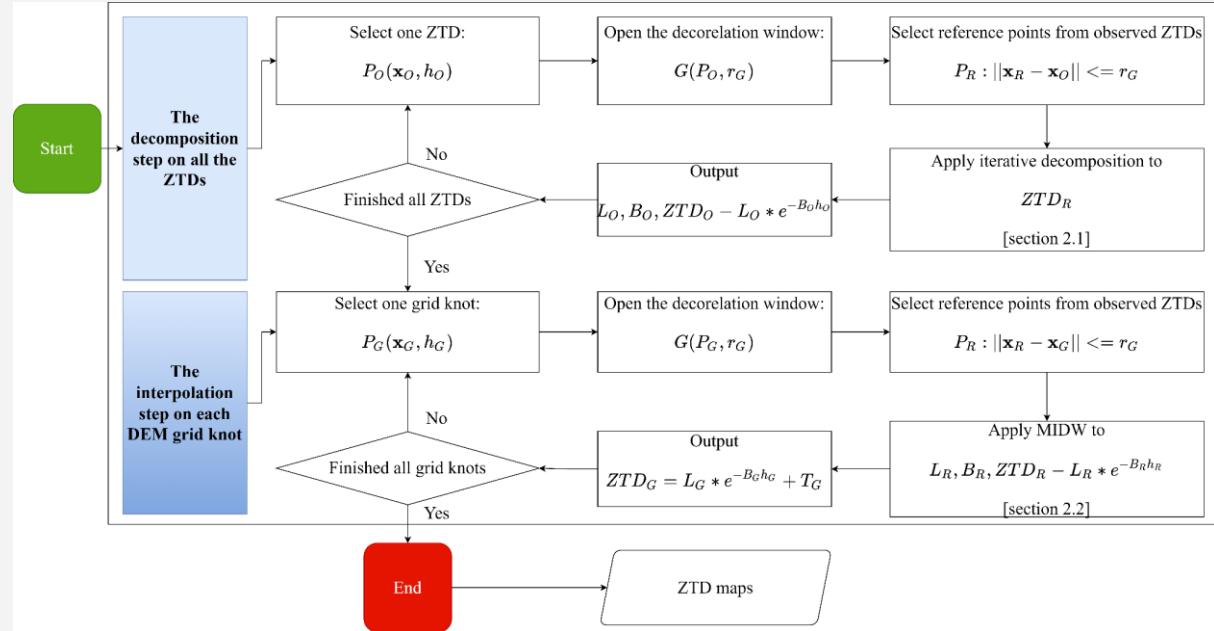


Figure 4: Workflow of the Augmented Iterative decomposition strategy

Advantages of the augmented strategy

- comparable accuracy
- reduced interpolation artifacts: bumps and moving window effects
- reduced computation time WRT ITD (20 time faster if used on ERA5 ZTDs)
- by enabling the use of GNSS only it allows for a better synchronization with SAR

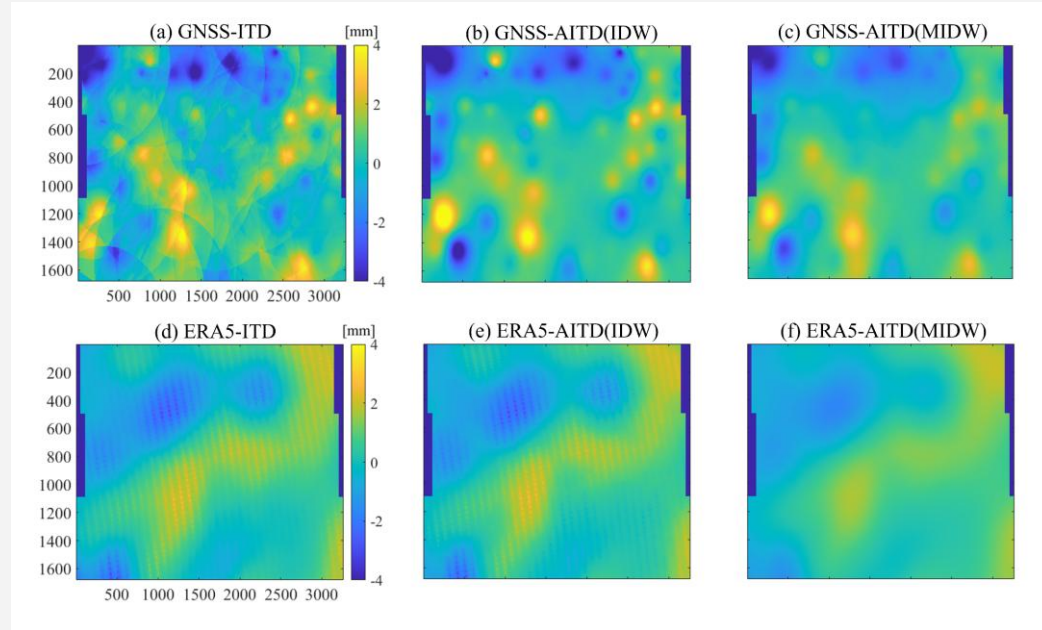


Figure 5: The tropospheric delay turbulent component obtained by using the original ITD and the AITD .

Comparison of the augmented strategy

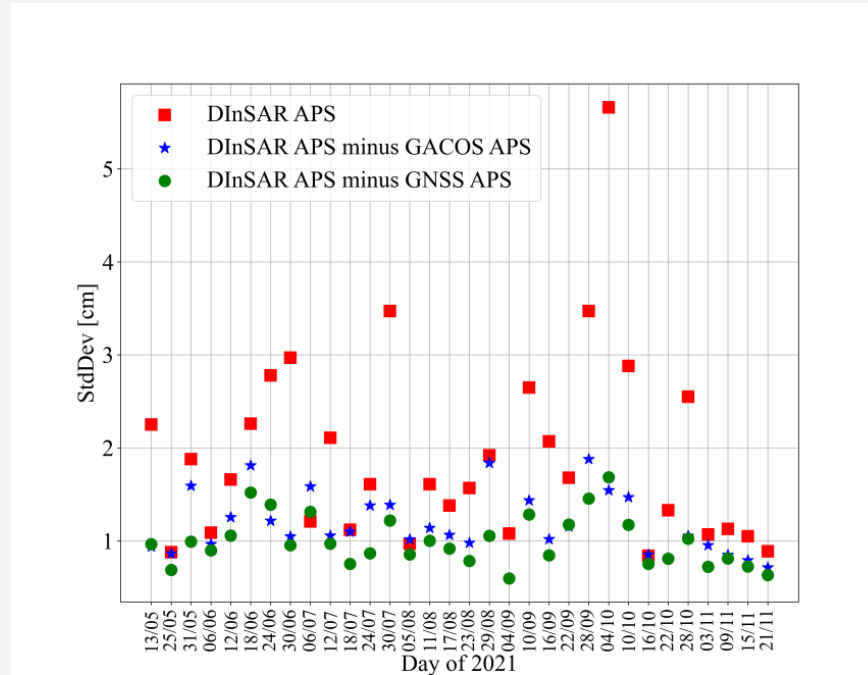


Figure 6: Standard Deviation of the InSAR-APSs, of the difference between InSAR APSs and GNSS APSs and of the difference between InSAR APSs and the GACOS APSs.

Other applications

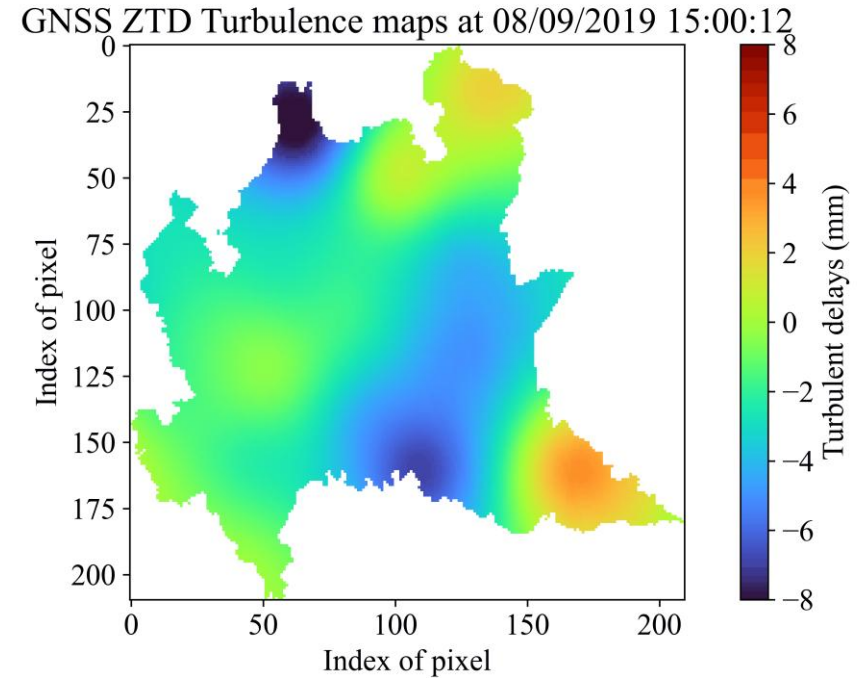


Figure 7: Time series GNSS ZTD Turbulence maps.



STD prediction based on GNSS only

03

STD predictions - DATA DRIVEN refinements

For a given epoch, once the ZTD is predicted and given the position of the satellites in view, the slant delays can be reconstructed by applying a proper mapping function.

The 'best' way to make the projection from the zenith to the slant directions is based on the Vienna Mapping Function.

Again, extra-GNSS data are required (VMF is based on NWM reanalysis data).

Question:

can we obtain STD based on GNSS data only, plus an ad-hoc data driven model?

Data driven STD prediction

Based on a pretrained Transformer-based foundation model **TabPFN** (Tabular Prior-Data Fitted Network), we fine tuned it to predict

slant total delay residual (actual STD – ZTD/sin(E)) **of one station – one satellite**

starting from

1. station position
2. satellite position
3. AITD interpolated ZTD

The prediction is done separately for 3 different clusters (unsupervised clustering- kmean - of all the LOS directions used for training).

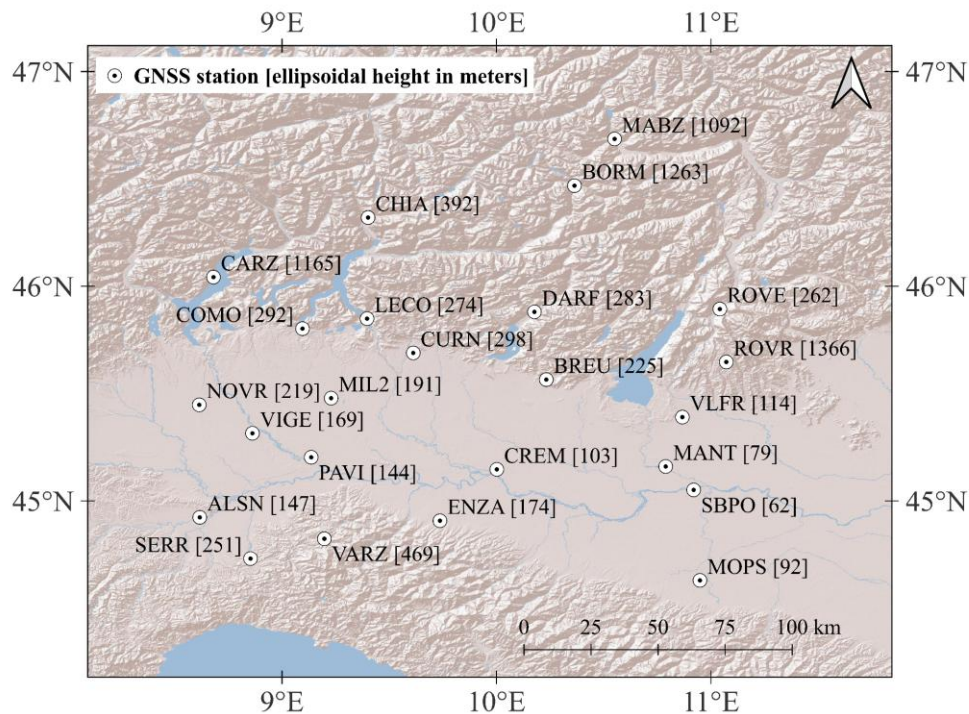


Figure 8: Distribution of 24 GNSS stations in the SPIN3 GNSS network in northern Italy. Stations' Spacing is from 9 km to 60 km. Terrain is from mean sea level to 4000 meters.

Accuracy assessment

A Leave-One-Out procedure was set up to assess the accuracy of the data driven approach.

We compared the predicted STD, based on AITD and the data driven approach, with the actual STD as determined in the PPP processing of the GNSS data (ZTD + projection with the VMF3).

Note: RMSE evaluations of the difference between raw GNSS STD and STD calculated from

GNSS-derived ZTD or AITD-interpolated ZTD considering gradient, VMF3 and NNs.

¹Modeled STD calculated from GNSS-derived ZTD using VMF3+grad and VMF3.

²Modeled STD calculated from AITD-interpolated ZTD using VMF3 and NNs residuals prediction.

³All date times refer to epoch 15:59:42.

⁴Satellite elevation angle in degrees.

⁵The total number of Line-Of-Sight (LOS) observations for all receivers at the current epoch.

Table 1: A part of comparison results.

Datetime ³	E ⁴	#LOS ⁵	GNSS-derived ZTD ¹		AITD-interpolated ZTD ²	
			mpf+grad	mpf	mpf	NNs
21 May	15-90	259	1.2	4.3	18.2	21.1
	30-90	233	1.1	2.2	13.3	15.5
	15-30	60	1.3	8.5	30.6	35.3
22 May	15-90	280	1.4	5.5	19	20.8
	30-90	223	1.1	2.2	13.4	14.3
	15-30	57	2.3	11.3	32.7	36.5
23 May	15-90	281	1.9	8.9	35.3	31.5
	30-90	220	1.6	4.5	27.1	25.5
	15-30	61	2.7	17.2	55.5	47.1
24 May	15-90	249	2.3	7.3	27.3	27.4
	30-90	174	1.8	3.8	21.9	23.6
	15-30	75	3.3	12	36.9	35.1
25 May	15-90	244	1.7	3.6	18	16.1
	30-90	187	1.5	2	14.7	13
	15-30	57	2.4	6.6	26.1	23.6
26 May	15-90	278	1.3	6.1	28	15.1
	30-90	174	1.2	3.5	21.5	11.8
	15-30	104	1.6	8.9	36.4	19.4
27 May	15-90	284	1.5	6.3	22.9	17.2
	30-90	176	1.4	2.1	15.1	12
	15-30	108	1.7	9.9	31.8	23.2
28 May	15-90	277	1.7	9.6	21.4	19.7
	30-90	176	1.5	1.8	11.7	10.5
	15-30	101	2.4	15.7	31.8	29.5
29 May	15-90	311	1.6	7.4	21.6	19.4
	30-90	208	1.4	2.5	13.9	13.9
	15-30	103	1.9	12.3	31.9	27.4
30 May	15-90	317	1.8	9.1	27.4	22.7
	30-90	219	1.1	2.2	15.2	13.9
	15-30	98	2.8	16.1	43.8	35.2



Future work

04

Future work

Refine the deep learning model to better account for the water vapor anisotropy.

Comparison against InSAR APS or ballon or drone based radio soundings.

Further studies are ongoing on stochastic tomography base on an ad hoc developed covariance function model for slant delay residuals.

References

- ❑ X. Song, G. Venuti, A. V. Monti-Guarnieri, M. Manzoni. Augmented iterative tropospheric decomposition strategy for GNSS-based zenith tropospheric delay map generation. Environmental Modelling & Software, 194, 106669 (2025). DOI: <https://doi.org/10.1016/j.envsoft.2025.106669>
- ❑ Hollmann, N., Müller, S., Purucker, L. et al. Accurate predictions on small data with a tabular foundation model. Nature 637, 319–326 (2025). DOI: <https://doi.org/10.1038/s41586-024-08328-6>
- ❑ C. Yu, Z. Li, N. T. Penna. Interferometric synthetic aperture radar atmospheric correction using a GPS-based iterative tropospheric decomposition model. Remote Sensing of Environment, 204, 109–121 (2018). DOI: <https://doi.org/10.1016/j.rse.2017.10.038>
- ❑ Landskron, D., Bohm, J.: VMF3/GPT3: refined discrete and empirical troposphere mapping functions. Journal of Geodesy 92(4), 349–360 (2018). DOI: <https://doi.org/10.1007/s00190-017-1066-2>

Websites:

- BREVA GNSS Software (GReD srl): <https://www.g-red.eu/>
- SPIN3 GNSS Network: <https://www.spingnss.it>
- TabPFN Model (Prior Labs): <https://github.com/PriorLabs/tabpfn>
- VMF Data Server: <https://vmf.geo.tuwien.ac.at/index.html>
- GACOS: <http://www.gacos.net>